Chapter 7. Stereoscopic Viewing

7-1. Depth Perception



Figure 7-1. Depth perception by relative sizes and hidden objects.

- Methods of judging depth may be classified as either *stereoscopic* or *monoscopic*
- Persons with normal vision (those capable of viewing with both eyes simultaneously) are said to have *binocular* vision, and <u>perception of depth through binocular vision</u> is called *stereoscopic viewing*
- Monocular vision is the term applied to viewing with only one eye, and methods of judging distances with one eye are termed monoscopic
- A person having normal binocular vision can, of course, view monocularly by covering one eye

7-1. Depth Perception



Figure 7-1. Depth perception by relative sizes and hidden objects.

- Distances to objects, or depths, can be perceived monoscopically on the basis of:
 - 1) relative sizes of objects
 - 2) hidden objects
 - 3) shadows
 - 4) differences in focusing of the eye required for viewing objects at varying distances
- Examples of the first two are shown in Fig. 7-1
 - Depth to the far end of the football field may be perceived on the basis of the relative sizes of the goalposts
 - The goalposts are actually the same size, of course, but one appears smaller because it is farther away
 - The stands at the far end of the stadium are quickly judged to be a considerable distance away from the goalposts in the foreground because they are partially obstructed by the goalposts

7-1. Depth Perception



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- Distances to objects, or depths, can be perceived monoscopically on the basis of:
 - 1) relative sizes of objects
 - 2) hidden objects
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 - 4) differences in focusing of the eye required for viewing objects at varying distances
- Stereoscopic depth perception is of fundamental importance in photogrammetry, for <u>it</u> enables the formation of a three-dimensional stereomodel by viewing a pair of overlapping photographs
- ✤ The stereomodel can then be studied, measured, and mapped

7-2. The Human Eye

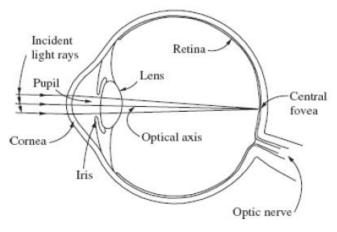
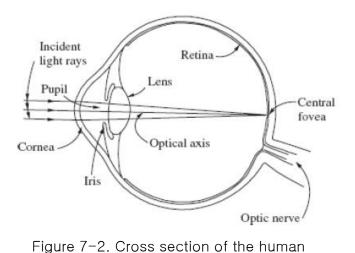


Figure 7-2. Cross section of the human eye.

- As shown in Fig. 7-2, the eye is essentially a spherical organ having a circular opening called the *pupil*
- The pupil is protected by a transparent coating called the *cornea*
- Incident light rays pass through the cornea, enter the eye through the pupil, and strike the *lens*, which is directly behind the pupil
- ✤ The cornea and lens refract the light rays according to Snell's law
- ◆ As with a camera, the eye must satisfy the lens formula for each different object distance
- When a distant object is viewed, the lens muscles relax, causing the spherical surfaces of the lens to become flatter
- ✤ When close objects are viewed, a reverse procedure occurs
- ✤ The eye's ability to focus for varying object distances is called *accommodation*

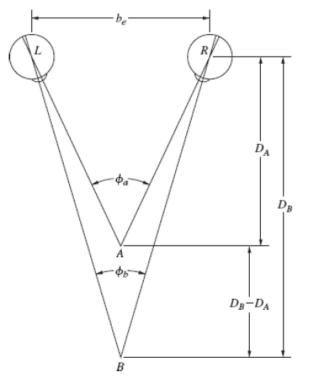
eye.

7-2. The Human Eye



- As with a camera, the eye has a diaphragm called the iris
- The iris (colored part of the eye) automatically contracts or expands to regulate the amount of light entering the eye
- The cornea partially refracts incident light rays before they encounter the lens
- The lens refracts them further and brings them to focus on the *retina*, thereby forming an image of the viewed object
- The most important region of the retina is the *central fovea*, a small pit near the intersection of the optical axis with the retina
- The central fovea is the area of sharpest vision and the retina performs a function similar to that performed by the emulsion of photographic film
- When it is stimulated by light, the sense of vision is caused, which is transmitted to the brain via the optic nerve

7-3. Stereoscopic Depth Perception

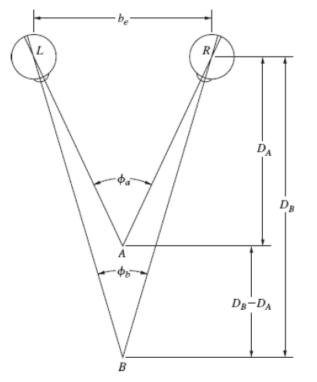


- With binocular vision, when the eyes fixate on a certain point, the optical axes of the two eyes converge on that point, <u>intersecting at an angle</u> called the *parallactic angle*
- The nearer the object, the greater the parallactic angle, and vice versa.
- In Fig. 7-3, the optical axes of the two eyes L and R are separated by a distance b_e , called the eye base
- When the eyes fixate on point A, the optical axes converge, forming parallactic angle ϕ_a

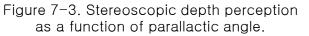
Figure 7-3. Stereoscopic depth perception as a function of parallactic angle.

- Similarly, when sighting an object at B, the optical axes converge, forming parallactic angle ϕ_b
- * The brain automatically and unconsciously associates distances D_A and D_B with corresponding parallactic angles ϕ_a and ϕ_b
- ✤ The depth between objects A and B is $D_B D_A$ and is perceived from the difference in these parallactic angles

7-3. Stereoscopic Depth Perception

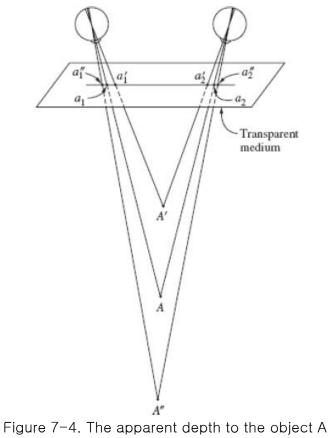


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- Similarly, when sighting an object at B, the optical axes converge, forming parallactic angle ϕ_b
- The ability of human beings to detect changes in parallactic angles, and thus judge differences in depth, is quite remarkable
- This means that photogrammetric procedures for determining heights of objects and terrain variations based on depth perception by comparisons of parallactic angles can be highly precise

7-4. Viewing Photographs Stereoscopically



- Suppose that while a person is gazing at object A of
 Fig. 7-4, a transparent medium containing image marks
 a₁ and a₂ is placed in front of the eyes as shown
- Assume further that the image marks are identical in shape to object *A*, and that they are placed on the optical axes so that the eyes are unable to detect whether they are viewing the object or the two marks
- As shown in Fig. 7-4, if the image marks are moved closer together to, say, a₁', and a₂', the parallactic angle increases and the object is perceived to be nearer the eyes at A'

can be changed by changing the spacing of the images.

If the marks are moved farther apart to a₁" and a₂", the parallactic angle decreases and the brain receives an impression that the object is farther away, at A"

7-4. Viewing Photographs Stereoscopically

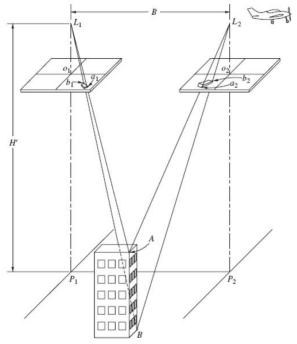
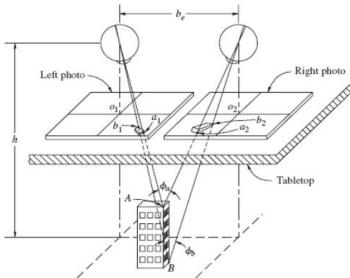


Figure 7-5. Photographs from two exposure stations with building in common overlap area.

- The phenomenon of creating the three-dimensional or stereoscopic impression of objects by viewing identical images of the objects can be achieved photographically
- Suppose that a pair of aerial photographs is taken from exposure stations L_1 and L_2 so that the building appears on both photos, as shown in Fig. 7-5
- ✤ Flying height above ground is H', and the distance between the two exposures is B, the air base
- Object points A and B at the top and bottom of the building are imaged at a₁ and b₁ on the left photo and at a₂ and b₂ on the right photo

7-4. Viewing Photographs Stereoscopically



- If the two photos are laid on a table and viewed so that the left eye sees only the left photo and the right eye sees only the right photo, as shown in Fig. 7-6, a three-dimensional impression of the building is obtained
- Tthree-dimensional impression appears to lie below the tabletop at a distance h from the eyes

Figure 7-6. Viewing the building stereoscopically.

- * The brain judges the height of the building by associating depths to points A and B with the parallactic angles ϕ_a and ϕ_b
- When the eyes gaze over the entire overlap area, the brain receives a continuous threedimensional impression of the terrain
- The three-dimensional model thus formed is called a *stereoscopic model* or simply a *stereomodel*, and <u>the overlapping pair of photographs</u> is called a *stereopair*

- One of the major problems associated with stereoviewing without optical aids is that <u>the eyes are focused on the photos</u>, while at the same time the brain perceives parallactic angles which tend to form the stereomodel at some depth beyond the photos
 - ⇒ Stereoscopic viewing may be achived through the use of instruments called stereoscopes
- The *lens* or *pocket stereoscope*, shown in Fig. 7-7, is the least expensive and most commonly used stereoscope

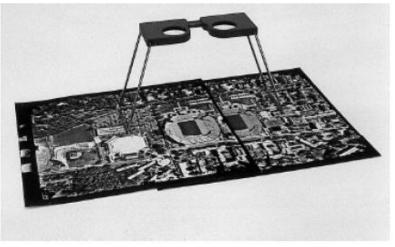


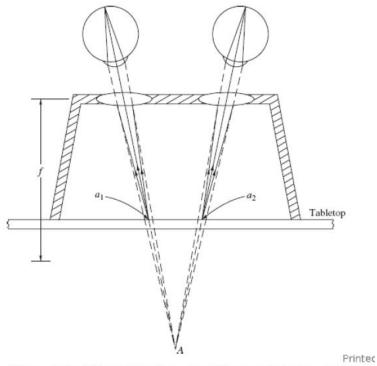
Figure 7–7. Lens or pocket stereoscope. (Courtesy University of Florida.)

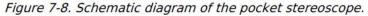
- The spacing between the lenses can be varied to accommodate various eye bases
- The legs fold or can be removed so that the instrument is easily stored or carried—a feature which renders the pocket stereoscope ideal for fieldwork

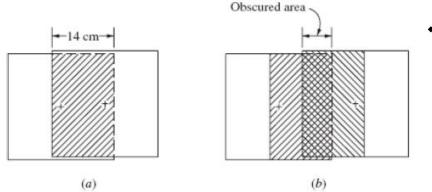
- A schematic diagram of the pocket stereoscope is given in Fig. 7-8
- The legs of the pocket stereoscope are slightly shorter than the focal length f of the lenses
- When the stereoscope is placed over the photos, light rays emanating from points such as a₁ and a₂ on the photos are refracted slightly as they pass through each lens
- The eyes receive the refracted rays, and on the basis of the eye focusing associated with these incoming rays, the brain receives the impression that the rays actually originate from a greater distance than that to the tabletop upon which the photos rest



 \Rightarrow The lenses also serve to magnify the images, thereby enabling details to be seen clearly







- For normal 23-cm-format photos taken with 60 percent end lap, the common overlap area of a pair of photos is a rectangular area about 14cm wide, as shown crosshatched in Fig. 7-9a
- Figure 7-9. (a) The common overlap area of a pair of 23cm-format photos taken with 60 percent end lap (corresponding images coincident). (b) Obscured area when photos are oriented for viewing with pocket stereoscope.
- If the photos are separated by 5 cm for stereoviewing with a pocket stereoscope, as shown in Fig. 7-9b, there is a rectangular area, shown double crosshatched, in which the top photo obscures the bottom photo, thereby preventing stereoviewing
- To overcome this problem, the top photo can be gently rolled up out of the way to enable viewing the corresponding imagery of the obscured area

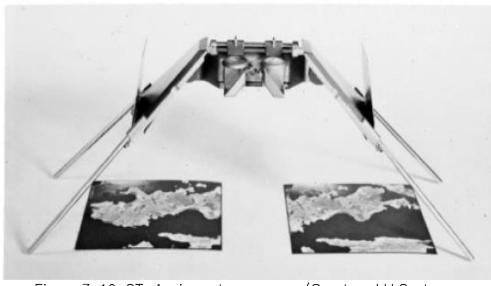


Figure 7-10. ST-4 mirror stereoscope. (Courtesy LH Systems, LLC.)

- The *mirror stereoscope* shown in Fig. 7-10 permits the two photos to be completely separated when viewed stereoscopically
- This <u>eliminates the problem of one photo obscuring part of the overlap of the other</u>, and it also <u>enables the entire width of the stereomodel to be viewed simultaneously</u>

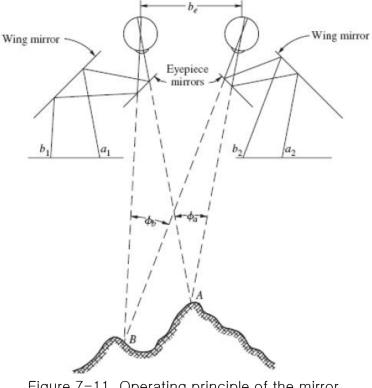


Figure 7–11. Operating principle of the mirror stereoscope.

- The operating principle of the mirror stereoscope is illustrated in Fig. 7-11
- The stereoscope has two large wing mirrors and two smaller eyepiece mirrors, all of which are mounted at 45° to the horizontal
- * Light rays emanating from image points on the photos such as a_1 and a_2 are reflected from the mirror surfaces, according to the principles of reflection, and are received at the eyes, forming parallactic angle ϕ_a
- The brain automatically associates the depth to point A with that parallactic angle
- ✤ The stereomodel is thereby created beneath the eyepiece mirrors, as illustrated in Fig. 7-11
- Mirror stereoscopes may be equipped with binoculars which fasten over the eyepiece mirrors
- The binoculars, which may be focused individually to accommodate each eye, permit viewing images at high magnification

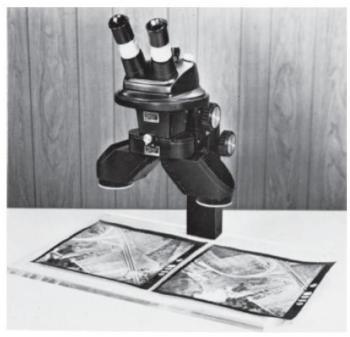
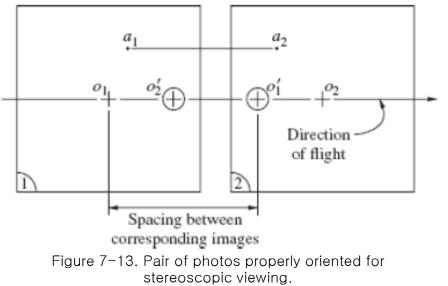


Figure 7-12. Zoom 95 stereoscope. (Courtesy Bausch and Lomb Co.)

- A different type of stereoscope called the *zoom* stereoscope is shown in Fig. 7-12
- A variety of these instruments are manufactured, affording a choice of special features such as continuous zoom magnification up to 120X, capability of rotating images optically (which permits convenient correction for crab or alignment), and differential enlargement so that two photos of different scales can be viewed stereoscopically

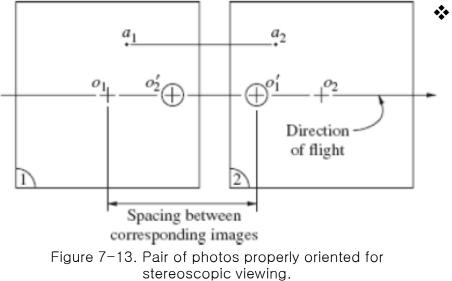
 For direct stereoscopic viewing of film negatives, <u>these stereoscopes may be obtained</u> <u>mounted on a light table and equipped with a special scanning mechanism</u>

- In stereoscopic viewing, it is important to orient the photos so that the left and right eyes see the left and right photos, respectively
- If <u>the photos are viewed in reverse</u>, a *pseudoscopic view* results in which ups and downs are reversed
- Accurate and comfortable stereoscopic viewing requires that the eye base, the line joining the centers of the stereoscope lenses, and the flight line all be parallel

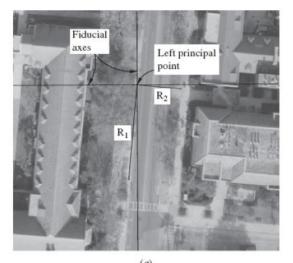


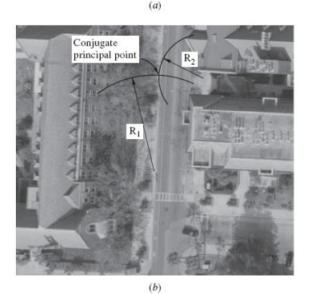
- In marking the flight line, the photo centers (principal points) are first located by joining opposite fiducial marks with straight lines
- Principal points are shown at o₁ and o₂ on
 Fig. 7-13

- In stereoscopic viewing, it is important to orient the photos so that the left and right eyes see the left and right photos, respectively
- If the photos are viewed in reverse, a pseudoscopic view results in which ups and downs are reversed; e.g., valleys appear as ridges and hills appear as depressions
- Accurate and comfortable stereoscopic viewing requires that the eye base, the line joining the centers of the stereoscope lenses, and the flight line all be parallel



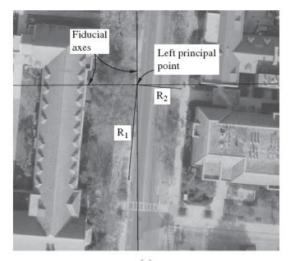
Corresponding principal points (also called conjugate principal points), which are the locations of principal points on adjacent overlapping photos, are marked next
 ⇒ This may be done satisfactorily by carefully observing images immediately surrounding the principal points, and then marking the corresponding principal points by estimating their positions with respect to these surrounding images





- If the area is level and there are identifiable features in the photos, the method illustrated in Fig. 7-14 can be used to find the conjuagte principal point with acceptable accuracy
- In Fig. 7-14a, intersecting lines joining opposite fiducial marks define the principal point of the left photo
- Since there is no distinct feature located at this intersection, distances R₁ and R₂ are measured to nearby features at approximately the same elevation
- ✤ Figure 7-14b shows the corresponding area on the right photo, where arcs centered on these features and having radii R₁ and R₂ are intersected to obtain the conjugate principal point

Figure 7-14. (a) Center portion of left photo showing its principal point and distances to two manhole covers.(b) Intersection of corresponding distances to the same features at the conjugate principal point in the right



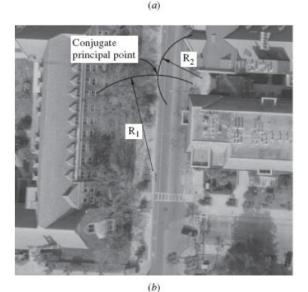
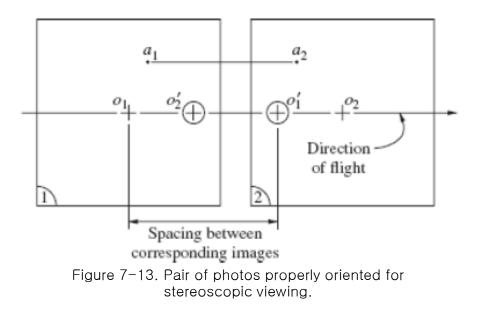
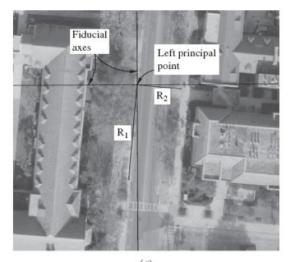


Figure 7-14. (a) Center portion of left photo showing its principal point and distances to two manhole covers.(b) Intersection of corresponding distances to the same features at the conjugate principal point in the right

- * The corresponding principal points are shown at o_1' and o_2' on Fig. 7-13
- The next step in orienting a pair of photos for stereoscopic viewing is to fasten the left photo down onto the table
- ✤ Then the right photo is oriented so that the four points defining the flight line (o₁, o₂', o₁', and o₂) all lie along a straight line, as shown in Fig. 7-13



Stereoscopic Viewing



- The right photo is retained in this orientation, and while being viewed through the stereoscope, it is moved sideways until the spacing between corresponding images produces a comfortable stereoscopic view
- Normally the required spacing between corresponding images is slightly more than 5 cm for a pocket stereoscope and about 25 cm for a mirror stereoscope

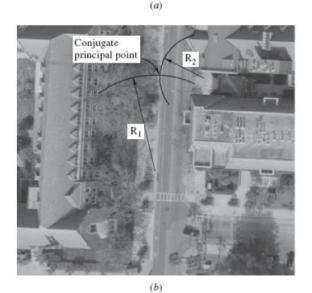
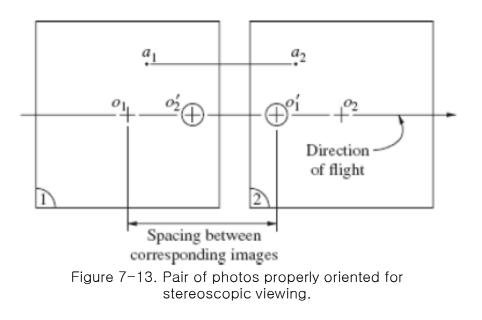
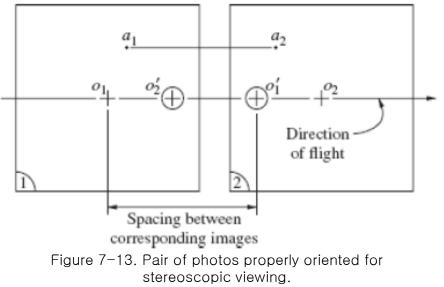


Figure 7-14. (a) Center portion of left photo showing its principal point and distances to two manhole covers.(b) Intersection of corresponding distances to the same features at the conjugate principal point in the right



Stereoscopic Viewing



- For casual stereoviewing, the geometry shown in Fig. 7-13 is normally achieved by a trial method in which the photos are simply shifted in position until a clear stereoscopic view is obtained
- If accuracy and eye comfort are considerations, however, orientation by the flight-line procedure is recommended
- Once the photos are properly oriented, the operator can easily align the stereoscope by simply rotating it slightly until the most comfortable viewing position is obtained
- The operator should look directly into the centers of the lenses, thereby holding the eye base parallel with the flight line

7-7. Causes of *Y* Parallax

- An essential condition which must exist for clear and comfortable stereoscopic viewing is that the line joining corresponding images be parallel with the direction of flight
- When corresponding images fail to lie along a line parallel to the flight line, y parallax, denoted by p_y , is said to exist
- Any slight amount of y parallax causes eyestrain, and excessive amounts prevent stereoscopic viewing altogether
- If a pair of truly vertical overlapping photos taken from equal flying heights is oriented perfectly, then no y parallax should exist anywhere in the overlap area

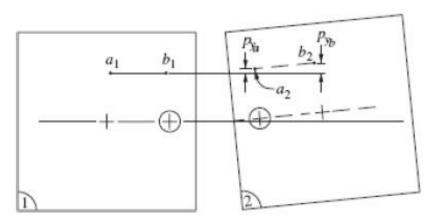


Figure 7–15. Here y parallax is caused by improper orientation of the photos.

- Failure of any of these conditions to be satisfied will cause y parallax
- In Fig. 7-15, for example, the photos are improperly oriented, and the principal points and corresponding principal points do not lie on a straight line
- As a result, *y* parallax exists at both points
 a and *b* ⇒ This condition can be prevented
 by careful orientation

7-7. Causes of *Y* Parallax

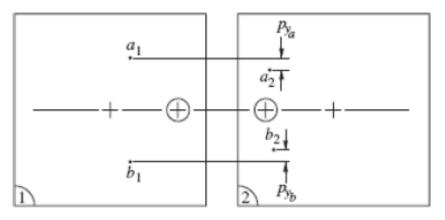


Figure 7-16. The y parallax is caused by variation in flying height.

- In Fig. 7-16 the left photo was exposed from a lower flying height than the right photo, and consequently its scale is larger than the scale of the right photo
- Even though the photos are truly vertical and properly oriented, y parallax exists at both points a and b due to variation in flying heights
- To obtain a comfortable stereoscopic view, the <u>y</u> parallax can be eliminated by sliding the right photo upward transverse to the flight line when viewing point a and sliding it downward when viewing point b

7-7. Causes of *Y* Parallax

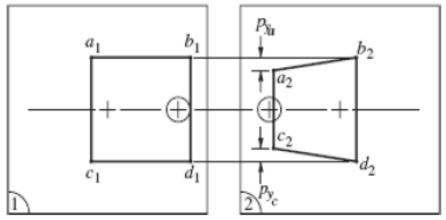


Figure 7-17. The y parallax is caused by tilt of the photos.

- The effect of tilted photos is illustrated in Fig. 7-17
- The left photo is truly vertical and shows positions of images *a* through *d* of a square parcel of property on flat terrain
- The right photo was tilted such that the same parcel appears as a trapezoid
- In this case, y parallax exists throughout the stereoscopic model as a result of the tilt, as indicated for points a and c
- In practice, the direction of tilt is random, and therefore small y parallaxes from this source are likely to exist in variable amounts throughout most stereomodels
- Most serious y parallaxes usually occur from improper orientation of the photos, a condition which can be easily corrected

- Under normal conditions, <u>the vertical scale of a stereomodel will appear to be greater</u> <u>than the horizontal scale</u>
- This apparent scale disparity is called vertical exaggeration
- Although other factors are involved, vertical exaggeration is caused primarily by the lack of equivalence of the photographic base-height ratio, B/H', and the corresponding stereoviewing base-height ratio, b_e/h
- * The term B/H' is the ratio of the *air base* (distance between the two exposure stations) to flying height above average ground, and b_e/h is the ratio of the *eye base* (distance between the two eyes) to the distance from the eyes at which the stereomodel is perceived

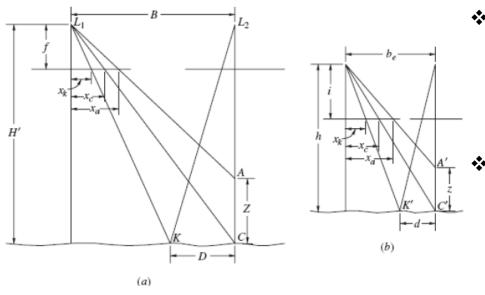


Figure 7-18. Simplistic diagrams for analyzing vertical exaggeration. (a) Geometry of overlapping aerial photography. (b) Geometry of stereoscopic viewing of the photos of part (a).

✤ In Fig. 7-18a, assume that Z is equal to D

- Figures 7-18a and b depict, respectively, the taking of a pair of vertical overlapping photographs and the stereoscopic viewing of those photos
- In Fig. 7-18a, the camera focal length is f, the air base is B, the flying height above ground is H', the height of ground object AC is Z, and the horizontal ground distance KC is D

- ✤ In Fig. 7-18b, *i* is the image distance from the eyes to the photos, *b_e* is the eye base, *h* is the distance from the eyes to the perceived stereomodel, *z* is the stereomodel height of object *A'C'*, and *d* is the horizontal stereomodel distance *K'C'*
- Note that while the ratio Z/D is equal to 1, the ratio z/d is greater than 1 due to vertical exaggeration

 An equation for calculating vertical exaggeration can be developed with reference to these figures

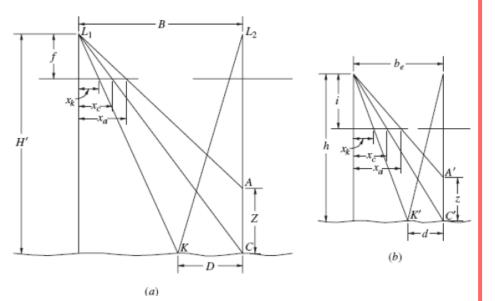


Figure 7–18. Simplistic diagrams for analyzing vertical exaggeration. (a) Geometry of overlapping aerial photography. (b) Geometry of stereoscopic viewing of the photos of part (a). From similar triangles of Fig. 7-18a, $\frac{x_a}{B} = \frac{f}{H' - Z}$ from which $x_a = \frac{Bf}{H' - Z}$ (a) $\frac{x_c}{B} = \frac{f}{H'}$ from which $x_c = \frac{Bf}{H'}$ (b)

Subtracting (b) from (a) and reducing gives

$$x_a - x_c = Bf \frac{Z}{(H')^2 - H'Z}$$
(c)

Also from similar triangles of Fig. 7-18b,

$$\frac{x_a}{b_e} = \frac{i}{h-z} \qquad \text{from which} \qquad x_a = \frac{b_e i}{h-z} \qquad \text{(d)}$$

$$\frac{x_c}{b_e} = \frac{i}{h} \qquad \text{from which} \qquad x_c = \frac{b_e i}{h} \qquad \text{(e)}$$
Subtracting (e) from (d) and reducing gives
$$x_a - x_c = b_e i \frac{z}{h^2 - hz} \qquad \text{(f)}$$

Stereoscopic Viewing

 An equation for calculating vertical exaggeration can be developed with reference to these figures

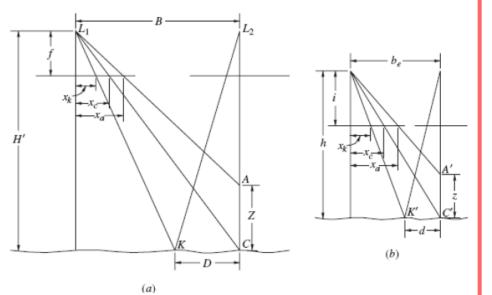


Figure 7-18. Simplistic diagrams for analyzing vertical exaggeration. (a) Geometry of overlapping aerial photography. (b) Geometry of stereoscopic viewing of the photos of part (a).

$$x_a - x_c = Bf \frac{Z}{(H')^2 - H'Z}$$
(c)

$$x_a - x_c = b_e i \frac{z}{h^2 - hz} \tag{f}$$

Equating (c) and (f) gives

$$Bf \frac{Z}{(H')^2 - H'Z} = b_e i \frac{Z}{h^2 - hz}$$

In the above equation, the values of Z and z are normally considereably smaller than the values of Hand h, respectively; thus

$$\frac{BfZ}{(H')^2} = \frac{b_e iz}{h^2} \quad \text{from which} \quad \frac{z}{Z} = \frac{fh}{H'i} \frac{Bh}{H'b_e} \quad \text{(g)}$$
Also from similar triangles of Fig. 7-18a, and b

$$\frac{x_c - x_k}{D} = \frac{f}{H'} \quad \text{from which} \quad D = (x_c - x_k) = \frac{h}{f} \quad \text{(h)}$$

and
$$\frac{x_c - x_k}{d} = \frac{r}{h} \quad \text{from which} \quad d = (x_c - x_k)\frac{h}{i} \quad \text{(i)}$$

Stereoscopic Viewing

 An equation for calculating vertical exaggeration can be developed with reference to these figures

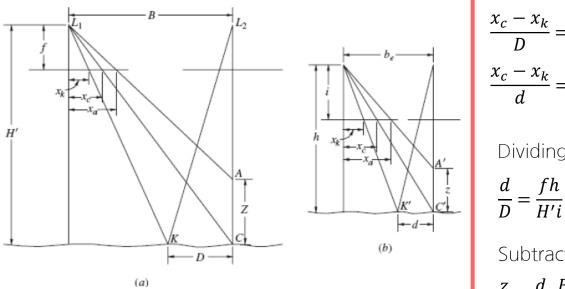


Figure 7-18. Simplistic diagrams for analyzing vertical exaggeration. (a) Geometry of overlapping aerial photography. (b) Geometry of stereoscopic viewing of the photos of part (a).

Subtracting (j) into (g) and reducing gives

$$\frac{z}{Z} = \frac{d}{D} \frac{Bh}{H'b_e} \tag{k}$$

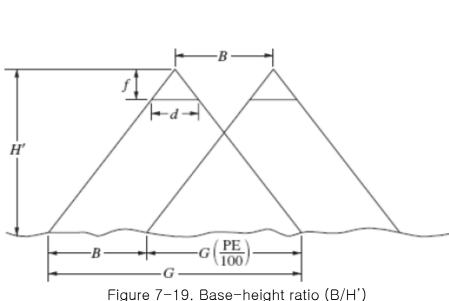
 $V = \frac{B}{H'} \frac{h}{b_{c}}$

7-8. Vertical Exaggeration in Stereoviewing

(k)

 $\frac{z}{Z} = \frac{d}{D} \frac{Bh}{H'b_e}$

- ✤ In Eq. (k), if the term Bh/(H'b_e) is equal to 1, there is no vertical exaggeration of the stereomodel (Recall that Z is equal to D)
- Thus an expression for the magnitude of vertical exaggeration V is given by



(7-1)

- From Eq. (7-1) it is seen that the magnitude of vertical exaggeration in stereoscopic viewing can be approximated by multiplying the *B/H'* ratio by the inverse of the *b_e/h* ratio
- ✤ An expression for the *B/H*′ ratio can be developed with reference to Fig. 7-19
- In this figure, G represents the total
 ground coverage of a vertical photo taken
 from an altitude of H' above ground
- Air base B is the distance between exposures

$$B = G - G \frac{PE}{100} = G(1 - \frac{PE}{100}) \tag{I}$$

- In Eq. (I), PE is the percentage of end lap, which gives the amount that the second photo overlaps the first
- ✤ Also by similar triangles of the figure,

$$\frac{H'}{G} = \frac{f}{d}$$
 from which $H' = \frac{fG}{d}$ (m)

- In Eq. (m), f is the camera focal length and d its format dimension
- Dividing Eq. (I) by Eq. (m) and reducing gives

$$\frac{B}{H'} = (1 - \frac{PE}{100})\frac{d}{f}$$
(7-2)

The stereoviewing base-height ratio varies due to differences in the distances * between the eyes of users and varying dimensions of stereoscopes

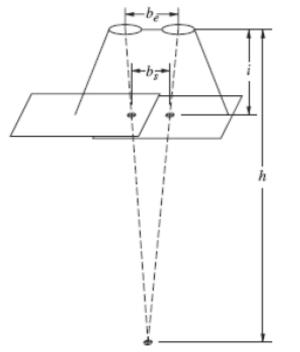


Figure 7-20. Eye base to perceived model height ratio.

- Figure 7-20 illustrates the relationships involved in ** this approximation
- \bullet With an eye base, b_e , averaging about 65 mm in humans, we need only to find the perceived distance from the eyes to the stereomodel, h to make an approximation
- * If the distance between the photos is b_s , and the distance of the stereoscope from the photos is i then we can use the following equation to estimate h by similar triangles: h = i

$$\frac{b_e}{b_e - b_s}$$
 (n)

Equation (n) can be used to form Eq. (7-3) to directly solve for the stereoviewing * base-height ratio:

$$\frac{b_e}{h} = \frac{b_e - b_s}{i} \tag{7-3}$$