Chapter 13 Oscillators

- 13.1 General Considerations
- 13.2 Ring Oscillators
- 13.3 LC Oscillators
- 13.4 Phase Shift Oscillator
- 13.5 Wien-Bridge Oscillator
- 13.6 Crystal Oscillators
- 13.7 Chapter Summary

Chapter Outline

General Considerations

- Barkhausen's Criteria
- Oscillatory Feedback System
- Startup Condition
- Oscillator Topologies

Ring Oscillators

- Feedback around One or Two Stages
- Three-Stage Ring Oscillator
- Internal Waveforms

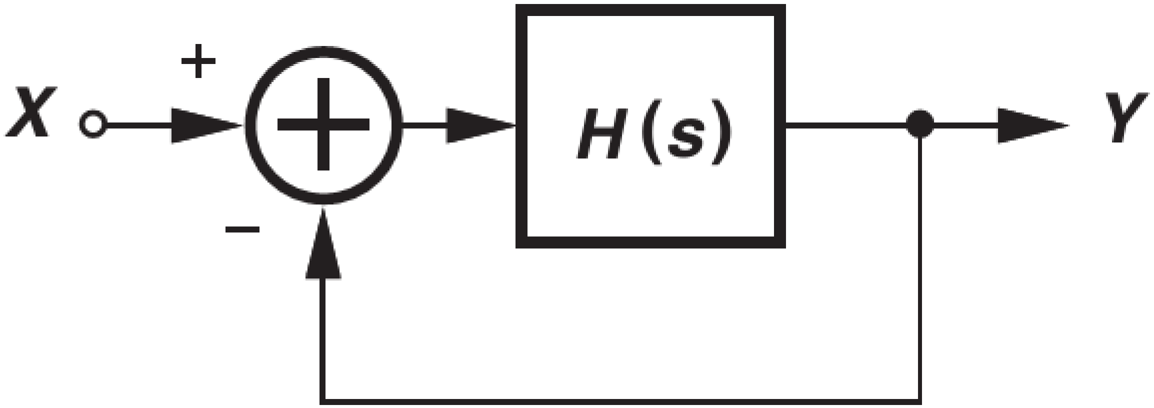
LC Oscillators

- Parallel LC Tank
- Cross-Coupled Oscillator
- Colpitts Oscillator

Other Oscillators

- Phase Shift Oscillator
- Wien-Bridge Oscillator
- Crystal Oscillator

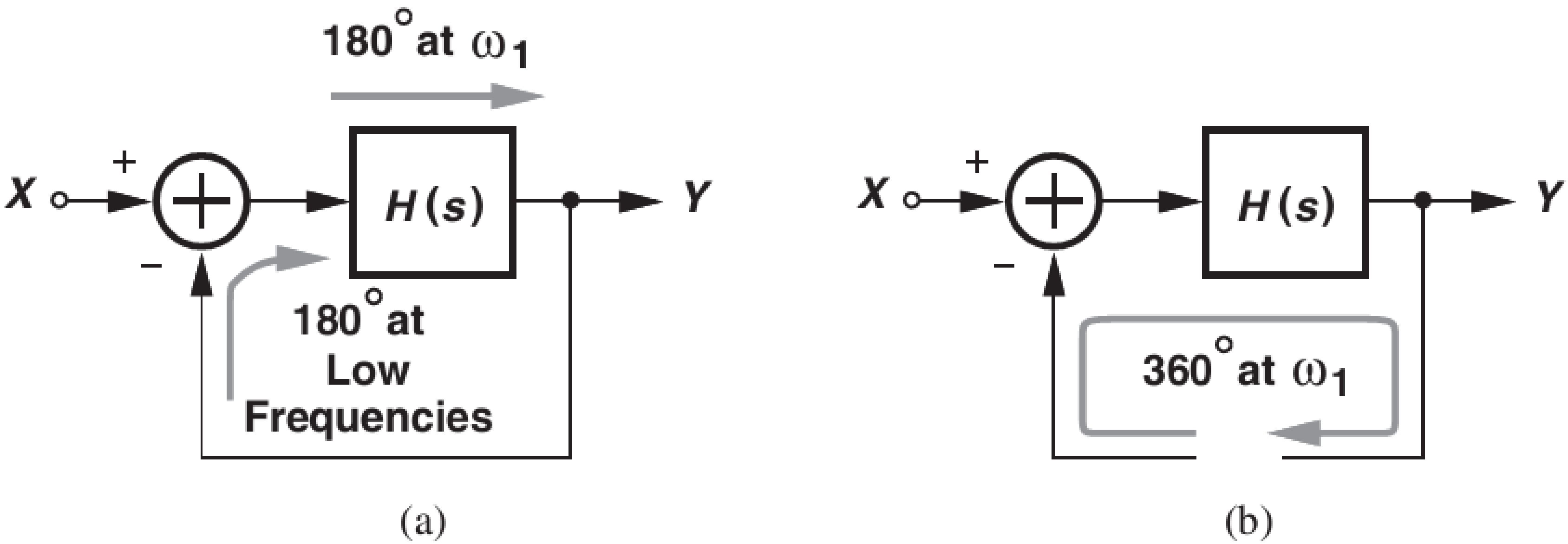
Negative-Feedback Circuit



$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$$

- **Barkhausen's criteria: Closed-loop transfer function goes to infinity at frequency ω_1 if $H(s = j\omega_1) = -1$, or, equivalently, $|H(j\omega_1)| = 1$ and $\angle H(j\omega_1) = 180^\circ$.**

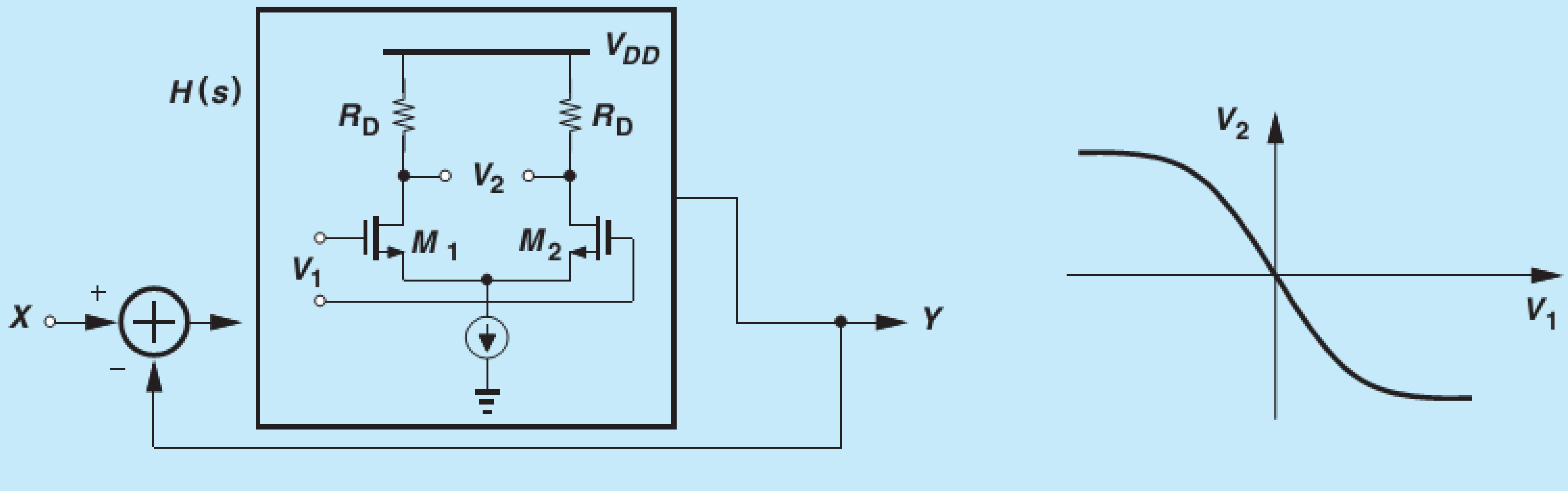
Phase Shift around an Oscillator



- Do NOT be confused with the frequency-dependent 180° phase shift stipulated by Barkhausen with the 180° phase shift necessary for negative feedback.
- The total phase shift around the loop reaches 360° at ω_1.

Example 13.1

An oscillator employs a differential pair. Explain what limits the output amplitude.



(a)

(b)

- **The gain of the differential pair drops and so does the loop gain as the input swing grows.**
- **The oscillation amplitude reaches its maximum when the tail current is steered completely to either side, i.e. swing from $-I_{SS}R_D$ to $I_{SS}R_D$.**

Summary of Oscillator topologies and applications

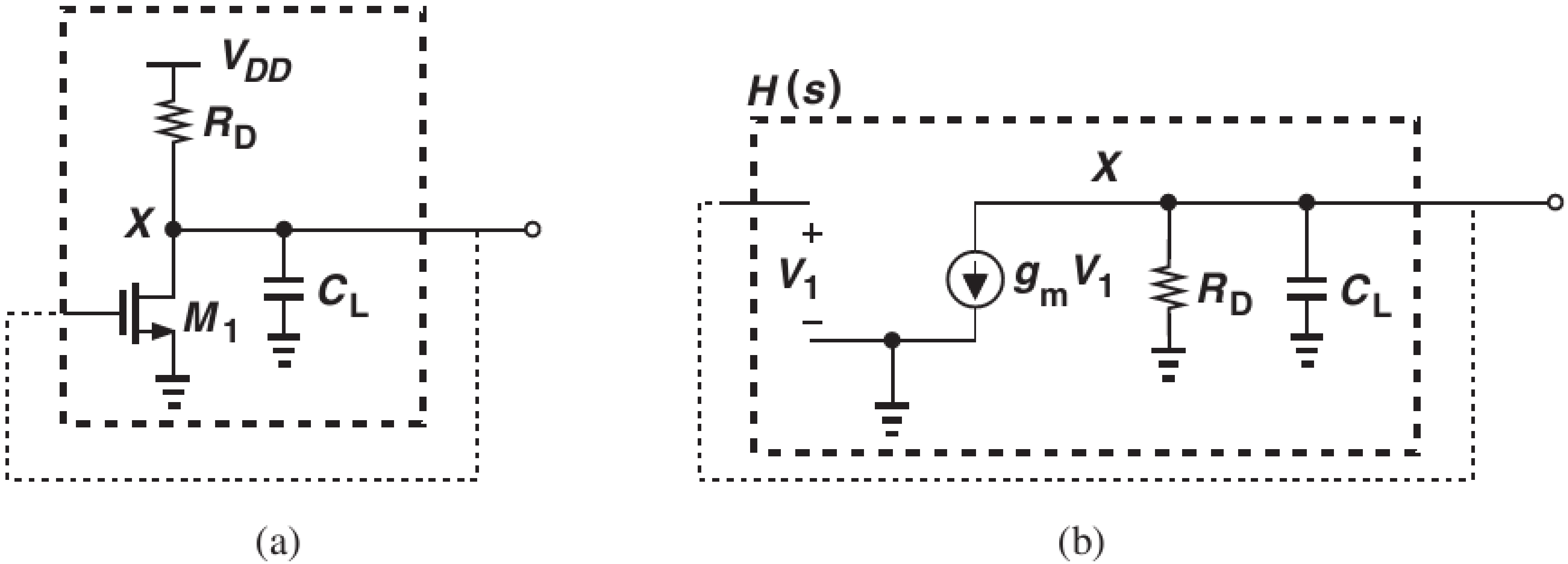
Oscillator Topology	Ring Oscillator	LC Oscillators: Cross-Coupled Oscillator	LC Oscillators: Colpitts Oscillator	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

- **Oscillators can be realized as either integrated or discrete circuits. The topologies are quite different in the two cases but still rely on Barkhausen's criteria.**

Startup Condition

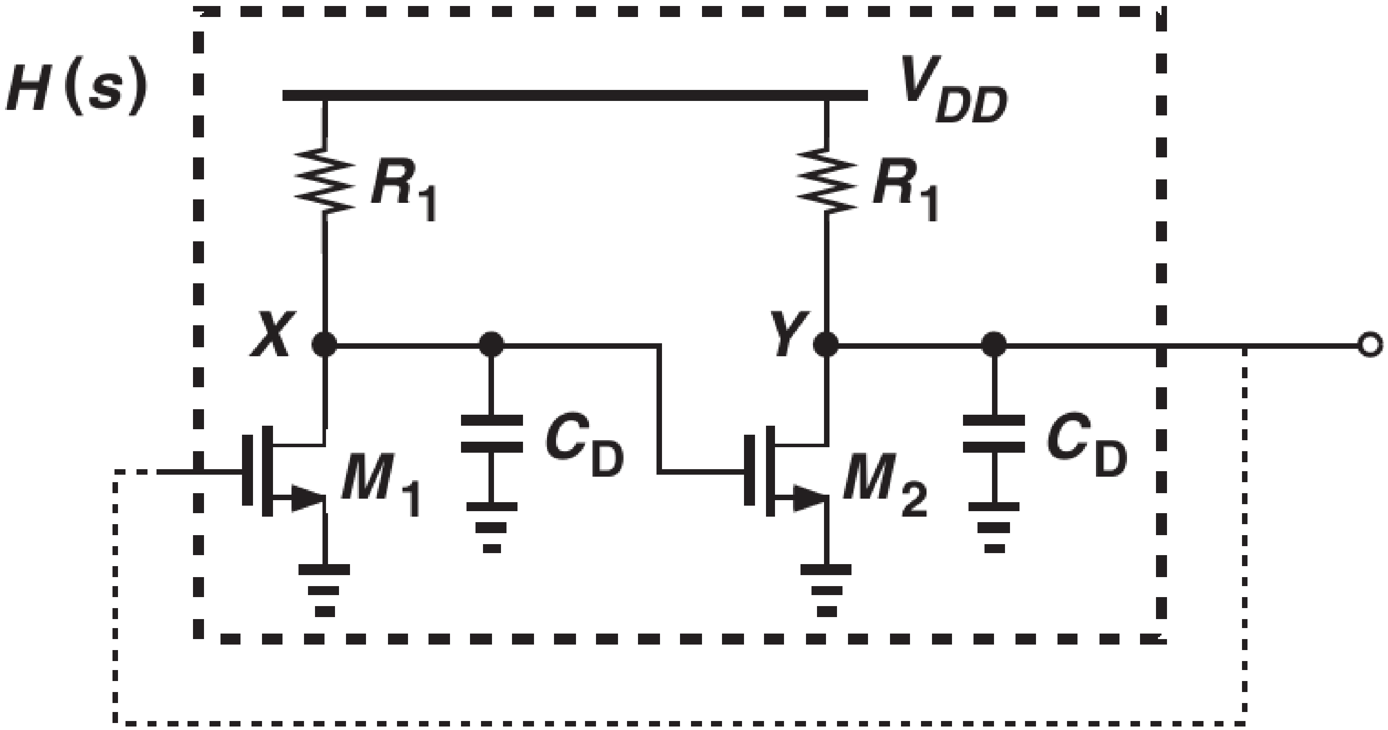
- **Startup condition: a unity loop gain at the desired oscillation frequency, ω_1.**
- **The loop gain is usually quite larger than unity to leave margin for process, temperature or supply voltage variation.**
- **Design specifications: oscillation frequency, output amplitude, power consumption, complexity and noise.**

Feedback Loop Using a Single CS Stage



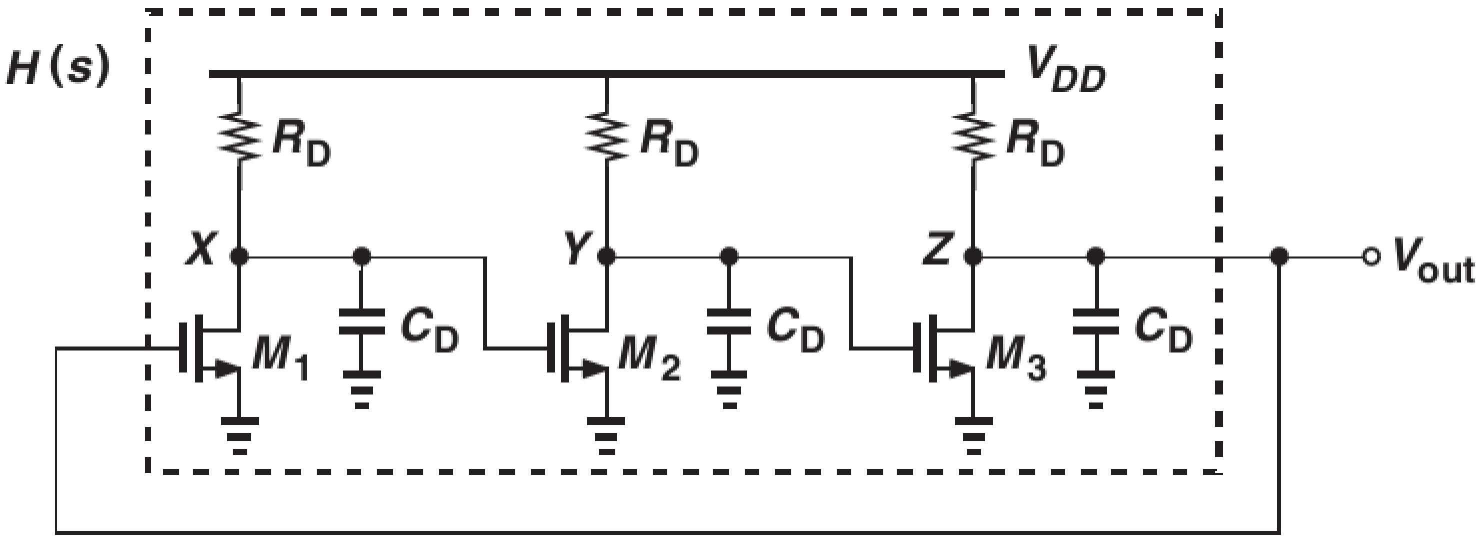
- **Will NOT oscillate.**
- **A single pole at node X ($\omega_{p,X} = -(R_D C_L)^{-1}$) can provide a maximum phase shift of -90° ($\omega = \infty$).**
- **The total phase shift around the loop cannot reach -90°.**

Feedback Loop Using Two CS Stages



- **Will NOT oscillate.**
- **Two poles exhibiting a maximum phase shift of 180° at $\omega = \infty$, but no gain at this frequency.**
- **We still cannot meet both of Barkhausen's criteria.**

Simple Three-Stage Ring Oscillator



$$\omega_1 = \frac{\sqrt{3}}{R_D C_D}$$

$$\left(\frac{g_m R_D}{\sqrt{1 + R_D^2 C_D^2 \omega_1^2}}\right)^3 = 1$$

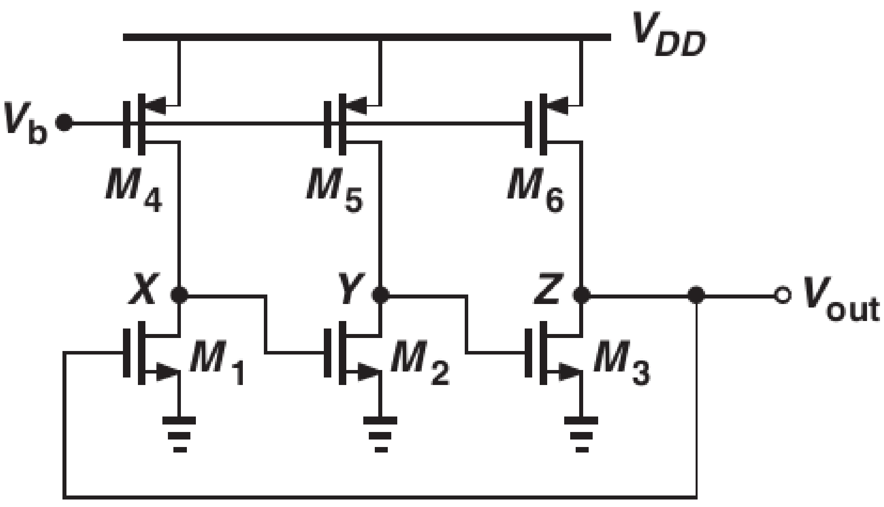
- Each pole provides a phase shift of 60°.
- The magnitude of the transfer function is equal to unity.

Example 13.2

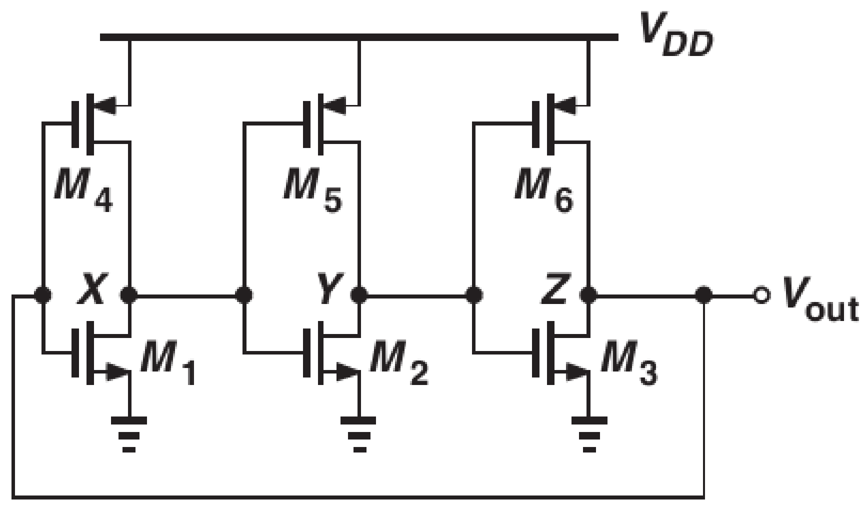
A student runs a transient SPICE simulation on the previous ring oscillator but observes that all three drain voltages are equal and the circuit does not oscillate. Explain why. Assume that the stages are identical.

- **With identical stages, SPICE finds equal drain voltages as the network solution and retains it.**
- **Compared to the simulated circuit, the device noise of the actual circuit will initiate oscillation.**
- **Therefore, we need to apply an initial condition to avoid the equilibrium point.**

Other Types of Ring Oscillator



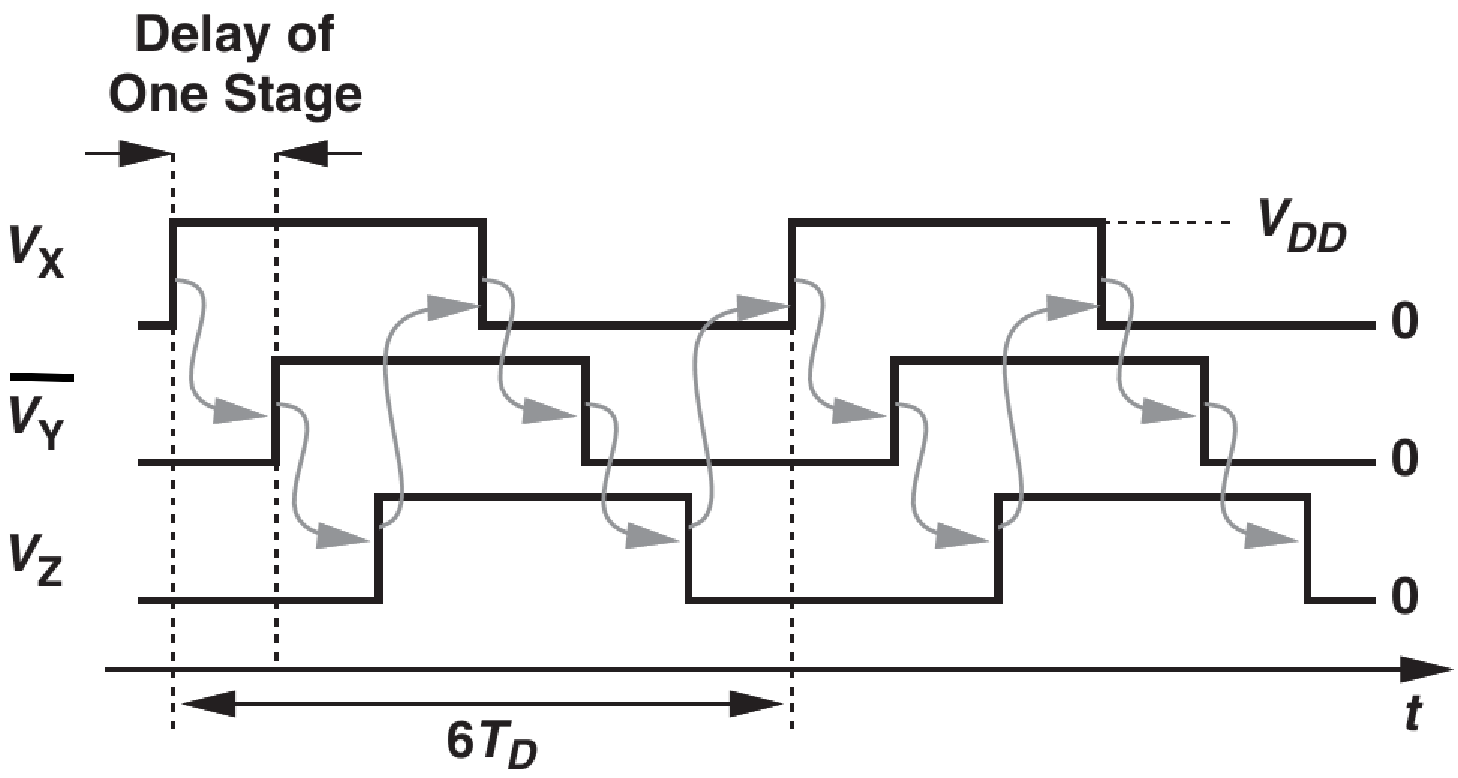
(a)



(b)

- **(a) Replace the load resistors with PMOS current sources.**
- **(b) Each stage is a CMOS inverter.**
- **The transistors themselves contribute capacitance to each node, limiting the speed.**

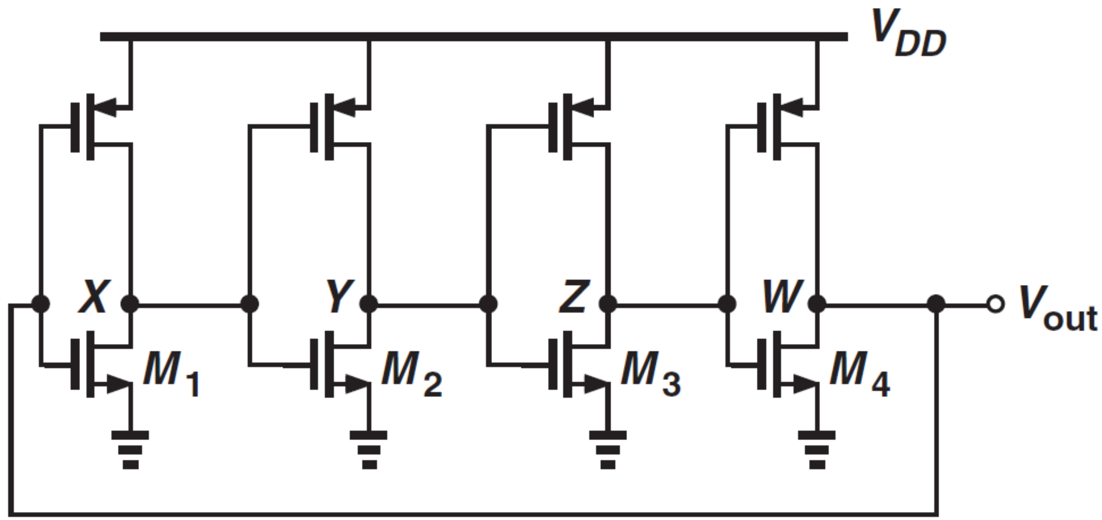
The Operation of the Inverter-Based Ring Oscillator



- If each inverter has a delay of T_D seconds, the oscillation frequency is $1/(6T_D)$.

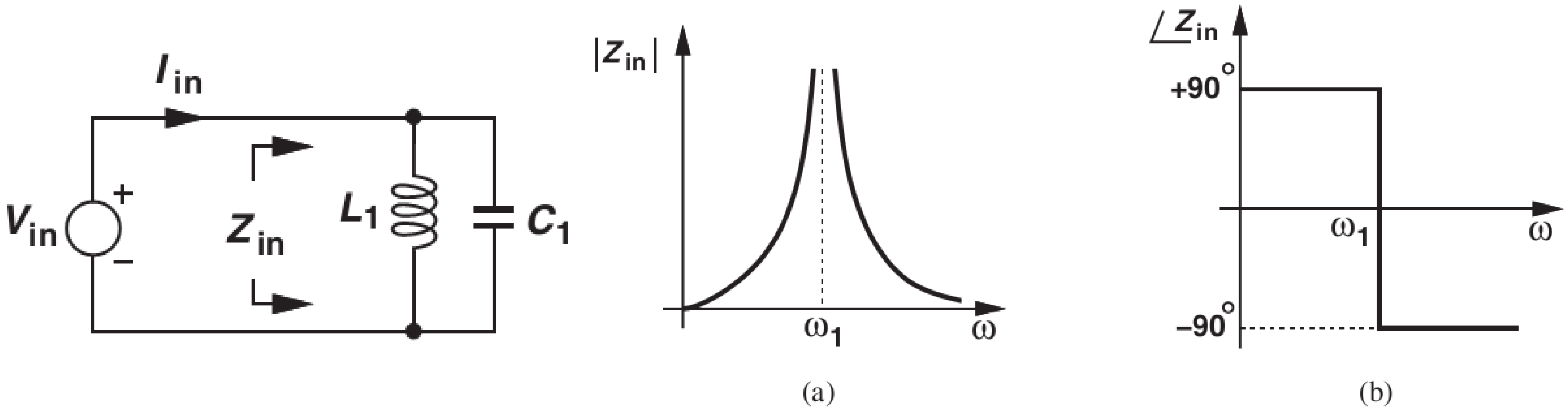
Example 13.3

Can we cascade four inverters to implement a four-stage ring oscillator?



- **Will NOT oscillate.**
- **The circuit will retain its initial value indefinitely.**
- **All of the transistors are either off or in deep triode region, yielding a zero loop gain and violate Barkhausen's criteria.**
- **A single-ended ring with an even number of inverters experiences latch-up.**

Ideal Parallel LC Tanks

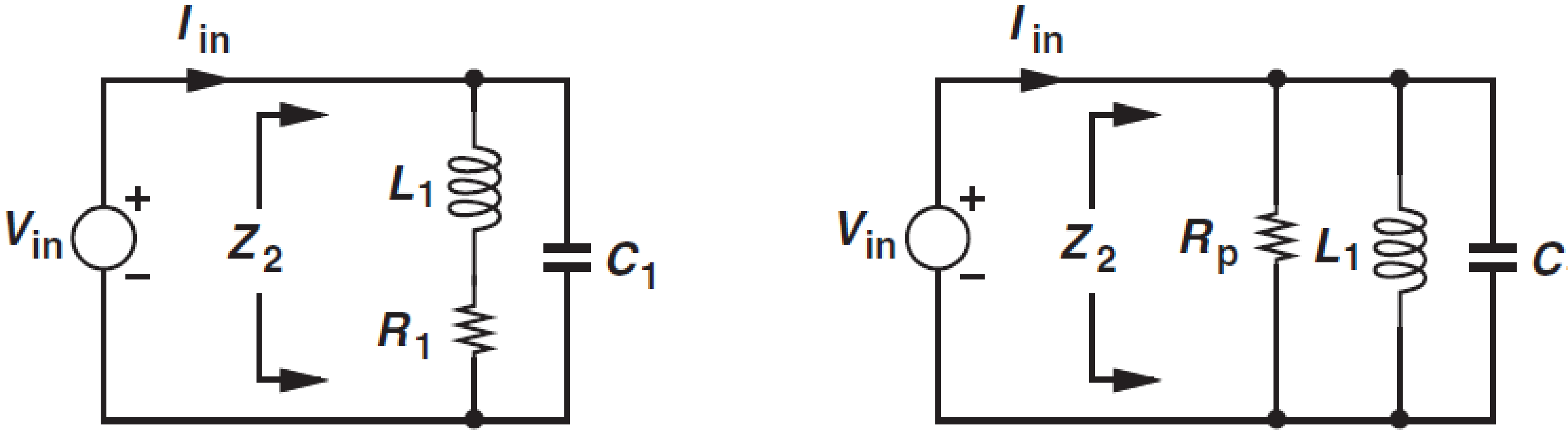


$$Z_{in}(j\omega) = \frac{jL_1\omega}{1 - L_1C_1\omega^2}$$

$$\omega_1 = 1/\sqrt{L_1C_1}$$

- **The impedance goes to infinity at ω_1, i.e. LC tank resonates.**
- **The tank has an inductive behavior for $\omega < \omega_1$ and a capacitive behavior for $\omega > \omega_1$.**

Lossy LC Tank (1)

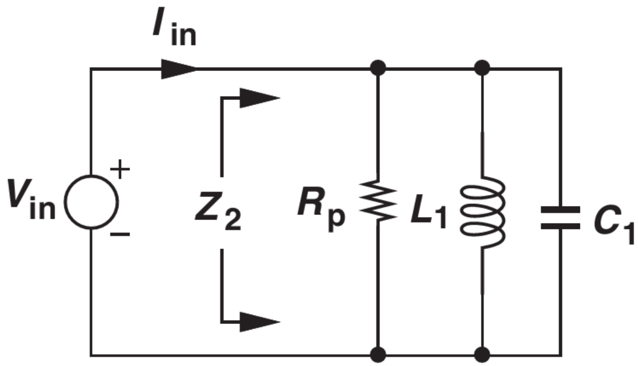


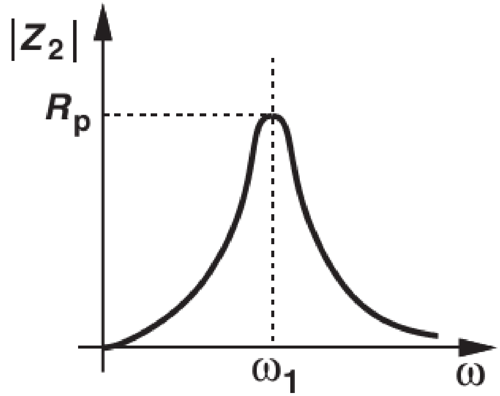
(a) (b)

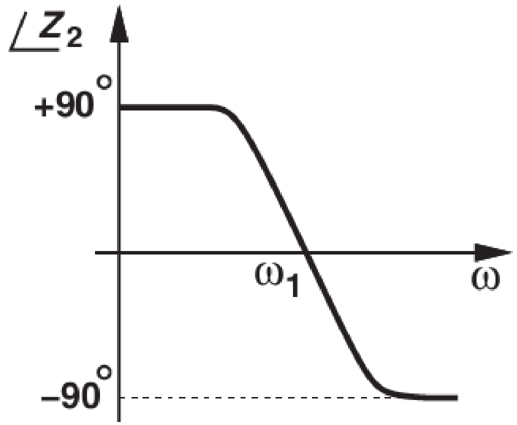
$$R_p = \frac{L_1^2 \omega^2}{R_1}$$

- In practice, the impedance of LC tank does not goes to infinity at the resonance frequency due to finite resistance of the inductor.
- Circuit (a) and (b) are only equivalent for a narrow range around the resonance frequency.

Lossy LC Tank (2)





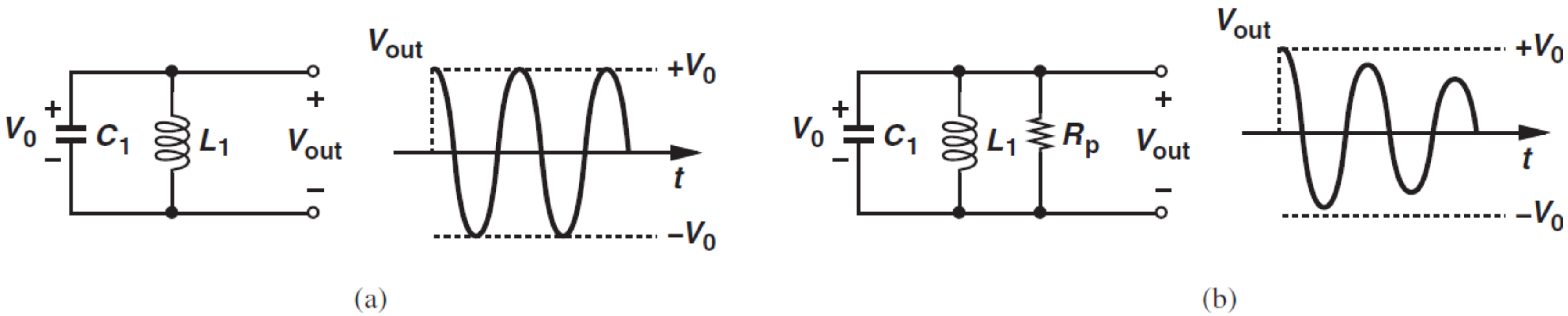


$$Z_2(j\omega) = \frac{jR_p L_1 \omega}{R_p(1 - L_1 C_1 \omega^2) + jL_1\omega}$$

- **In the analysis of LC oscillators, we prefer to model the loss of the tank by a parallel resistance, R_p.**
- **Z_2 reduces to a single resistance, R_p, at ω_1.**
- **At very low frequency, $Z_2 \approx jL_1\omega$; at very high frequency, $Z_2 \approx 1/(jC_1\omega)$.**

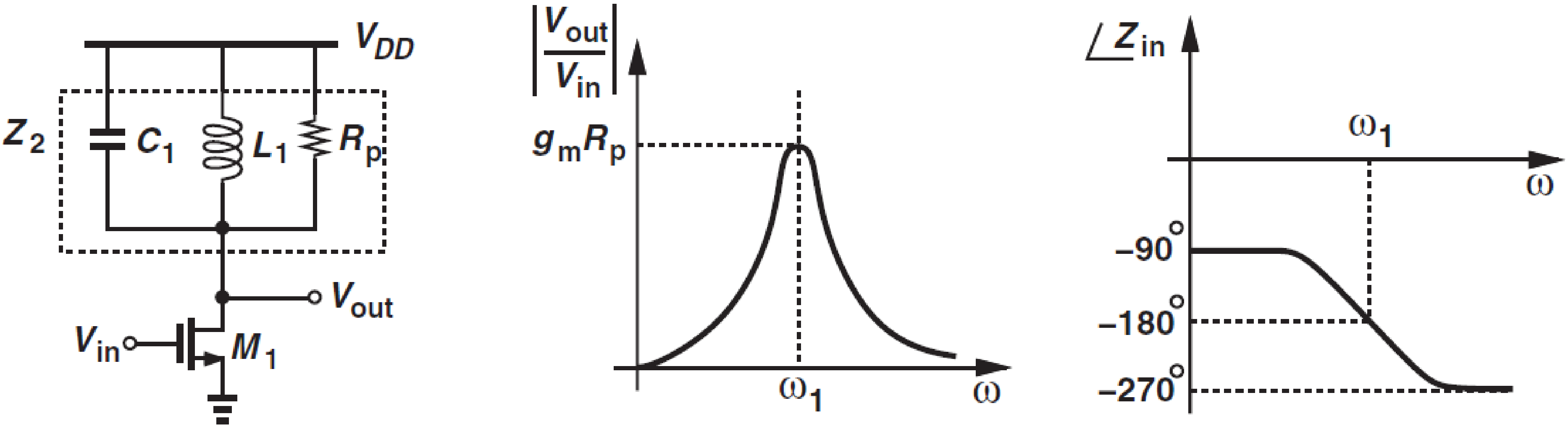
Example 13.6

Suppose we apply an initial voltage of V_0 across the capacitor in an isolated parallel tank. Study the behavior of the circuit in the time domain if the tank is ideal or lossy.



- **For ideal tank, the transfer of energy between C_1 and L_1 repeats and the tank oscillates indefinitely.**
- **For lossy tank, the current flowing through R_p dissipates energy and thus the tank loses some energy each cycle, producing a decaying oscillatory output.**

Single CS Stage with a Tank Load

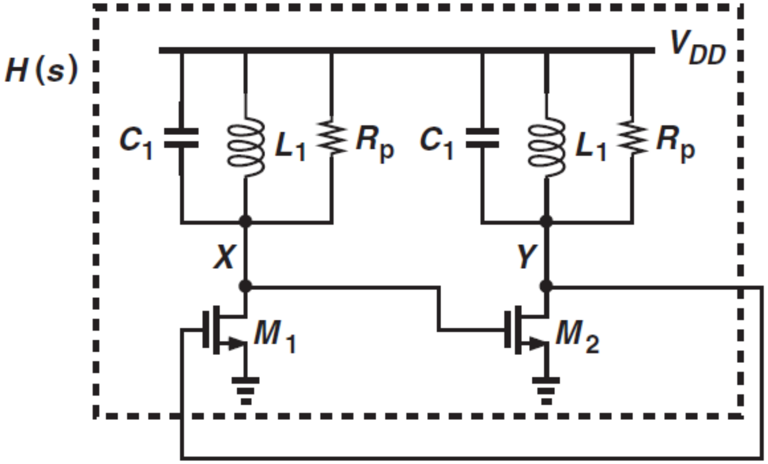


(a) (b)

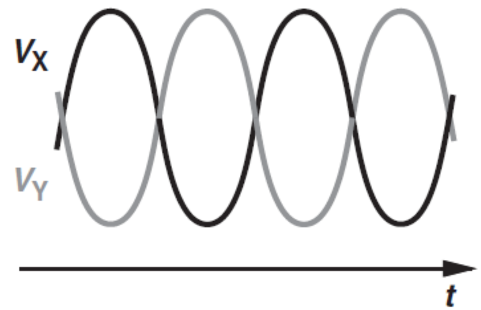
$$\frac{V_{out}}{V_{in}} = -g_m Z_2(s)$$

- The gain reaches a maximum of at resonance and approaches zero at very low or very high frequencies.
- The phase shift at resonance frequency is equal to 180°.

Two LC-load CS Stages in a Loop



(a)



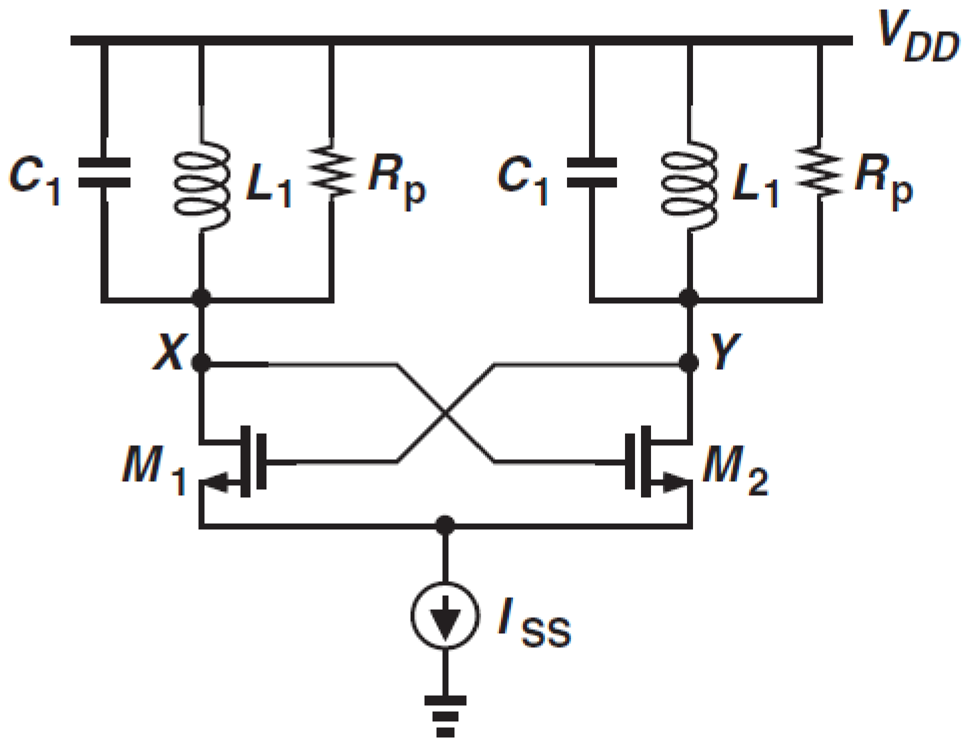
(b)

$$(g_m R_p)^2 \geq 1$$

$$\omega_1 = 1/\sqrt{L_1 C_1}$$

- Each stage provides 180° at ω_1 to achieve the total phase shift of 360°.
- Differential signals at nodes X and Y.
- However, the bias current of the transistors is poorly defined.

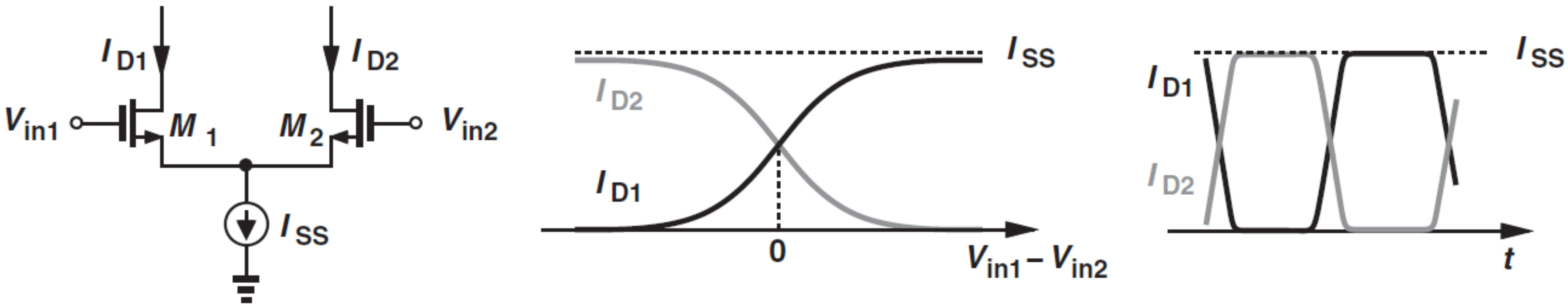
Cross-Coupled Oscillator



- A tail current source is added to set bias condition for the transistors.
- Most popular and robust LC oscillator used in integrated circuits.

Example 13.7

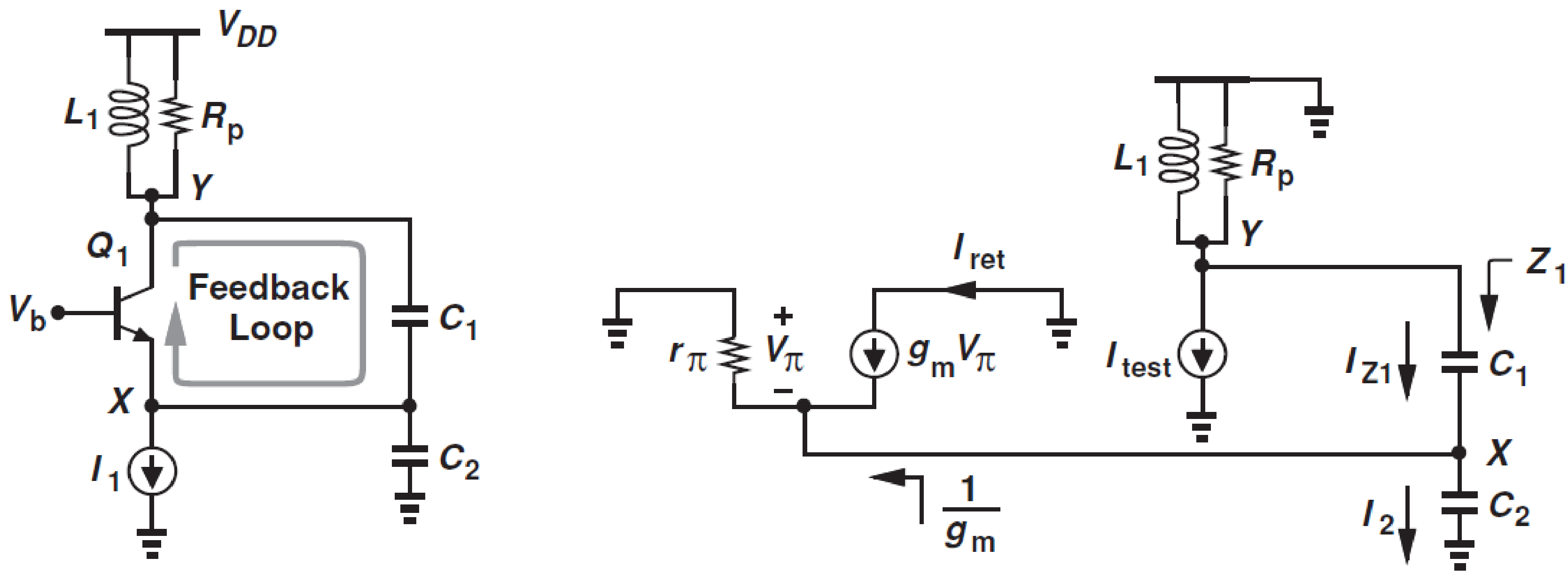
Plot the drain currents of M_1 and M_2 of cross-coupled oscillator if the voltage swings at X and Y are large.



(a) (b) (c)

- **With large input voltage swings, the entire current is steered to the left or to the right.**
- **Therefore, the drain current swings between zero and I_{SS} .**

Colpitts Oscillator (1)



(a)

(b)

$$\omega_1^2 = \frac{(C_1 + C_2)}{L_1 C_1 C_2} + \frac{g_m}{R_p C_1 C_2} \approx \left(L_1 \frac{C_1 C_2}{C_1 + C_2}\right)^{-1}$$

$$g_m R_p = \frac{(C_1 + C_2)^2}{C_1 C_2} = 4 \; (If \; C_1 = C_2)$$

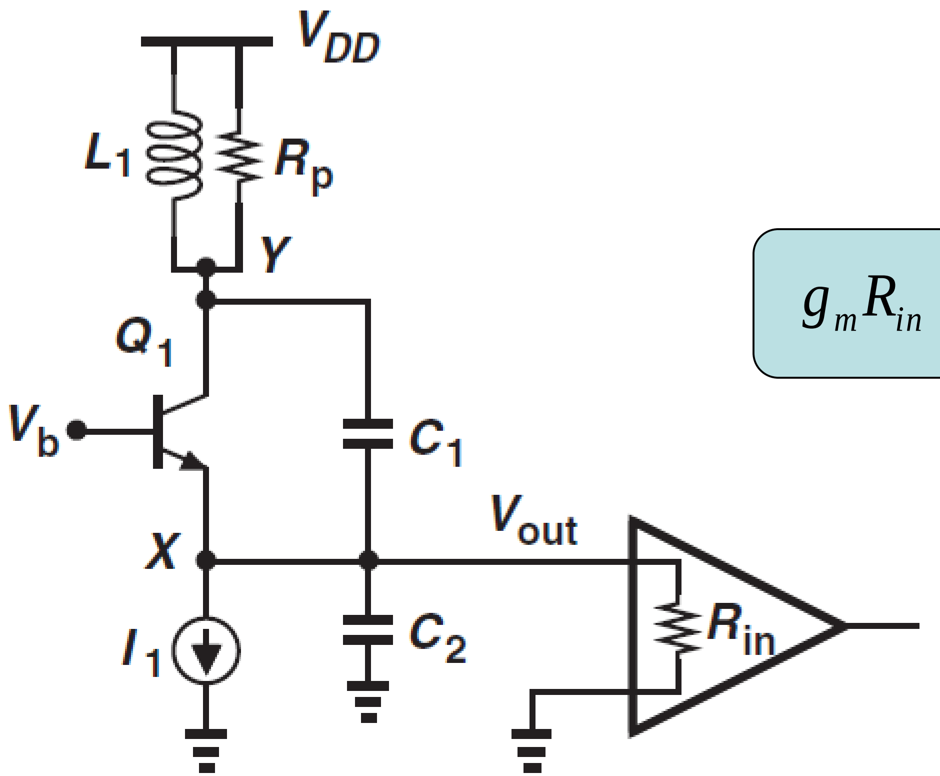
- Break feedback loop at node Y.
- I_{ret} / I_{test} must exhibit a phase of 360° and a magnitude of at least unity at the oscillation frequency.
- Wide application in discrete design.

Example 13.8

Compare the startup conditions of cross-coupled and Colpitts oscillators.

- **Cross-coupled topology requires a minimum $g_m R_p$ of 1, which means it can tolerate a lossier inductor than the Colpitts oscillator can.**
- **Compared to the differential output of cross-coupled oscillator, Colpitts topology provides only a single-ended output.**

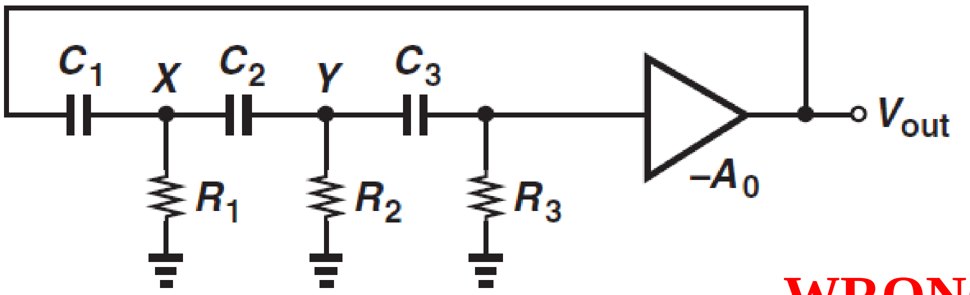
Colpitts Oscillator (2)



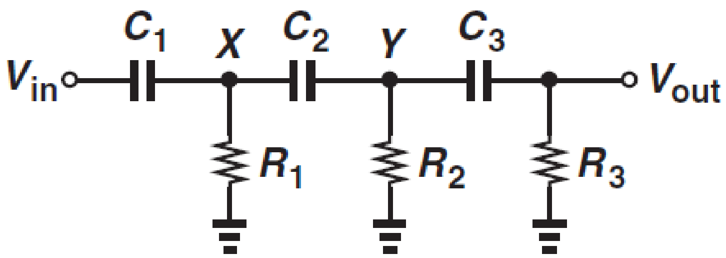
$$g_m R_{in} = 1 \; (if \; R_p \to \infty)$$

- **The preferable output of the oscillator is the emitter.**
- **Compared to output sensed at collector, the oscillator can (1) drive a lower load resistance; (2) have more relaxed startup condition which simplifies the design.**

Phase Shift Oscillator



(a)



(b)

WRONG!!
Loading Effect Missing!!

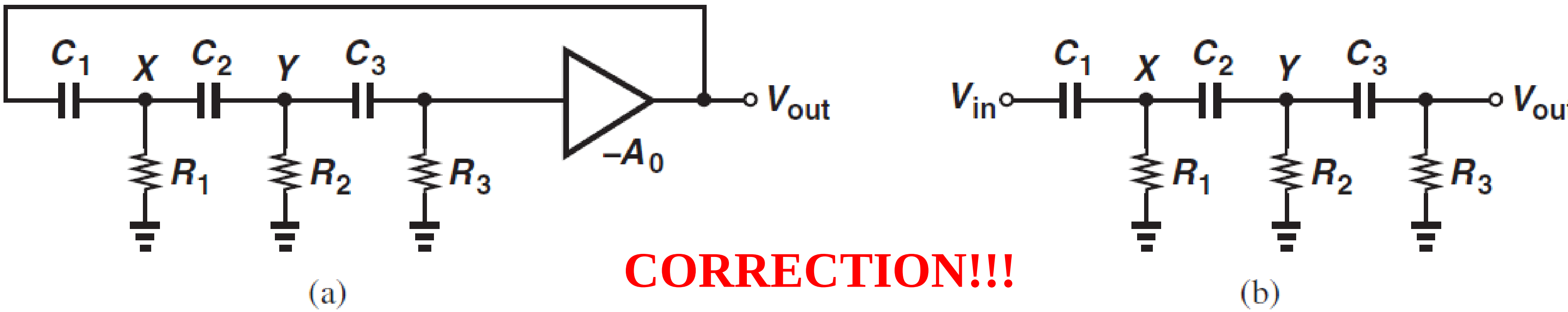
$$\frac{V_{out}}{V_{in}} = \frac{(RCs)^3}{(RCs+1)^3}$$

$$\omega_1 = \frac{1}{\sqrt{3}RC}$$

$$\frac{ARC\omega_1}{\sqrt{R^2C^2\omega_1^2+1}} = 1$$

- **Three RC sections can provide 180° phase shift at oscillation frequency.**
- **The signal attenuation of the passive stages must be compensated by the amplifier to fulfill the startup condition.**
- **Occasionally used in discrete design.**

Phase Shift Oscillator



CORRECTION!!!

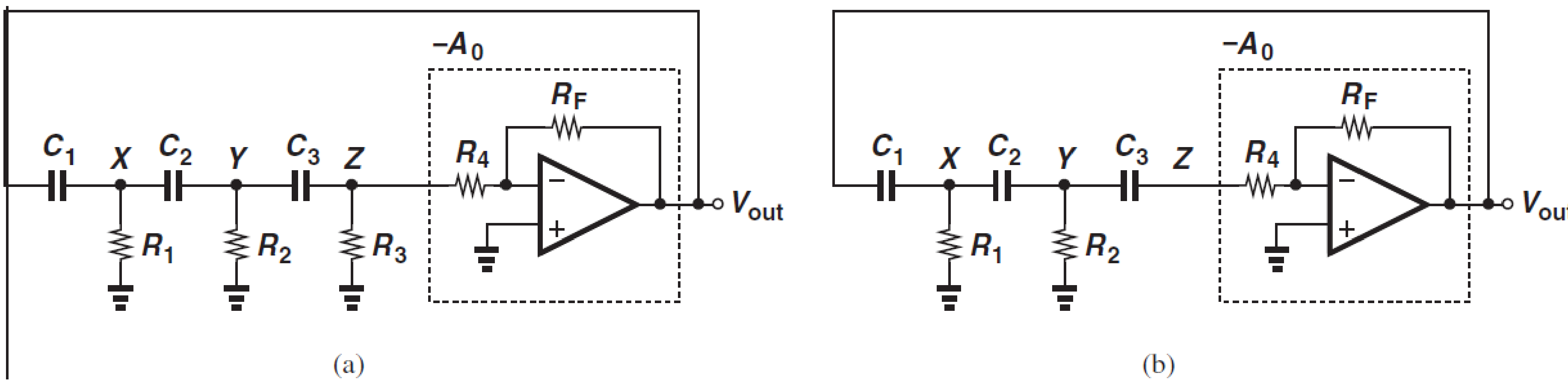
$$\frac{V_{out}}{V_{in}} = \frac{(RCs)^3}{(RCs)^3 + 6(RCs)^2 + 5(RCs) + 1}$$

$$\omega_1 = \frac{1}{\sqrt{6}RC}$$

$$\frac{-A(\omega_1 RC)^3}{-(\omega_1 RC)^3 + 5(\omega_1 RC)} = 1$$

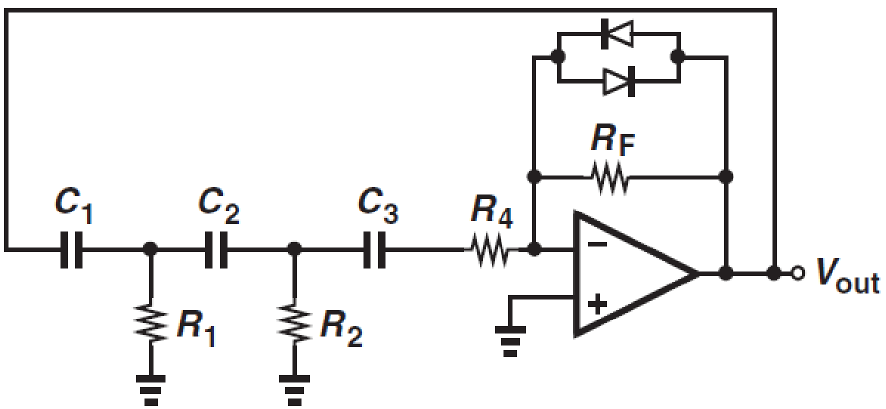
Example 13.9

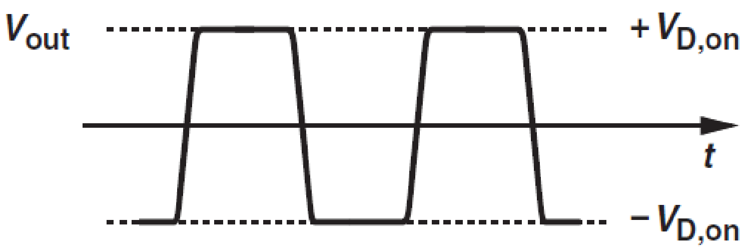
Design the phase shift oscillator using an op amp.



- **The op amp is configured as an inverting amplifier.**
- **Due to R_4 equivalently shunting R_3, we must choose $R_3||R_4 = R_2 = R_1 = R$.**
- **Alternatively, we may simply eliminate R_3 and set R_4 to be equal to R.**

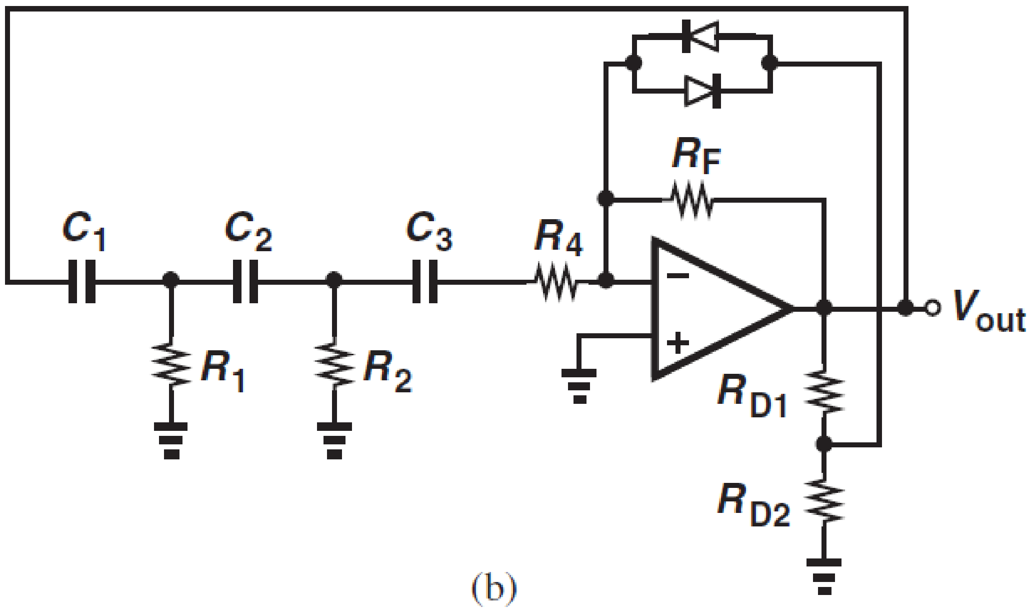
Stabilize Oscillation Amplitude (1)





- Replace the feedback resistor with two "anti-parallel" diodes to speed up op amp response.
- The output swings by one diode drop (700 to 800 *mV*) below and above its average value.
- This technique may prove inadequate in many applications.

Stabilize Oscillation Amplitude (2)

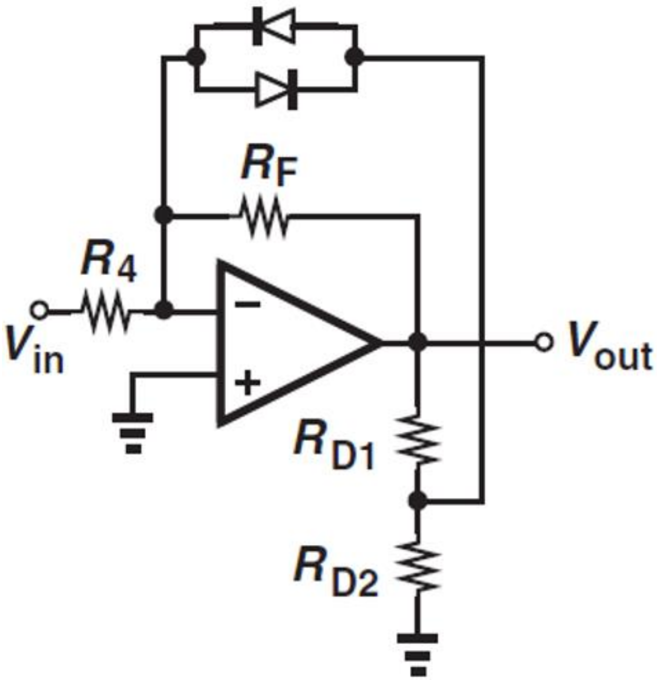


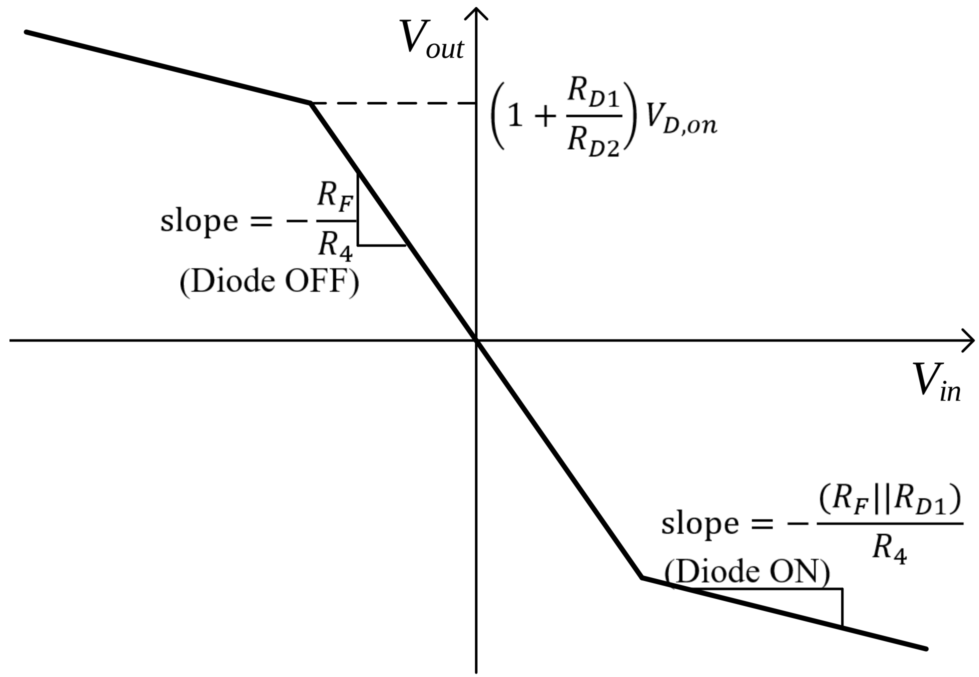
(b)

$$V_{out} = (1 + \frac{R_{D1}}{R_{D2}})V_{D,on}$$

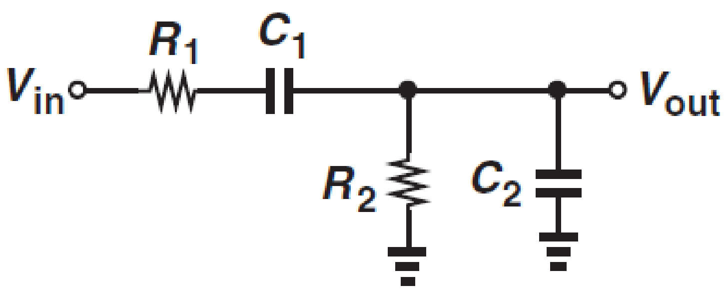
- **In order to achieve larger amplitude, we divide V_{out} down and feed the result to the diodes.**

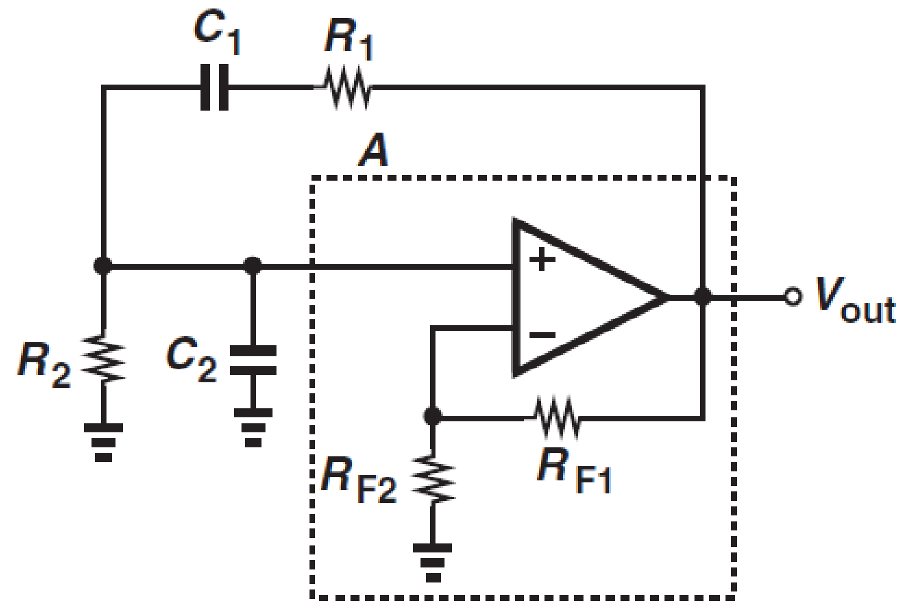
Stabilize Oscillation Amplitude (Supplementary)





Wien-Bridge Oscillator





(b)

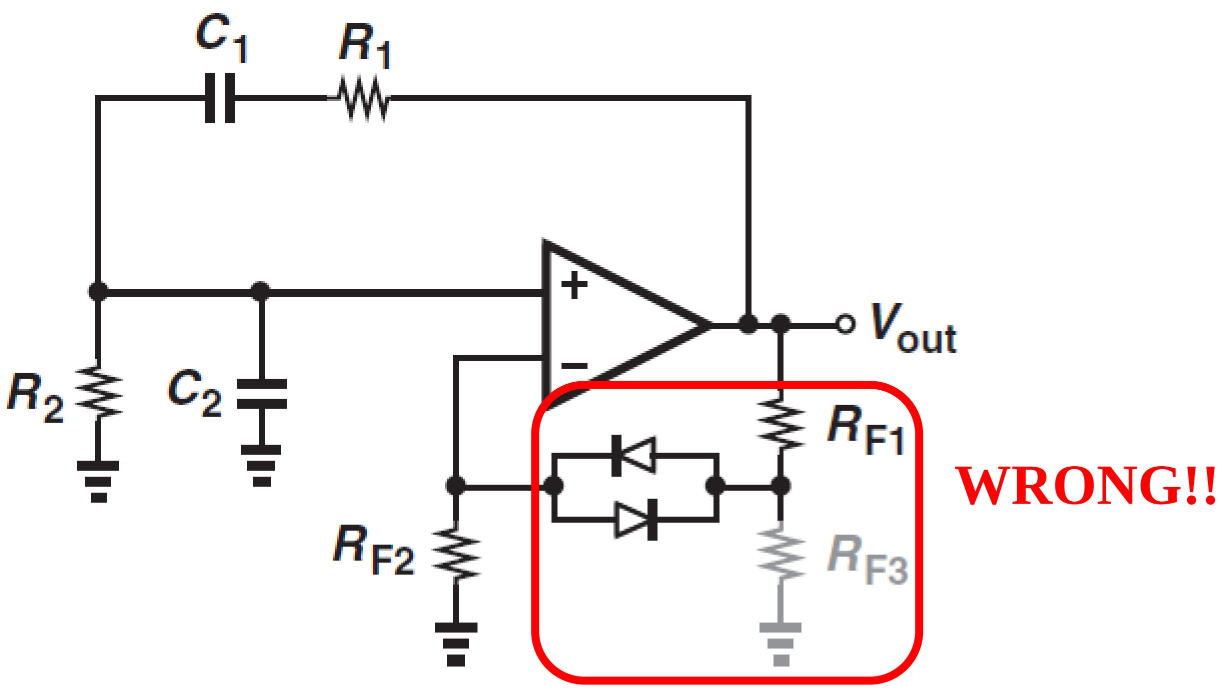
$$\frac{V_{out}}{V_{in}} = \frac{RCs}{R^2C^2s^2 + 3RCs + 1}$$

$$\omega_1 = \frac{1}{RC}$$

$$R_{F1} \geq 2R_{F2}$$

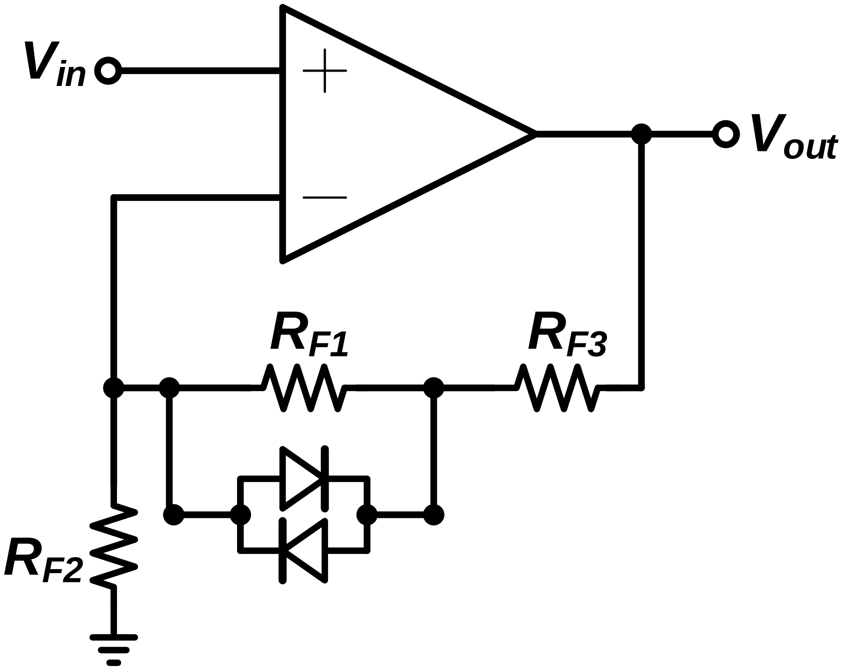
- **Passive feedback network provides zero phase shift.**
- **The amplifier is non-inverting.**

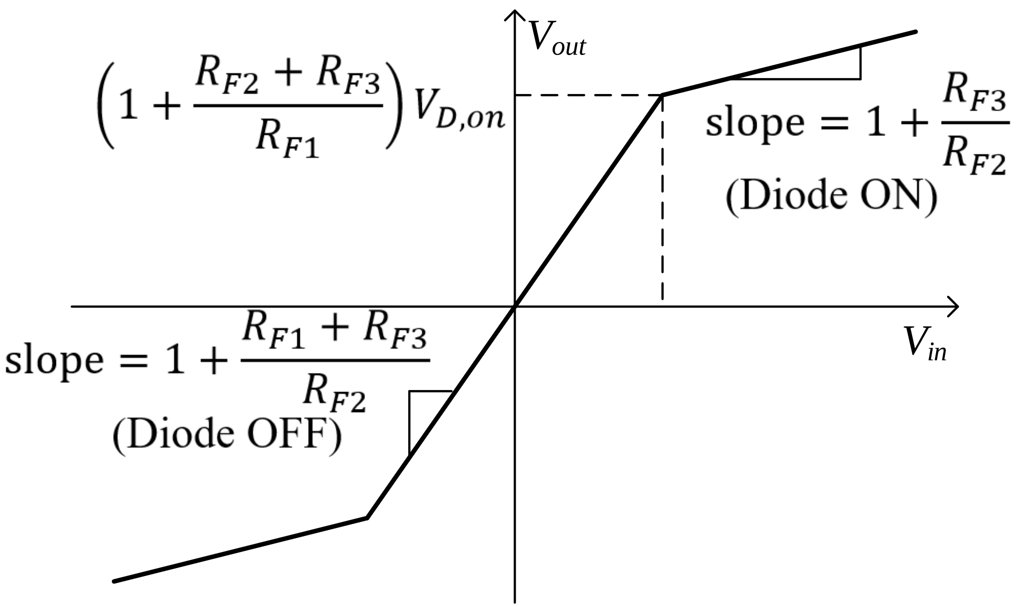
Stabilize Oscillation Amplitude



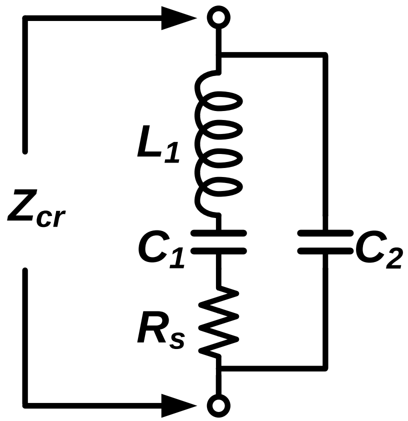
- Two anti-parallel diodes are inserted in series with R_{F1} to create strong feedback as $|V_{out}|$ exceeds $V_{D,on}$.
- To achieve larger amplitude, resistor R_{F3} can be added to divide V_{out} and apply the result to the diodes.

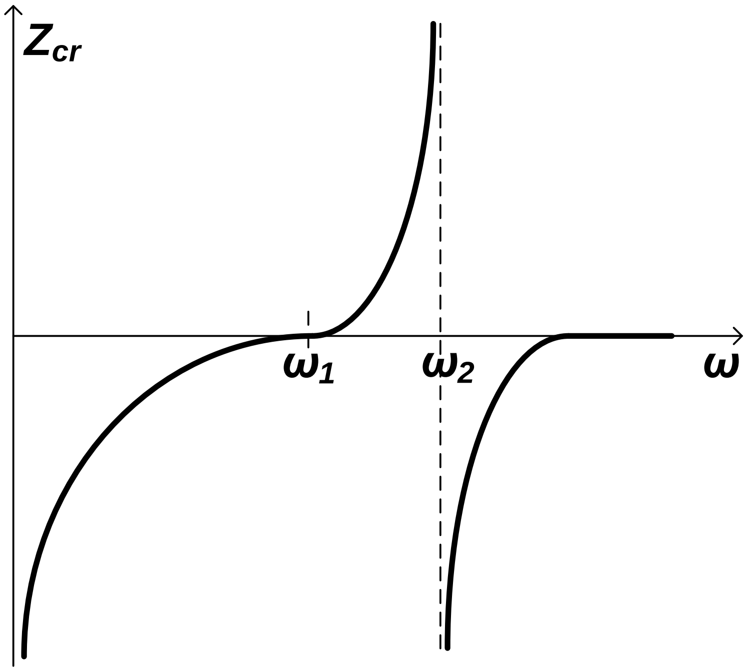
Stabilize Oscillation Amplitude (Supplementary)





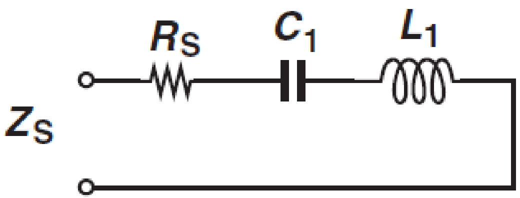
Crystal Model (1)

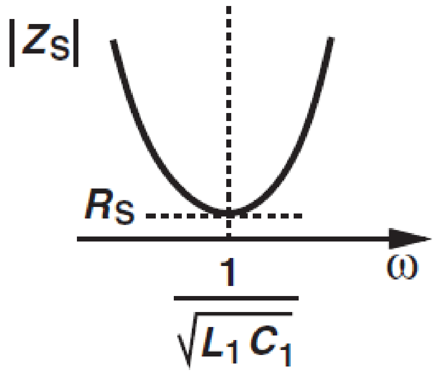


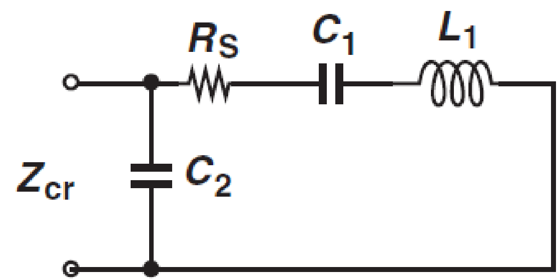


- **Attractive as frequency reference: (1) vibration frequency extremely stable; (2) easy to be cut to produce a precise frequency; (3) very low loss.**
- **The impedance falls to nearly zero at ω_1 and rises to a very high value at ω_2.**

Crystal Model (2)







$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_2 = \left(\sqrt{L_1 \frac{C_1 C_2}{C_1 + C_2}}\right)^{-1}$$

$$Z_{cr} \approx \frac{1 - L_1 C_1 \omega^2}{j\omega(C_1 + C_2 - L_1 C_1 C_2 \omega^2)}$$

- **At ω_1 the device experiences series resonance, while at ω_2 it experiences parallel resonance.**
- **In practice, ω_1 and ω_2 are very close which means $C_2 \gg C_1$.**

Example 13.10

If $C_2 \gg C_1$, find a relation between the series and parallel resonance frequencies.

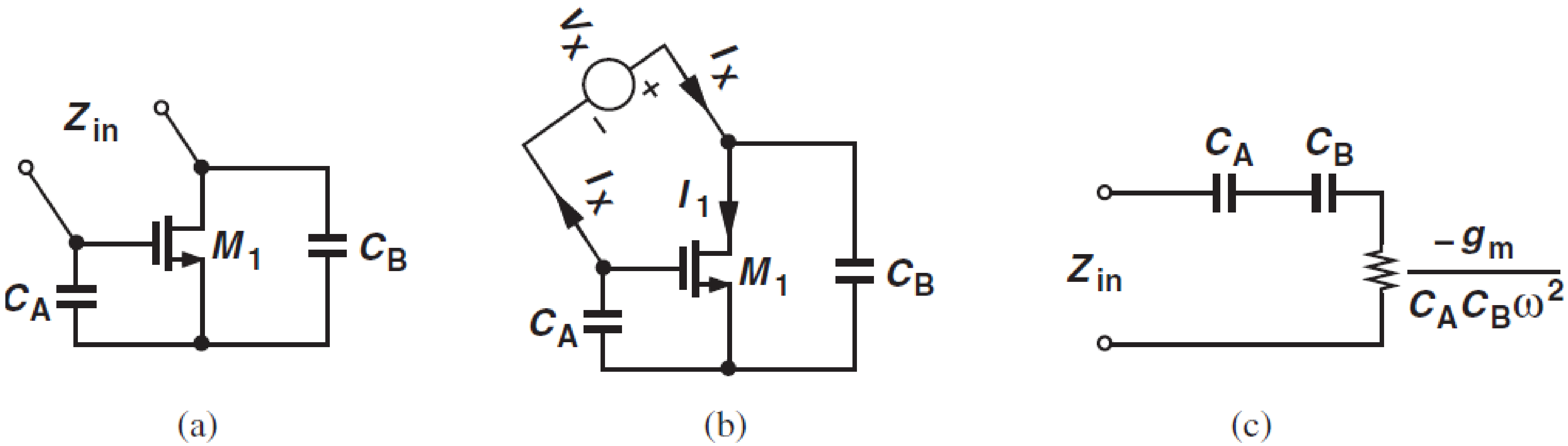
$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_2 = \left(\sqrt{L_1 \frac{C_1 C_2}{C_1 + C_2}}\right)^{-1}$$

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{C_1 + C_2}{C_2}} \approx 1 + \frac{C_1}{2C_2}$$

- **At ω_1 the device experiences series resonance, while at ω_2 it experiences parallel resonance.**
- **In practice, ω_1 and ω_2 are very close which means $C_2 \gg C_1$.**

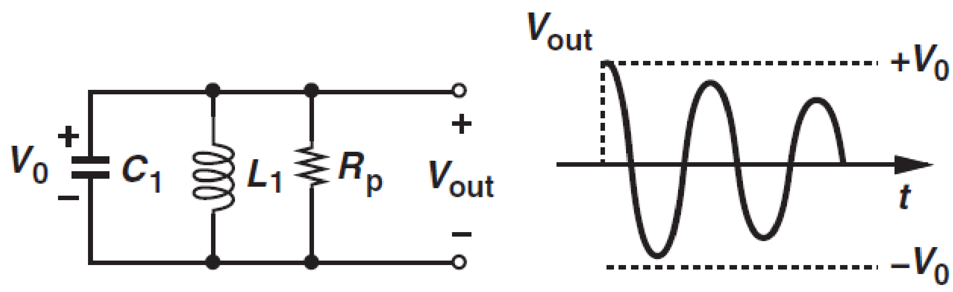
Negative-Resistance Circuit (1)



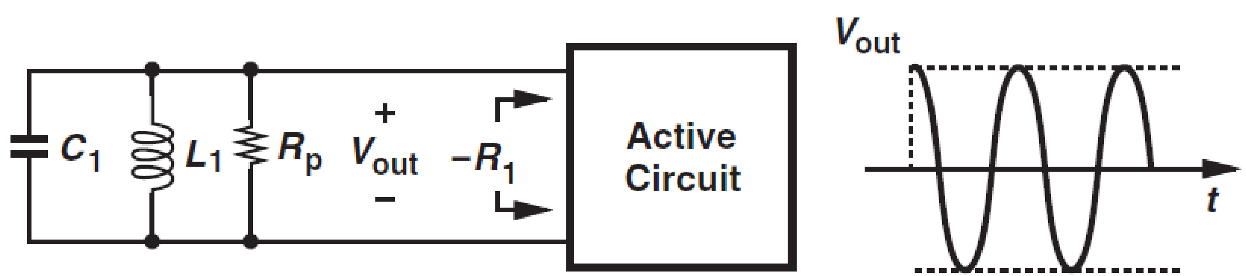
$$Z_{in}(j\omega) = \frac{1}{jC_A\omega} + \frac{1}{jC_B\omega} - \frac{g_m}{C_A C_B \omega^2}$$

- **The first two terms represent two capacitors in series and the third is a negative resistance.**
- **A small-signal negative resistance means if the voltage across the device increases, the current through it decreases.**

Negative-Resistance Circuit (2)



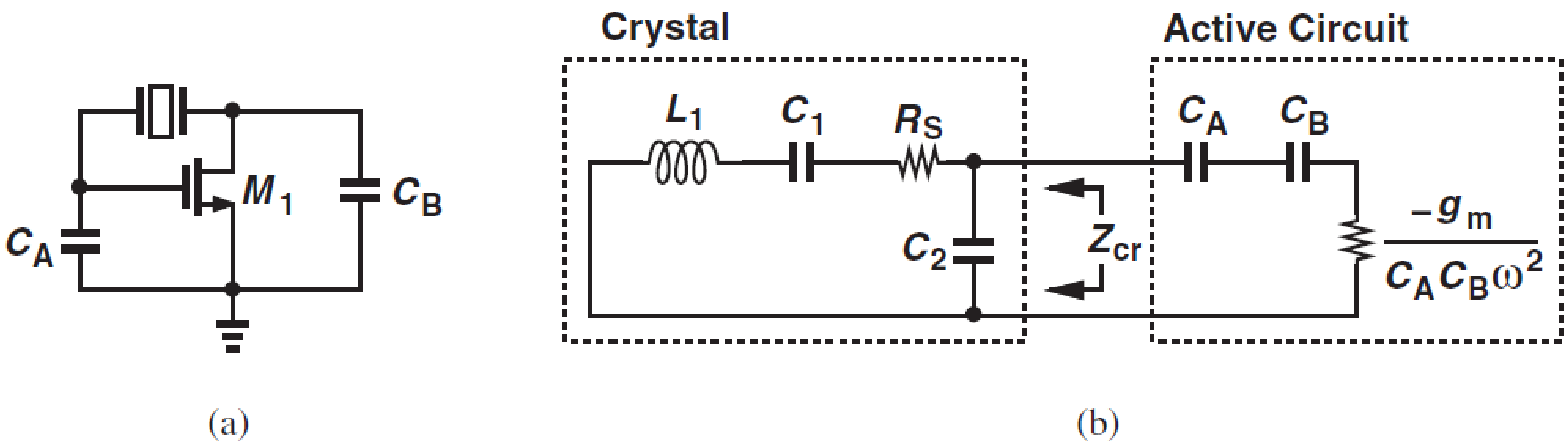
(a)



(b)

- A negative resistance can help sustain oscillation.
- The energy lost by R_p in every cycle is replenished by the active circuit.

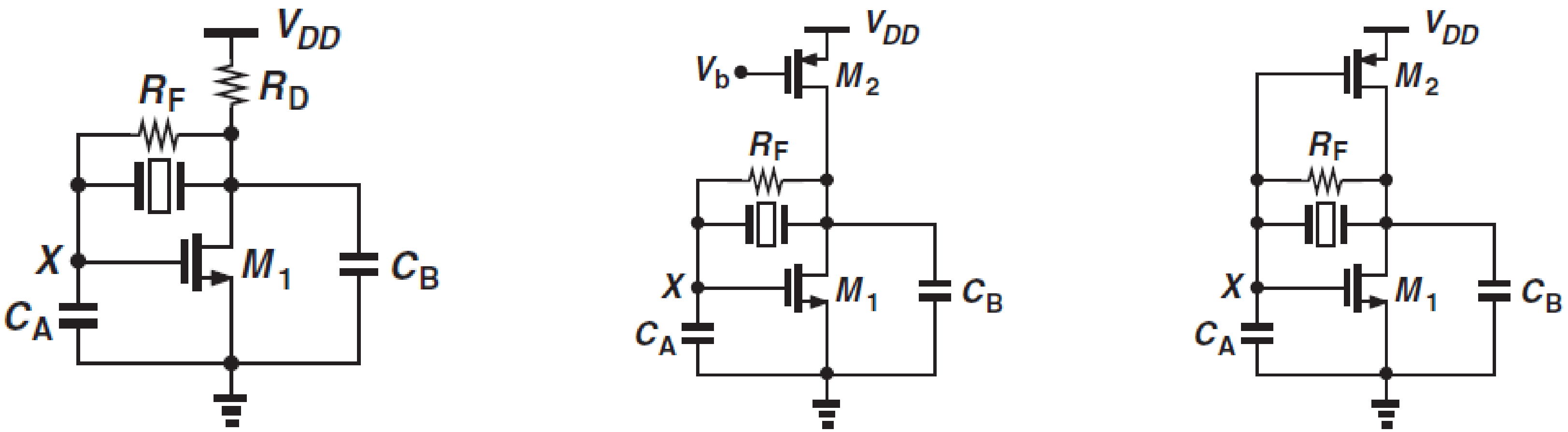
Crystal Oscillator



$$L_1 C_1 \omega^2 - 1 \le g_m R_S \frac{C_1 C_2}{C_A C_B} \ (Parallel\,resonance)$$

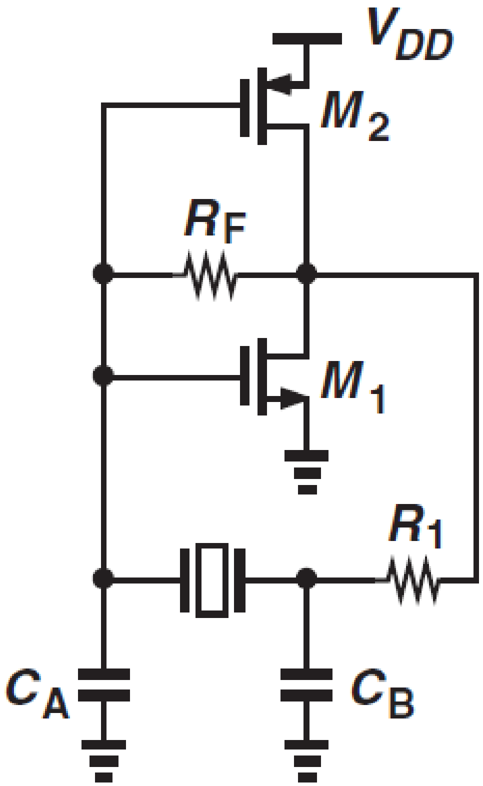
- Attach a crystal to a negative-resistance circuit to form an oscillator.
- C_A and C_B are chosen 10 to 20 times smaller than C_2 to minimize their effect on the oscillation frequency and to make negative resistance strong enough to cancel the loss.

Crystal Oscillator with Proper Bias (1)



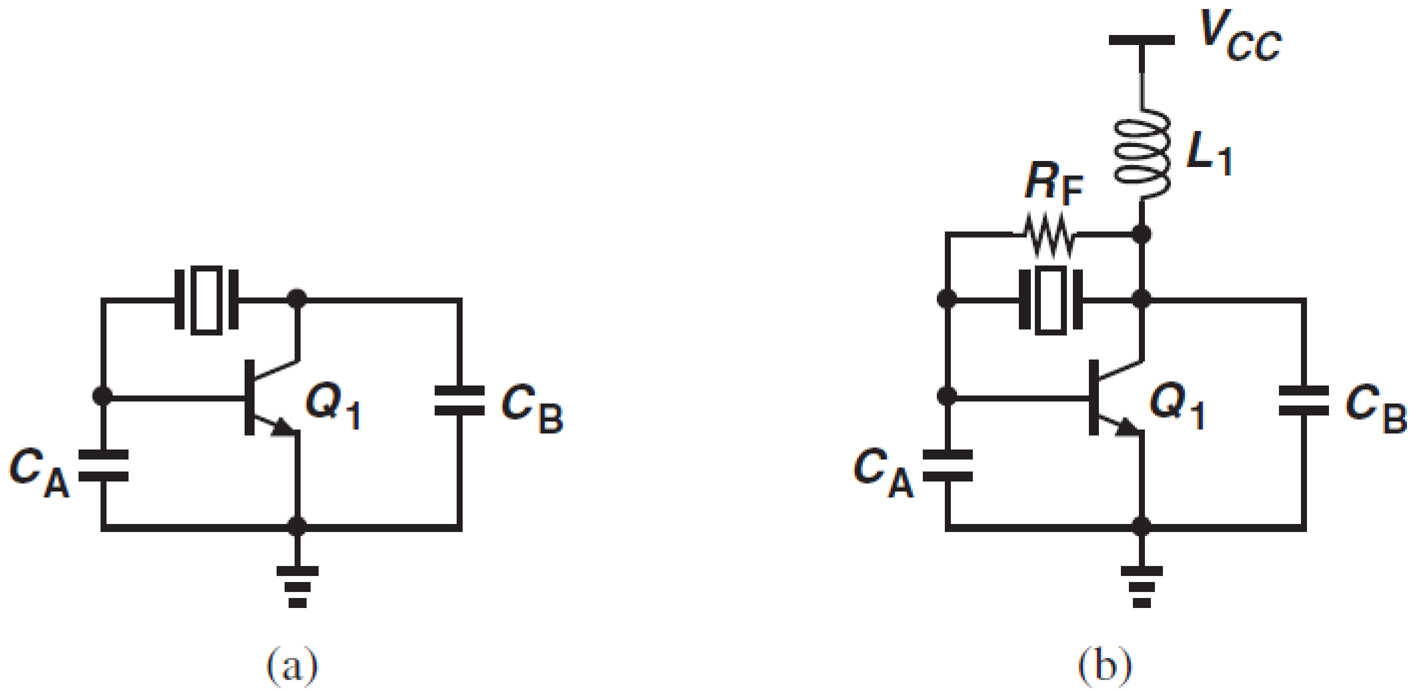
- Add a feedback resistor R_F (very large) to realize a self-biased stage.
- R_D can be replaced with a current source or an amplifying device.
- The third topology is popular in integrated circuits because (1) both transistors are biased in saturation and amplify the signal; (2) it can be viewed as an inverter biased at trip point.

Crystal Oscillator with Proper Bias (2)



- A low-pass filter (R_1 and C_B) is inserted in the feedback loop to suppress higher harmonic frequencies.
- The pole frequency $1/(2\pi R_1 C_B)$ is chosen slightly above the oscillation frequency.

Crystal Oscillator Using Bipolar Device



- L_1 provides the bias current of Q_1 but should not affect the oscillation frequency.
- Therefore, we choose L_1 large enough that $L_1\omega$ is a high impedance (approx. an open circuit).
- L_1 is called a "radio-frequency choke" (RFC).

Chapter Summary

- Negative-feedback system
- Startup condition
- Oscillation amplitude limited by nonlinearity of devices
- Ring oscillators
- Ideal and lossy LC tank
- Cross-coupled oscillator with differential output
- Colpitts LC oscillator with single-ended output
- Phase shift oscillator
- Wien-bridge oscillator
- Crystal oscillator