Chapter 6.
Vertical Photographs

6-1. Geometry of Vertical Photographs

- Photographs taken from an aircraft with the optical axis of the camera vertical or as nearly vertical as possible are called ***vertical photographs***
- If the optical axis is exactly vertical, the resulting photograph is termed *truly vertical*
- In this chapter, equations are developed **assuming truly vertical photographs**
- In spite of precautions taken to keep the camera axis vertical, small tilts are invariably present
- For photos intended to be vertical, however, tilts are usually less than 1° and rarely exceed 3°
- Photographs containing these small unintentional tilts are called ***near-vertical*** or ***tilted photographs***, and for many practical purposes these photos may be analyzed using the relatively simple "truly vertical" equations of this chapter without serious error

In this chapter, besides assuming truly vertical photographs, other assumptions are that the photo coordinate axis system has its origin at the photographic principal point and that all photo coordinates have been corrected for shrinkage, lens distortion, and atmospheric refraction distortion

6-1. Geometry of Vertical Photographs

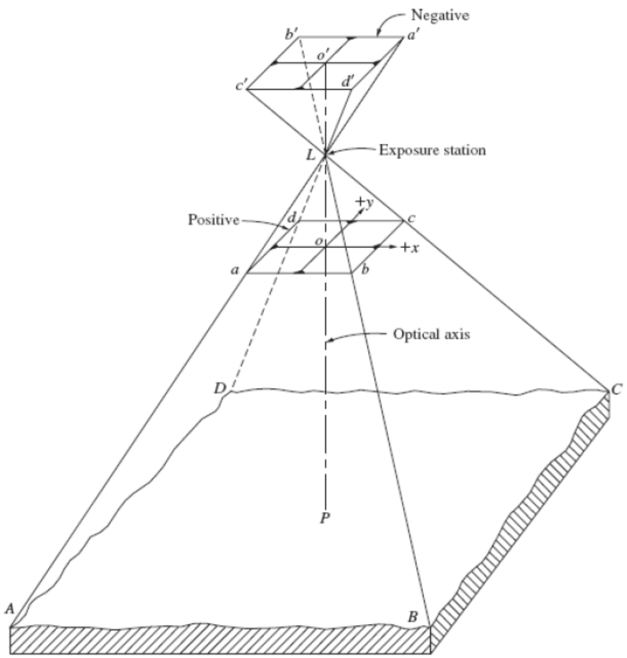

Figure 6-1. The geometry of a vertical photograph.

- ❖ Figure 6-1 illustrates the geometry of a vertical photograph taken from an exposure station L
- ❖ The ***negative***, which is a reversal in both tone and geometry of the object space, is situated a distance equal to the focal length (distance $o'L$ in Fig. 6-1) above the rear nodal point of the camera lens
- ❖ The ***positive*** may be obtained by direct emulsion-to-emulsion "contact printing" with the negative
- ❖ This process produces a reversal of tone and geometry from the negative, and therefore the tone and geometry of the positive are exactly the same as those of the object space
- ❖ Geometrically the plane of a contact-printed positive is situated a distance equal to the focal length (distance oL in Fig. 6-1) below the front nodal point of the camera lens

6-1. Geometry of Vertical Photographs

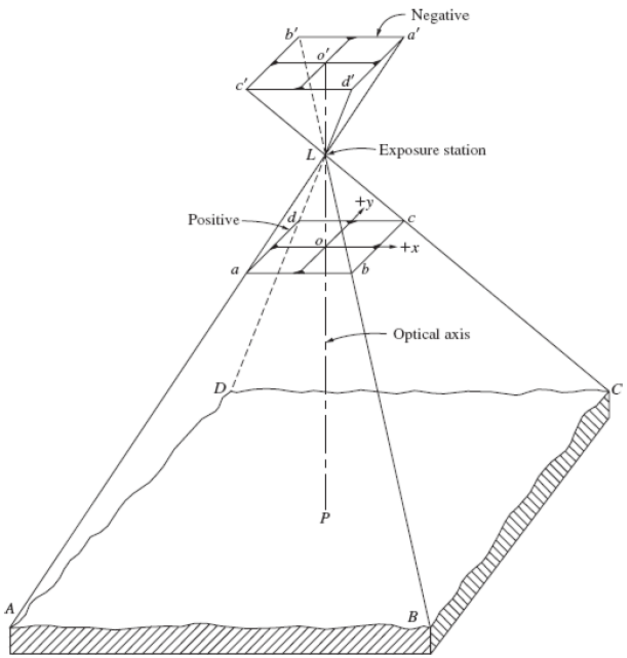

Figure 6–1. The geometry of a vertical photograph.

- The same is true for an image obtained with a frame-type digital camera
- The reversal in geometry from object space to negative is readily seen in Fig. 6-1 by comparing the positions of object points $A, B, C,$ and D with their corresponding negative positions a', b', c', and d'
- The correspondence of the geometry of the object space and the positive is also readily apparent
- The photographic coordinate axes x and y are shown on the positive of Fig. 6-1

6-2. Scale

- *Map scale* is ordinarily interpreted as the ratio of a map distance to the corresponding distance on the ground
- In a similar manner, the *scale of a photograph* is the ratio of a distance on the photo to the corresponding distance on the ground
- Due to the nature of map projections, map scale is not influenced by terrain variations
- A vertical aerial photograph, however, is a perspective projection, and its scale varies with variations in terrain elevation
- Scales may be represented as *unit equivalents, unit fractions, imensionless representative fractions*, or *dimensionless ratios*
- For example, if 1 inch (in) on a map or photo represents 1000 ft (12,000 in) on the ground, the scale expressed in the aforementioned four ways is

> * By convention, the first term in a scale expression is always chosen as 1
> 1. Unit equivalents: 1 in = 1000 ft
> 2. Unit fraction: 1 in/1000 ft
> 3. Dimensionless representative fraction: 1/12,000
> 4. Dimensionless ratio: 1:12,000

- A large number in a scale expression denotes a small scale, and vice versa;
  ⇨ for example, 1:1000 is a larger scale than 1:5000.

6-3. Scale of a Vertical Photograph Over Flat Terrain

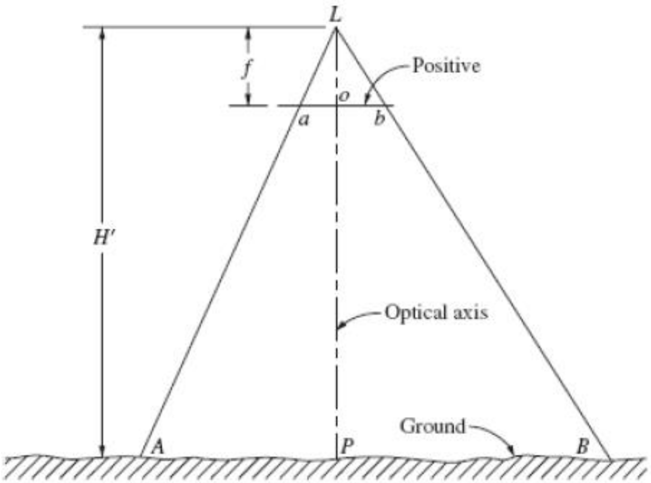

Figure 6-2. Two-dimensional view of a vertical photograph taken over flat terrain.

- Figure 6-2 shows the side view of a vertical photograph taken over flat terrain
- The scale of a vertical photograph over flat terrain is simply the ratio of photo distance ab to corresponding ground distance AB
- That scale may be expressed in terms of camera focal length f and flying height above ground H' by equating similar triangles Lab and LAB as follows:

$$S = \frac{ab}{AB} = \frac{f}{H'} \qquad (6\text{-}1)$$

- From Eq. (6-1) it is seen that the scale of a vertical photo is directly proportional to camera focal length (image distance) and inversely proportional to flying height above ground (object distance)

6-4. Scale of a Vertical Photograph Over Variable Terrain

❖ For any given vertical photo scale increases with increasing terrain elevation and decreases with decreasing terrain elevation

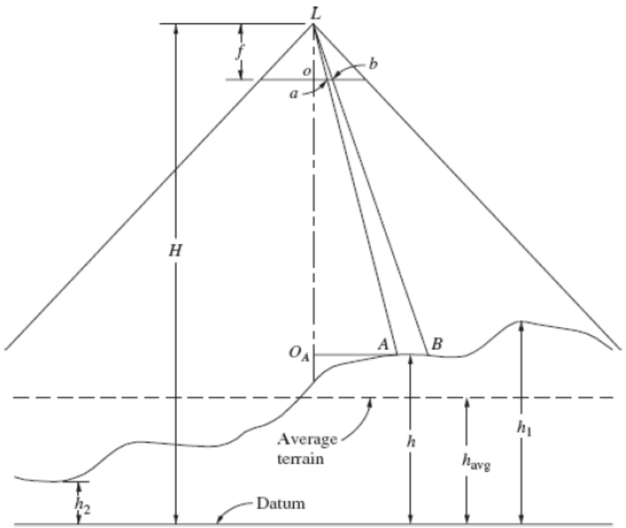

Figure 6–3. Scale of a vertical photograph over variable terrain.

❖ Suppose a vertical aerial photograph is taken over variable terrain from exposure station L of Fig. 6-3

❖ Ground points A and B are imaged on the positive at a and b, respectively

❖ Photographic scale at h, the elevation of points A and B, is equal to the ratio of photo distance ab to ground distance AB

❖ By similar triangles Lab and LAB, an expression for photo scale S_{AB} is

$$S_{AB} = \frac{ab}{AB} = \frac{La}{LA} \qquad \text{(a)}$$

❖ Also, by similar triangles LO_AA and Loa,

$$\frac{La}{LA} = \frac{f}{H-h} \qquad \text{(b)}$$

Substituting Eq. (b) into Eq. (a) gives

$$S_{AB} = \frac{ab}{AB} = \frac{La}{LA} = \frac{f}{H-h} \qquad \text{(c)}$$

6-4. Scale of a Vertical Photograph Over Variable Terrain

- Considering line AB to be infinitesimal, we see that Eq. (c) reduces to an expression of photo scale at a point

- In general, by dropping subscripts, the scale at any point whose elevation above datum is h may be expressed as

$$S = \frac{f}{H - h} \qquad (6\text{-}2)$$

- In Eq. (6-2), the denominator $H - h$ is the object distance

- In this equation, as in Eq. (6-1), scale of a vertical photograph is seen to be simply the ratio of image distance to object distance

$$S = \frac{ab}{AB} = \frac{f}{H'} \qquad (6\text{-}1)$$

- The shorter the object distance (the closer the terrain to the camera), the greater the photo scale, and vice versa

- For vertical photographs taken over variable terrain, there are an infinite number of different scales
  - ⇨ <u>The principal differences between a photograph and a map</u>

6-5. Average Photo Scale

- It is often convenient and desirable to use an ***average scale*** to define the overall mean scale of a vertical photograph taken over variable terrain

- Average scale is the scale at the average elevation of the terrain covered by a particular photograph and is expressed as

$$S_{avg} = \frac{f}{H - h_{avg}} \qquad (6\text{-}3)$$

- When an average scale is used, it must be understood that it is exact only at those points that lie at average elevation, and it is an approximate scale for all other areas of the photograph

- In each of Eqs. (6-1), (6-2), and (6-3), it is noted that Flying height appears in the denominator ⇨ Thus, for a camera of a given focal length, if flying height increases, object distance $H - h$ increases and scale decreases

6-5. Average Photo Scale

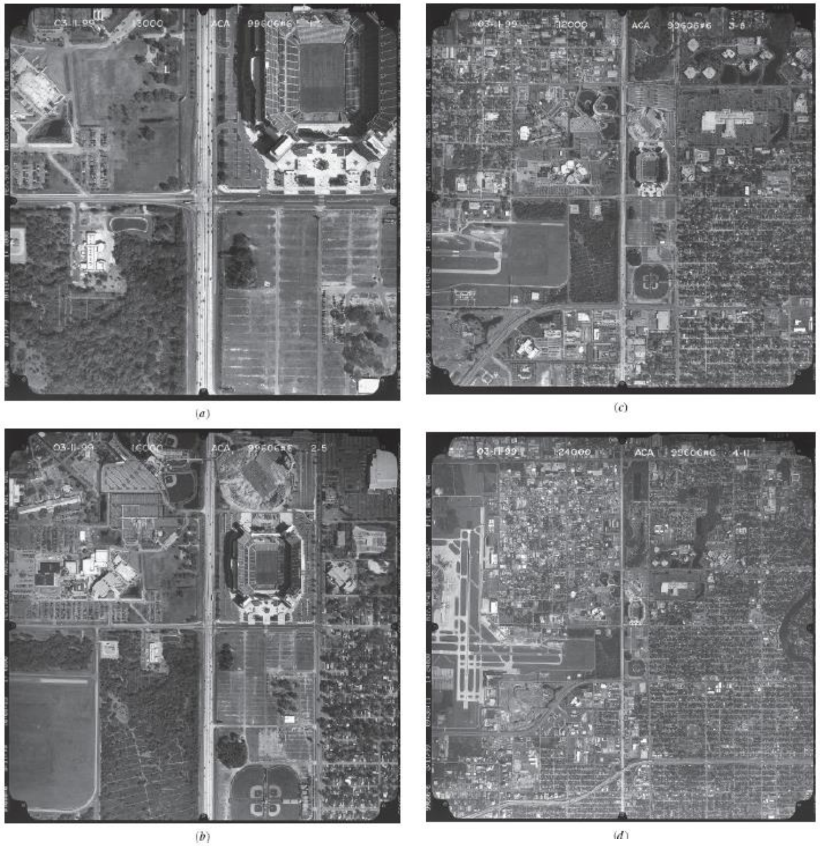

Figure 6–4. Four vertical photos taken over Tampa, Florida, illustrating scale variations due to changing flying heights. (Courtesy Aerial Cartographics of America, Inc.)

- Figures 6-4a through d illustrate this principle vividly
- Each of these vertical photos was exposed using the very same 23-cm format and 152-mm-focal-length camera
- The photo of Fig.6-4a had a flying height of 460m above ground, resulting in an average photo scale of 1:3000
- The photos of Fig. 6-4b, c, and d had flying heights above average ground of 910 m, 1830 m, and 3660 m, respectively, producing average photo scales of 1:6000, 1:12,000, and 1:24,000, respectively.

6-6. Other Methods of Determining Scale of Vertical Photographs

- ❖ A ground distance may be measured in the field between two points whose images appear on the photograph
- ❖ After the corresponding photo distance is measured, the scale relationship is simply the ratio of the photo distance to the ground distance
- ❖ The resulting scale is exact only at the elevation of the ground line, and if the line is along sloping ground, the resulting scale applies at approximately the average elevation of the two endpoints of the line
- ❖ The scale of a vertical aerial photograph may also be determined if a map covering the same area as the photo is available
- ❖ In this method it is necessary to measure, on the photograph and on the map, the distances between two welldefined points that can be identified on both photo and map
- ❖ Photographic scale can then be calculated from the following equation:

$$S = \frac{photo\ distance}{map\ distance} \times map\ scale \qquad (6\text{-}4)$$

- ❖ The scale of a vertical aerial photograph can also be determined without the aid of a measured ground distance or a map if lines whose lengths are known by common knowledge appear on the photo

6-7. Ground Coordinates from a Vertical Photograph

❖ The ground coordinates of points whose images appear in a vertical photograph can be determined with respect to an arbitrary ground coordinate system.

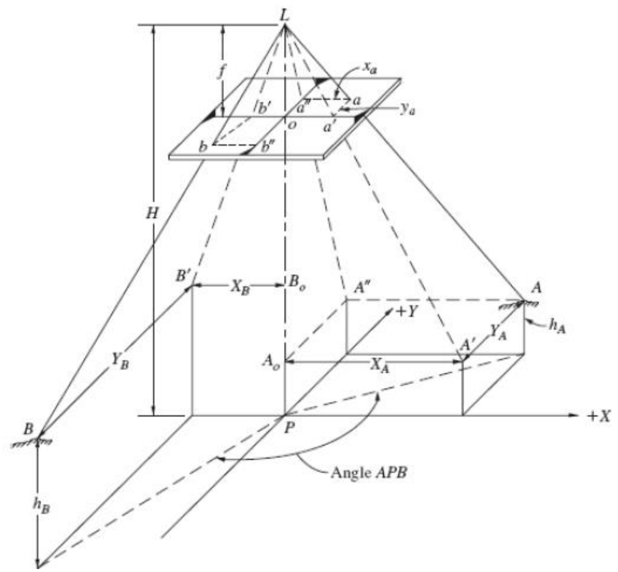

Figure 6–5. Ground coordinates from a vertical photograph.

❖ The arbitrary X and Y ground axes are in the same vertical planes as the photographic x and y axes, and the origin of the system is at the datum principal point (point in the datum plane vertically beneath the exposure station)

❖ Figure 6-5 shows a vertical photograph taken at a flying height H above datum

- ❖ Images a and b of the ground points A and B appear on the photograph, and their measured photographic coordinates are x_a, y_a, x_b, y_b

❖ The arbitrary ground coordinate axis system is X and Y, and the coordinates of points A and B in that system are X_A, Y_A, X_B, Y_B

❖ From similar triangles $La'o$ and $LA'A_o$, the following equation may be written:

$$\frac{oa''}{A_oA''} = \frac{f}{H - h_A} = \frac{y_a}{X_A} \qquad \text{from which} \qquad X_A = x_a\left(\frac{H - h_A}{f}\right) \qquad (6\text{-}5)$$

6-7. Ground Coordinates from a Vertical Photograph

- Also, from similar triangles $La''o$ and $LA''A_o$,

$$\frac{oa''}{A_oA''} = \frac{f}{H - h_A} = \frac{y_a}{Y_A} \quad \text{from which} \quad Y_A = y_a\left(\frac{H - h_A}{f}\right) \qquad \text{(6-6)}$$

- Similarly, the ground coordinates of point B are

$$X_B = x_b\left(\frac{H - h_B}{f}\right) \qquad \text{(6-7)}$$

$$Y_B = y_b\left(\frac{H - h_B}{f}\right) \qquad \text{(6-8)}$$

- Upon examination of Eqs. (6-5) through (6-8), it is seen that X and Y ground coordinates of any point are obtained by simply multiplying x and y photo coordinates by the inverse of photo scale at that point

- From the ground coordinates of the two points A and B, the horizontal length of line AB can be calculated, using the pythagorean theorem, as

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \qquad \text{(6-9)}$$

- Also, horizontal angle APB may be calculated as

$$APB = 90° + \tan^{-1}\left(\frac{X_B}{Y_B}\right) + \tan^{-1}\left(\frac{Y_A}{X_A}\right) \qquad \text{(6-10)}$$

6-7. Ground Coordinates from a Vertical Photograph

$$X_A = x_a\left(\frac{H - h_A}{f}\right) \quad (6\text{-}5) \qquad Y_A = y_a\left(\frac{H - h_A}{f}\right) \quad (6\text{-}6) \qquad X_B = x_b\left(\frac{H - h_B}{f}\right) \quad (6\text{-}7) \qquad Y_B = y_b\left(\frac{H - h_B}{f}\right) \quad (6\text{-}8)$$

- To solve Eqs. (6-5) through (6-8) it is necessary to know the camera focal length, flying height above datum, elevations of the points above datum, and photo coordinates of the points
- The photo coordinates are readily measured, camera focal length is commonly known from camera calibration, and flying height above datum is calculated by methods
- Elevations of points may be obtained directly by field measurements, or they may be taken from available topographic maps
- Ground coordinates calculated by Eqs. (6-5) through (6-8) are in an abritrary rectangular coordinate system, as previously described
- If arbitrary coordinates are calculated for two or more "control" points (points whose coordinates are also known in an absolute ground coordinate system such as the state plane coordinate system), then the arbitrary coordinates of all other points for that photograph can be transformed to the ground system
- Using Eqs. (6-5) through (6-8), an entire planimetric survey of the area covered by a vertical photograph can be made

6-8. Relief Displacement on a Vertical Photograph

- *Relief displacement* is the shift or displacement in the photographic position of an image caused by the relief of the object, i.e., its elevation above or below a selected datum
- With respect to a **datum**, **relief displacement** is outward for points whose elevations are above datum and inward for points whose elevations are below datum

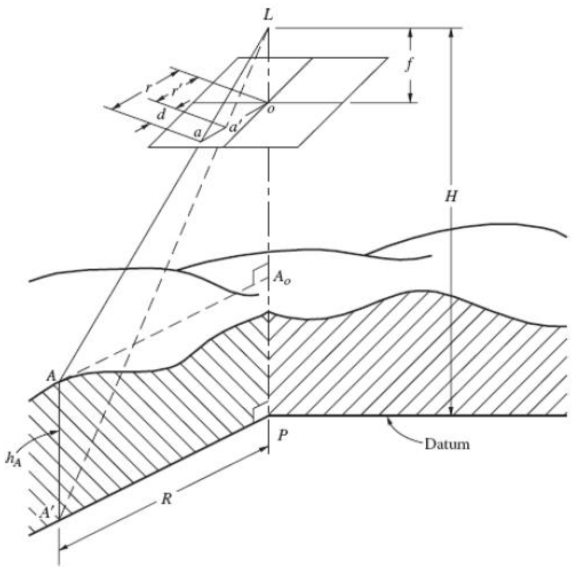

Figure 6-6. Relief displacement on a vertical photograph.

- The concept of relief displacement is illustrated in Fig. 6-6, which represents a vertical photograph taken from flying height H above datum
- Camera focal length is f , and o is the principal point
- The image of terrain point A, which has an elevation h_A above datum, is located at a on the photograph
- An imaginary point A' is located vertically beneath A in the datum plane, and its corresponding imaginary image position is at a'
- On the figure, both $A'A$ and PL are vertical lines, and therefore $A'AaLoP$ is a vertical plane, and Plane $A'a'LoP$ is also a vertical plane which is coincident with $A'AaLoP$
- Since these planes intersect the photo plane along lines oa and oa', respectively, line aa' (relief displacement of point A due to its elevation h_A) is radial from the principal point

6-8. Relief Displacement on a Vertical Photograph

❖ An equation for evaluating relief displacement may be obtained by relating similar triangles

❖ First consider planes Lao and LAA_o in Fig. 6-6:

$$\frac{r}{R} = \frac{f}{H - h_A} \quad \text{(d)} \qquad \text{Rearranging gives} \quad r(H - h_A) = fR$$

❖ Also, from similar triangles $La'o$ and $LA'P$,

$$\frac{r'}{R} = \frac{f}{H} \quad \text{(e)} \qquad \text{or} \qquad r'H = fR$$

❖ Equating expressions (d) and (e) yields

$$r(H - h_A) = r'H$$

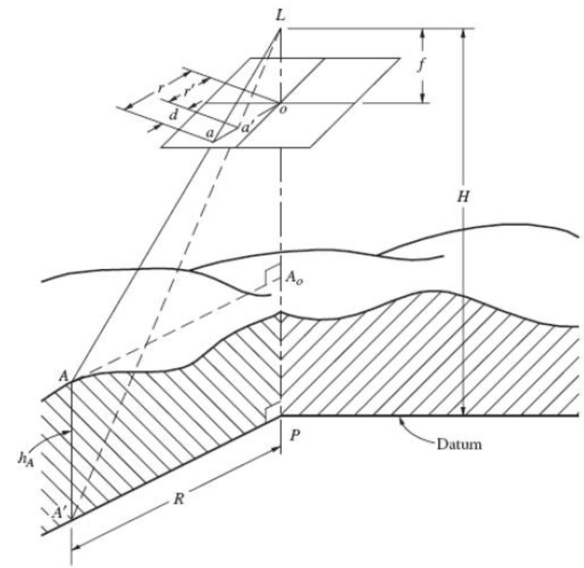

Figure 6–6. Relief displacement on a vertical photograph.

❖ Rearranging the above equation, dropping subscripts, and substituting the symbol d for $r – r'$ gives

$$d = \frac{rh}{H} \qquad \text{(6-11)}$$

where d=relief displacement
h = height above datum of object point whose image is displaced
r = radial distance on photograph from principal point to displaced image
(The units of d and r must be the same.)
H = flying height above same datum selected for measurement of h

6-8. Relief Displacement on a Vertical Photograph

$$d = \frac{rh}{H} \qquad (6\text{-}11)$$

where d=relief displacement

h = height above datum of object point whose image is displaced

r = radial distance on photograph from principal point to displaced image (The units of d and r must be the same.)

H = flying height above same datum selected for measurement of h

- Equation (6-11) is the basic relief displacement equation for vertical photos
- Examination of this equation shows that relief displacement increases with increasing radial distance to the image, and it also increases with increased elevation of the object point above datum
- On the other hand, relief displacement decreases with increased flying height above datum
- It has also been shown that relief displacement occurs radially from the principal point

6-8. Relief Displacement on a Vertical Photograph

Figure 6–7. Vertical photograph of Tampa, Florida, illustrating relief displacements. (Courtesy US Imaging, Inc.)

- Figure 6-7 is a vertical aerial photograph which vividly illustrates relief displacement
- Note in particular the striking effect of **relief displacement** on the tall buildings in the upper portion of the photo
- Notice also that the **relief displacement** occurs radially from the center of the photograph (principal point)
- This radial pattern is also readily apparent for the relief displacement of all the other vertical buildings in the photo
- The building in the center is one of the tallest imaged on the photo (as evidenced by the length of its shadow); however, its relief displacement is essentially zero due to its proximity to the principal point.

6-8. Relief Displacement on a Vertical Photograph

Figure 6–7. Vertical photograph of Tampa, Florida, illustrating relief displacements. (Courtesy US Imaging, Inc.)

- Figure 6-7 is a vertical aerial photograph which vividly illustrates relief displacement
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- This radial pattern is also readily apparent for the relief displacement of all the other vertical buildings in the photo

- Relief displacement often causes straight roads, fence lines, etc., on rolling ground to appear crooked on a vertical photograph
- The severity of the crookedness will depend on the amount of terrain variation
- Relief displacement causes some imagery to be obscured from view
- Several examples of this are seen in Fig. 6-7; e.g., the street in the upper portion of the photo is obscured by relief displacement of several tall buildings adjacent to it

6-8. Relief Displacement on a Vertical Photograph

❖ Vertical heights of objects such as buildings, poles, etc., appearing on aerial photographs can be calculated from relief displacements

❖ For this purpose, Eq. (6-11) is rearranged as follows:

$$d = \frac{rh}{H} \qquad (6\text{-}11) \qquad \Rightarrow \qquad h = \frac{dH}{r} \qquad (6\text{-}12)$$

❖ To use Eq. (6-12) for height determination, it is necessary that the images of both the top and bottom of the vertical object be visible on the photograph, so that d can be measured

❖ Datum is arbitrarily selected at the base of the vertical object

❖ Equation (6-12) is of particular import to the photo interpreter, who is often interested in relative heights of objects rather than absolute elevations

6-9. Flying Height of a Vertical Photograph

- It is apparent that **flying height** above datum is an important quantity which is often needed for solving basic photogrammetric equations
- For rough computations, flying height may be taken from altimeter readings
- Flying heights also may be obtained by using either Eq. (6-1) or Eq. (6-2) if a ground line of known length appears on the photograph
- In general, <u>the greater the difference in elevation of the endpoints, the greater the error in the computed flying height</u>
  - ⇨ Therefore the ground line should lie on fairly level terrain

6-9. Flying Height of a Vertical Photograph

- Accurate flying heights can be determined even though the endpoints of the ground line lie at different elevations, regardless of the locations of the endpoints in the photo
- This procedure requires knowledge of the elevations of the endpoints of the line as well as of the length of the line
- Suppose ground line AB has its endpoints imaged at a and b on a vertical photograph
- Length AB of the ground line may be expressed in terms of ground coordinates, by the pythagorean theorem, as follows:

  $$(AB)^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2$$

  Substituting Eqs. (6-5) through (6-8) into the previous equation gives

  $$(AB)^2 = \left[\frac{x_b}{f}(H - h_B) - \frac{x_a}{f}(H - h_A)\right]^2 + \left[\frac{y_b}{f}(H - h_B) - \frac{y_a}{f}(H - h_A)\right]^2 \qquad \text{(6-13)}$$

- The only unknown in Eq. (6-13) is the flying height H
- When all known values are inserted into the equation, it reduces to the quadratic form of $aH^2 + bH + c = 0$
- The direct solution for H in the quadratic is

  $$H = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(6-14)}$$

6-10. Error Evaluation

- ❖ Answers obtained in solving the various equations presented in this chapter will inevitably contain errors
- ❖ It is important to have an awareness of the presence of these errors and to be able to assess their approximate magnitudes
- ❖ Some of the more significant sources of errors in calculated values are
  1) Errors in photographic measurements, e.g., line lengths or photo coordinates
  2) Errors in ground control
  3) Shrinkage and expansion of film and paper
  4) Tilted photographs where vertical photographs were assumed
- ❖ Sources1 and 2 can be minimized if precise, properly calibrated equipment and suitable caution are used in making the measurements
- ❖ Source3 can be practically eliminated by making corrections
- ❖ Magnitudes of error introduced by Source4 depend upon the severity of the tilt
- ❖ For the methods described in this chapter, errors caused by lens distortions and atmospheric refraction are relatively small and can generally be ignored

6-10. Error Evaluation

- A simple and straightforward approach to calculating the combined effect of several random errors is to use **statistical error propagation**
- This approach involves calculating rates of change with respect to each variable containing error and requires the use of differential calculus
- As an example of this approach, assume that a vertical photograph was taken with a camera having a focal length of 152.4 mm
- Assume also that a ground distance AB on flat terrain has a length of 1524 m and that its corresponding photo distance ab measures 127.0 mm
- Flying height above ground may be calculated, using Eq. (6-1), as follows:

$$H' = f\left(\frac{AB}{ab}\right) = 152.4\left(\frac{1524}{127.0}\right) = 1829m$$

- It is required to calculate the expected error dH' caused by errors in measured quantities AB and ab
- Suppose that the error σ_{AB} in the ground distance is ±0.50 m and that the error σ_{ab} in the measured photo distance is ±0.20 mm

6-10. Error Evaluation

- The rate of change of error in H' caused by the error in the ground length can be evaluated by taking the partial derivative $\partial H'/\partial AB$ as

$$\frac{\partial H'}{\partial AB} = \frac{f}{ab} = \frac{152.4mm}{127.0mm} = 1.200$$

- The rate of change of error in H' caused by the error in the measured image length can be evaluated by taking the partial derivative $\partial H'/\partial ab$ as

$$\frac{\partial H'}{\partial ab} = \frac{-f(AB)}{(ab)^2} = \frac{-152.4mm(1524m)}{(127.0mm)^2} = -14.40m/mm$$

- A useful interpretation of these derivative terms is that an error of 1m in ground distance AB will cause an error of approximately 1.2m in the flying height, whereas an error of 1mm in image distance ab will cause an error of approximately 14m in the flying height

- Substitution of these derivative terms into the error propagation along with the error terms σ_{AB} and σ_{ab} gives

$$\sigma_{H'} = \sqrt{(1.200)^2(0.50m)^2+(-14.40m/mm)^2(0.20mm)^2} = \sqrt{0.36m^2 + 8.29m^2} = \pm 2.9m$$

- Note that the error in H' caused by the error in the measurement of photo distance ab is the more severe of the two contributing sources

- Therefore, to increase the accuracy of the computed value of H', it would be more beneficial to refine the measured photo distance to a more accurate value