Chapter 4.
Image Measurements and Refinements
4-1. Introduction

- The solution of most photogrammetric problems generally requires some type of photographic measurement.

- Photographic measurements are usually made on positives printed on paper, film, or glass, or in digital images manipulated on a computer.

- Photographic measurements could also be made directly on the negatives.

- Equipment used for making photographic measurements varies from inexpensive, simple scales to very precise and complex machines that provide computer-compatible digital output.

In this chapter,

- Various types of instruments and the manner in which they are used are described.

- The sources of systematic errors and the manners by which they are eliminated are also discussed.
4-2. Coordinate Systems for Image Measurements

- For metric cameras with side fiducial marks, the commonly adopted reference system for photographic coordinates, is the **rectangular axis system** formed by joining opposite fiducial marks with straight lines, as shown in Fig. 4-1.

- The $x$ axis is usually arbitrarily designated as the fiducial line most nearly parallel with the direction of flight, positive in the direction of flight.

- The positive $y$ axis is 90° counterclockwise from positive $x$.

- The origin of the coordinate system is the intersection of fiducial lines. This point is often called the **indicated principal point**.

- For the most precise work, since the lines joining opposite fiducials are not exactly perpendicular, the fiducial marks serve as control points from which the photo coordinate axis system can be determined.
4-2. Coordinate Systems for Image Measurements

- The position of any image on a photograph, such as point $a$ of Fig. 4-1, is given by its rectangular coordinates $x_a$ and $y_a$, where $x_a$ is the perpendicular distance from the $y$ axis to $a$ and $y_a$ is the perpendicular distance from the $x$ axis to $a$.

- Similarly, the photographic position of image point $b$ is given by its rectangular coordinates $x_b$ and $y_b$.

- It is common for **aerial cameras** to have eight fiducials installed, in both side and corner locations.

- On **digital images**, coordinates are expressed as row and column numbers of individual pixels.

- If pixel dimensions are known, the dimensionless row and column values can be converted to linear measurements.

- Note that for many common image manipulation programs, pixel coordinates are given in the order of column (abscissa) followed by row (ordinate).

  - This results in a "**left-handed**" coordinate system, which may lead to problems in calculations.
4-2. Coordinate Systems for Image Measurements

- **Rectangular coordinates** are a very basic and useful type of photographic measurement.
- Rectangular coordinates can be used to calculate the photo distances between points by using simple analytic geometry.
- Photographic distance \( ab \) of Fig. 4-1, for example, may be calculated from rectangular coordinates as follows:

\[
ab = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}
\]

Equation (4-1)
4-3. Simple Scales for Photographic Measurements

- There are a variety of **simple scales** available for photographic measurements.

**Engineer’s scales**, are available in both metric and English units.
- They have several different graduation intervals so as to accommodate various nominal scales.
- Precision and accuracy can be enhanced by using a magnifying glass and the finest set of graduations, as long as suitable care is taken in aligning and reading the scale.

**Glass scales** have fine graduations etched on the bottom surface and are equipped with magnifying eyepieces which can slide along the scale.
- When greater accuracy is desired, a device such as the glass scale may be used.
- With a glass scale, readings may be estimated quite readily to one-tenth of the smallest division, but these scales cannot be used to lay off distances.
4-4. Measuring Photo Coordinates with Simple Scales

- The conventional procedure for measuring photo coordinates when using an engineer's scale generally consists of **first marking the photo coordinate axis system**.

- This may be done by **carefully aligning a straightedge across the fiducial marks and lightly making a line with a razor blade, pin, or very sharp 4H or 5H pencil**.

- **Rectangular coordinates are then obtained** by direct measurement of the perpendicular distances from these axes.

- It is important to **affix the proper algebraic sign to measured rectangular coordinates**; failure to do so will result in frustrating mistakes in solving photogrammetry problems.

- Points situated to the right of the $y$ axis have positive $x$ coordinates, and points to the left have negative $x$ coordinates.

- Points above the $x$ axis have positive $y$ coordinates, and those below the $x$ axis have negative $y$ coordinates.
4-5. Comparator Measurement of Photo Coordinates

- For direct measurement of film, the ultimate in photo coordinate measurement accuracy can be achieved with precise instruments called comparators.

- There are two basic types of comparators, monocomparators and stereocomparators.

- Monocomparators make measurements on one photograph at a time, and with stereocomparators, image positions are measured by simultaneously viewing an overlapping stereo pair of photographs.

- Comparators are used primarily to obtain precise photo coordinates necessary for camera calibration and for analytical photogrammetry.

- Accuracy capability of comparators is typically in the range of from 2 to 3 micrometers (\( \mu m \)).

- The usual approach for measuring photographic coordinates with a comparator is to measure the coordinates of all image points as well as all fiducial marks. Then an affine or other two-dimensional coordinate transformation is performed in order to relate the arbitrary comparator coordinates to the axis system related to the fiducials.
4-5. Comparator Measurement of Photo Coordinates

- Traditional monocomparators are no longer being manufactured today, although specialized devices are still being produced for limited markets.

- A more versatile device, the analytical stereoplotter, is commonly used to perform the function of both monocomparators and stereocomparators when working directly with film.

- Although monocomparators are generally very precise, small systematic errors do occur as a result of imperfections in their measurement systems.

- The magnitudes of these errors can be determined by measuring coordinates of a precise grid plate and then comparing the results with known coordinate values of the grid plate.

- Measured photo coordinates can then be processed through the polynomial to effectively eliminate the systematic errors of the comparator.
4-6. Photogrammetric Scanners

- Photogrammetric scanners are devices used to convert the content of photographs from analog form (a continuous-tone image) to digital form (an array of pixels with their gray levels quantified by numerical values).

- Once the image is in digital form, coordinate measurement can take place in a computer environment, either through a manual process involving a human operator who points at features displayed on a computer screen, or through automated image-processing algorithms.

- It is essential that a photogrammetric scanner have sufficient geometric and radiometric resolution as well as high geometric accuracy.

- **Geometric/spatial resolution of a scanner** is an indication of the pixel size of the resultant image. The smaller the pixel size, the greater the detail that can be detected in the image. High-quality photogrammetric scanners should be capable of producing digital images with minimum pixel sizes on the order of 5 to 15 μm.

- **Radiometric resolution of a scanner** is an indication of the number of quantization levels (corresponding to image density differences) associated with a single pixel. Minimum radiometric resolution should be 256 levels (8-bit) with most scanners being capable of 1024 levels (10-bit) or higher.
4-6. Photogrammetric Scanners

- The geometric quality of a scanner can be expressed by the positional accuracy of the pixels in the resultant image.

- If a digital image is to produce the same level of accuracy as is attainable by using film images and a comparator, the positions of the pixels in the digital image need to be at the same spatial accuracy.
  - Hence, the geometric positional accuracy of a high-quality photogrammetric scanner should be at the 2- to 3-μm level.

- Measurements from scanned photos is similar to those using a comparator.
4.7. Refinement of Measured Image Coordinates

- Photo coordinates will still contain **systematic errors** from various other sources.

**Major sources of systematic errors**

1. (a) Film distortions due to shrinkage, expansion, and lack of flatness
   (b) CCD array distortions due to electrical signal timing issues or lack of flatness of the chip surface
2. (a) Failure of photo coordinate axes to intersect at the principal point
   (b) Failure of principal point to be aligned with center of CCD array
3. Lens distortions
4. Atmospheric refraction distortions
5. Earth curvature distortion

- **Corrections** may be applied to eliminate the effects of these systematic errors.

- However, not all corrections need to be made for all photogrammetric problems; in fact, for work of coarse accuracy they may all be ignored.

- If precise measurements for an analytical photogrammetry problem have been made with a comparator, all the corrections may be significant.

- **Corrections can be made after considering required accuracy versus magnitude of error caused by neglecting the correction.**
4.8. Distortions of Photographic Films and Papers

- In photogrammetric work, true positions of images in the picture are required.
- Photo coordinates will unavoidably contain small errors due to shrinkage or expansion of the photographic materials that support the emulsion of the negative and positive.
- In addition, since photogrammetric equations derived for applications involving frame cameras assume a flat image plane, any lack of flatness will likewise cause errors.
- These errors can also be categorized as film distortions, and they are generally the most difficult to compensate for, due to their nonhomogeneous nature.
- Photo coordinates must be corrected for these errors before they are used in photogrammetric calculations.
- The magnitude of error in computed values will depend upon the severity of the film distortions, which depends upon the type of emulsion support materials used and the flatness of the camera platen.
4.8. Distortions of Photographic Films and Papers

- Most photographic films used to produce negatives for photogrammetric work have excellent dimensional stability, but some **small changes in size do occur during processing and storage**

- **Dimensional change** during storage may be **held to a minimum by maintaining constant temperature and humidity in the storage room**

- The actual amount of distortion present in a film is a **function of several variables**, including the type of film and its thickness

- Paper media are generally much less stable than film

- Whether the images are photographically reproduced or are printed from a scanned image, paper copies often require corrections to measured distances
4.9. Image Plane Distortion

- Photo coordinates can be corrected if discrepancies exist, and the approach differs depending on the necessary level of accuracy.

- For lower levels of accuracy (corresponding to measurements with an engineer’s scale on paper prints) the following approach may be used:

- If \( x_m \) and \( y_m \) are measured fiducial distances on the positive, and \( x_c \) and \( y_c \) are corresponding calibrated fiducial distances, then the corrected photo coordinates of any point \( a \) may be calculated as:

\[
\begin{align*}
    x'_a &= \left( \frac{x_c}{x_m} \right) x_a \\
    y'_a &= \left( \frac{y_c}{y_m} \right) y_a
\end{align*}
\]

Equation (4-2)  
Equation (4-3)

- \( x'_a, y'_a \) = corrected photo coordinates
- \( x_a, y_a \) = measured coordinates
- \( \frac{x_c}{x_m}, \frac{y_c}{y_m} \) = scale factors in the \( x \) and \( y \) directions

- Equations (4-2) and (4-3) can be used by making CAD-unit measurements of the calibrated distances and applying the correction factors to subsequent measurements.
4.10. Reduction of Coordinates to an Origin at the Principal Point

- Photogrammetric equations that utilize photo coordinates are based on projective geometry and assume an origin of photo coordinates at the principal point.
  - Therefore it is theoretically correct to reduce photo coordinates from the measurement or photo coordinate axis system to the axis system whose origin is at the principal point.

- For precise analytical photogrammetric work, it is necessary to make the correction for the coordinates of the principal point.
  - The correction is applied after a two-dimensional coordinate transformation (e.g., affine) is made to the coordinates measured by comparator or from a scanned image.
  - The principal point coordinates $x_p$ and $y_p$ from the camera calibration report are subtracted from the transformed $x$ and $y$ coordinates.
  - The correction for the principal point offset is applied in conjunction with lens distortion corrections.
4.11. Correction for Lens Distortions

- The mathematical equations that are used to model lens distortions are typically comprised of two components: **symmetric radial distortion** and **decentering distortion**

- **Symmetric radial lens distortion** is an unavoidable product of lens manufacture, although with careful design its effects can be reduced to a very small amount.

- **Decentering distortion**, on the other hand, is primarily a function of the imperfect assembly of lens elements, not the actual design.

- Traditional camera calibration procedures provided information regarding only the symmetric radial component.

- The radial distortion value was the radial displacement from the ideal location to the actual image of the collimator cross, with positive values indicating outward displacements.

- The approach used for **determining radial lens distortion values** for these older calibration reports was to fit a polynomial curve to a plot of the displacements (on the ordinate) versus radial distances (on the abscissa).
4.11. Correction for Lens Distortions

- The form of the polynomial, based on lens design theory, is

\[ \Delta r = k_1 r^1 + k_2 r^3 + k_3 r^5 + k_4 r^7 \]  

Equation (4-4)

- In Eq. (4-4), \( \Delta r \) is the amount of radial lens distortion, \( r \) is the radial distance from the principal point, and \( k_1, k_2, k_3, \) and \( k_4 \) are coefficients of the polynomial.

- The coefficients of the polynomial are solved by least squares using the distortion values from the calibration report.

- To correct the \( x, y \) position of an image point, the distance \( r \) from the image point to the principal point is computed and used to compute the value of \( \Delta r \) from Eq. (4-4).

- This is done by first converting the fiducial coordinates \( x \) and \( y \), to coordinates \( \bar{x} \) and \( \bar{y} \), relative to the principal point, by Eqs. (4-5) and (4-6).

- Eq. (4-7) is used to compute the value of \( r \) to use in Eq. (4-4).

\[
\bar{x} = x - x_p \\
\bar{y} = y - y_p \\
r = \sqrt{\bar{x}^2 + \bar{y}^2}
\]

Equation (4-5)  
Equation (4-6)  
Equation (4-7)
4.11. Correction for Lens Distortions

- After the radial lens distortion value of $\Delta r$ is computed, its $x$ and $y$ components (corrections $\delta x$ and $\delta y$) are computed and subtracted from $\bar{x}$ and $\bar{y}$, respectively.

- The $\delta x$ and $\delta y$ corrections are based on a similar-triangle relationship, as shown in Fig. 4-4. By similar triangles of that figure:

$$\frac{\Delta r}{r} = \frac{\delta x}{\bar{x}} = \frac{\delta y}{\bar{y}}$$

from which:

$$\delta x = \bar{x} \frac{\Delta r}{r} \quad \text{Equation (4-8)}$$

$$\delta y = \bar{y} \frac{\Delta r}{r} \quad \text{Equation (4-9)}$$

- The corrected coordinates $x$ and $y$ are then computed by:

$$x_c = \bar{x} - \delta x \quad \text{Equation (4-10)}$$

$$y_c = \bar{y} - \delta y \quad \text{Equation (4-11)}$$

Figure 4-4. Relationship between radial lens distortion and corrections to $x$ and $y$ coordinates.
4.12. Correction for Atmospheric Refraction

- Density (and hence the index of refraction) of the atmosphere decreases with increasing altitude. Light rays do not travel in straight lines through the atmosphere, but rather they are bent according to Snell's law, as shown in Fig. 4-6.

- The incoming light ray from point A of the figure makes an angle $\alpha$ with the vertical.

- If refraction were ignored, the light ray would appear to be coming from point B rather than from point A.

- Photogrammetric equations assume that light rays travel in straight paths, and to compensate for the known refracted paths, corrections are applied to the image coordinates.

- In Fig. 4-6, if a straight path had been followed by the light ray from object point A, then its image would have been at $\alpha'$.

*Figure 4–6. Atmospheric refraction in aerial photography*
4.12. Correction for Atmospheric Refraction

- The angular distortion due to refraction is $\Delta \alpha$, and the linear distortion on the photograph is $\Delta r$.

- **Refraction** causes all imaged points to be displaced outward from their correct positions.

- The magnitude of refraction distortion increases with increasing flying height and with increasing $\alpha$ angle.

- **Refraction distortion** occurs radially from the photographic nadir point (principal point of a vertical photo) and is zero at the nadir point.

Figure 4-6. Atmospheric refraction in aerial photography.
4.12. Correction for Atmospheric Refraction

- The usual approach to the atmospheric refraction correction is based on the assumption that change in the refractive index of air is directly proportional to change in height.

- Snell’s law can be solved continuously along the ray path for each infinitesimal change in angle due to refraction.

- When all the infinitesimal changes are summed, the total is proportional to the tangent of the incident angle.

- The proportionality constant is based on the values of the refractive indices at ground level and at the camera position, which are related to elevation.

- The relationship that expresses the angular distortion $\Delta \alpha$ as a function of $\alpha$ is

$$\Delta \alpha = K \tan \alpha$$

Equation (4–18)

- In this equation, $\alpha$ is the angle between the vertical and the ray of light, as shown in Fig. 4-6.
4.12. Correction for Atmospheric Refraction

\[ \Delta \alpha = K \tan \alpha \]  
\text{Equation (4-18)}

- \( K \) is a value which depends upon the flying height above mean sea level and the elevation of the object point.

- A convenient method for calculating a value of \( K \), adapted from the Manual of Photogrammetry, is to compute \( K \) by

\[ K = (7.4 \times 10^{-4})(H - h)[1 - 0.02(2H - h)] \]  
\text{Equation (4-19)}

- In Eq. (4-19), \( H \) is the flying height of the camera above mean sea level in kilometers, and \( h \) is the elevation of the object point above mean sea level in kilometers.

- The units of \( K \) are degrees.

- The procedure for computing atmospheric refraction corrections to image coordinates on a vertical photo begins by computing radial distance \( r \) from the principal point to the image, using Eq. (4-20)

\[ r = \sqrt{r^2 + y^2} \]  
\text{Equation (4-20)}
4.12. Correction for Atmospheric Refraction

- Also from Fig. 4-6,
  \[ \tan \alpha = \frac{r}{f} \]  
  Equation (4–21)

- The values of \( K \) and \( \tan \alpha \) from Eqs. (4-19) and (4-21) are then substituted into Eq. (4-18) to compute refraction angle \( \Delta \alpha \)

  \[ \Delta \alpha = K \frac{r}{f} \]  
  Equation (4–22)

- The radial distance \( r' \) from the principal point to the corrected image location can then be computed by

  \[ r' = f \tan (\alpha - \Delta \alpha) \]  
  Equation (4–23)

- The change in radial distance \( \Delta r \) is then computed by

  \[ \Delta r = r - r' \]  
  Equation (4–24)

- The \( x \) and \( y \) components of atmospheric refraction distortion corrections (\( \delta x \) and \( \delta y \)) can then be computed by Eqs. (4-8) and (4-9), using the values of \( x \) and \( y \) in place of \( \bar{x} \) and \( \bar{y} \)

- To compute corrected coordinates \( x' \) and \( y' \), the corrections \( \delta x \) and \( \delta y \) are subtracted from \( x \) and \( y \)
4.13. Correction for Earth Curvature

- Traditionally, in analytical photogrammetry, corrections were commonly applied to measured photo coordinates to compensate for the effects of earth curvature.

- The rationale for this notion is that elevations of points are referenced to an approximately spherical datum whereas photogrammetric equations assume that the zero-elevation surface is a plane.

- The primary problem with the earth curvature correction is that because of the nature of map projection coordinates, correcting photo coordinates for earth curvature will degrade the accuracy of either X or Y object space coordinates, depending upon the map projection used.

- The proper approach, which avoids the need for any sort of earth curvature correction, employs a three-dimensional orthogonal object space coordinate system such as local vertical coordinate system.

- Under magnification or image zoom, the edges of the object will appear to be somewhat indistinct although the center can still be identified to a high precision as shown in Fig. 4-7.

- In a situation where a point feature is designated to be at the angle point of a painted chevron target, it is important to use the intersections of the centerlines of the stripes rather than the outside or inside corner as the point.

- Edges that may appear clear and sharp at a glance will become blurred and indistinct when viewed under magnification as shown in Fig. 4-8.

- It is important to make a best estimate of where the true edge of the feature is located.

- A certain amount of error can be expected in the measurement due to blurred edges.

- For example, it is very difficult to identify the precise location of where the paint meets the underlying pavement as shown in Fig.4-8.