Chapter 8.
Stereoscopic Parallax

8-1. Introduction

- **Parallax** is the apparent displacement in the position of an object, with respect to a frame of reference, caused by a shift in the position of observation
- If a person looked through the viewfinder of an aerial camera as the aircraft moved forward, images of objects would be seen to move across the field of view
- This image motion is another example of parallax caused by shifting the location of the observation point
- The change in position of an image from one photograph to the next caused by the aircraft's motion is termed stereoscopic parallax, x parallax, or simply parallax
- Parallax exists for all images appearing on successive overlapping photographs

8-1. Introduction

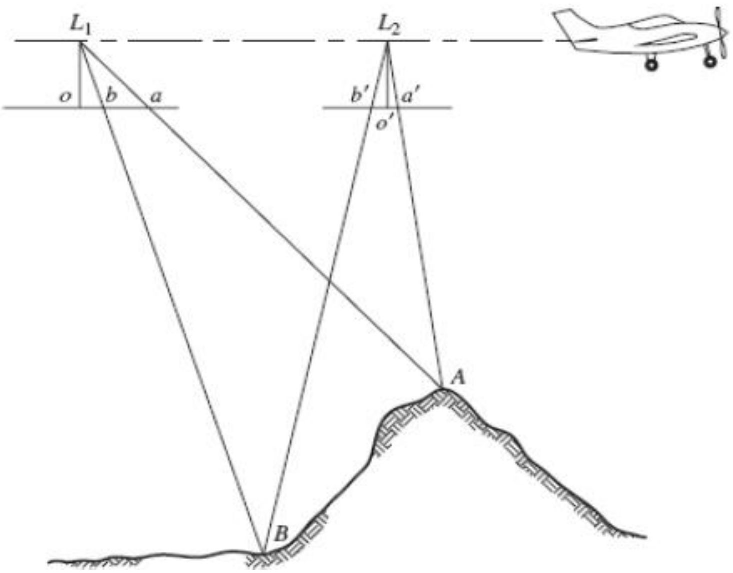
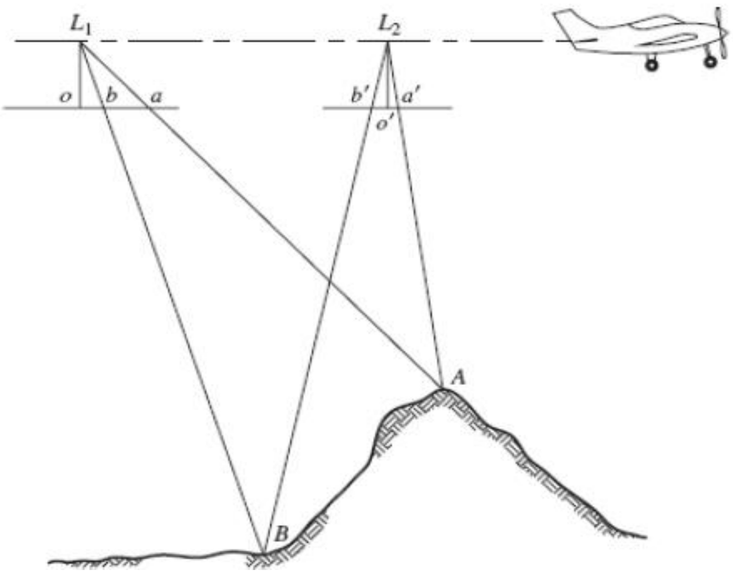


Figure 8-1. Stereoscopic parallax of vertical aerial photographs.

- In Fig. 8-1, for example, images of object points A and B appear on a pair of overlapping vertical aerial photographs which were taken from exposure stations L_1 and L_2
- Points A and B are imaged at a and b on the left-hand photograph
- Forward motion of the aircraft between exposures, however, caused the images to move laterally across the camera focal plane parallel to the flight line, so that on the right-hand photo they appear at a' and b'
- Because point A is higher (closer to the camera) than point B, the movement of image a across the focal plane was greater than the movement of image b
  - ⇨ In other words, the parallax of point A is greater than the parallax of point B

8-1. Introduction



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- ❖ Forward motion of the aircraft between exposures, however, caused the images to move laterally across the camera focal plane parallel to the flight line, so that on the right-hand photo they appear at a' and b'

- ❖ This calls attention to two important aspects of **stereoscopic parallax**:
  (1) The parallax of any point is directly related to the elevation of the point
  (2) Parallax is greater for high points than for low points

- ❖ Variation of parallax with elevation provides the fundamental basis for determining elevations of points from photographic measurements

8-1. Introduction

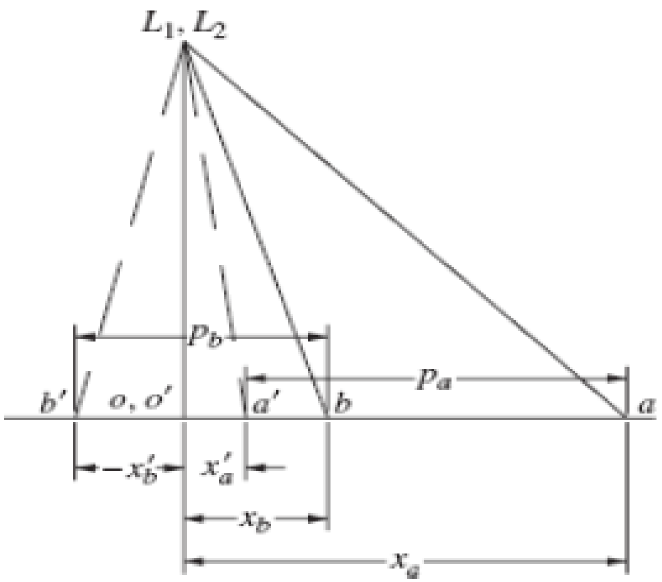

Figure 8–2. The two photographs of Fig. 8–1 are shown in superposition.

- ❖ Figure 8-2 shows the two photographs of Fig. 8-1 in superposition
- ❖ Parallaxes of object points A and B are p_a and p_b
- ❖ Stereoscopic parallax for any point such as A whose images appear on two photos of a stereopair, expressed in terms of ***flight-line*** **photographic coordinates**, is

$$p_a = x_a - x'_a \qquad (8\text{-}1)$$

- ❖ In Eq. (8-1), p is the stereoscopic parallax of object point A, x is the measured photo coordinate of image a on the left photograph of the stereopair, and x' is the photo coordinate of image a' on the right photo
- ❖ In Eq. (8-1), it is imperative that proper algebraic signs be given to measured photo coordinates to obtain correct values for stereoscopic parallax.

8-1. Introduction

Figure 8−3. Ovelapping vertical photographs taken over the University of Florida campus illustrating stereoscopic parallax. (Photos courtesy Hoffman and Company, Inc.)

- ❖ Figure 8-3 is a portion of a stereopair of vertical photographs taken over the University of Florida campus with a 153-mm-focal-length camera at a flying height of 462 m above ground
- ❖ On these photos, note how all images moved laterally with respect to the y axis from their positions on the left photo to their positions on the right photo
- ❖ Note also how clearly the bell tower (Century Tower) illustrates the increase in parallax with higher points; i.e., the top of the tower has moved farther across the focal plane than the bottom of the tower
- ❖ In Fig. 8-3, the tower affords an excellent example for demonstrating the use of Eq. (8-1) for finding parallaxes($p_a = x_a - x'_a$ (8-1))
- ❖ The top of the tower has an x coordinate (x = 48.2 mm) and an x' coordinate (x' = −53.2 mm)

8-1. Introduction

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- On these photos, note how all images moved laterally with respect to the y axis from their positions on the left photo to their positions on the right photo

$$p_a = x_a - x'_a \qquad \text{(8-1)}$$

- By Eq. (8-1), the parallax p = 48.2−(−53.2) = 101.4 mm
- Also, the bottom of the tower has an x coordinate (x = 42.7 mm) and an x' coordinate (x' = −47.9 mm)
- Again by Eq. (8-1), p = 42.7−(−47.9) = 90.6 mm

8-2. Photographic Flight-Line Axes for Parallax Measurement

- Since parallax occurs parallel to the direction of flight, the photographic x and x' axes for parallax measurement must be parallel with the flight line for each of the photographs of a stereopair
- For a vertical photograph of a stereopair, the flight line is the line connecting the principal point and corresponding (conjugate) principal point
- The flight line in this context is always a straight line even though the aircraft rarely travels in a perfectly straight line from one camera position to the next
- All photographs except those on the ends of a flight strip may have two sets of flight axes for parallax measurements—one to be used when the photo is the left photo of the stereopair and one when it is the right photo were exposed as shown

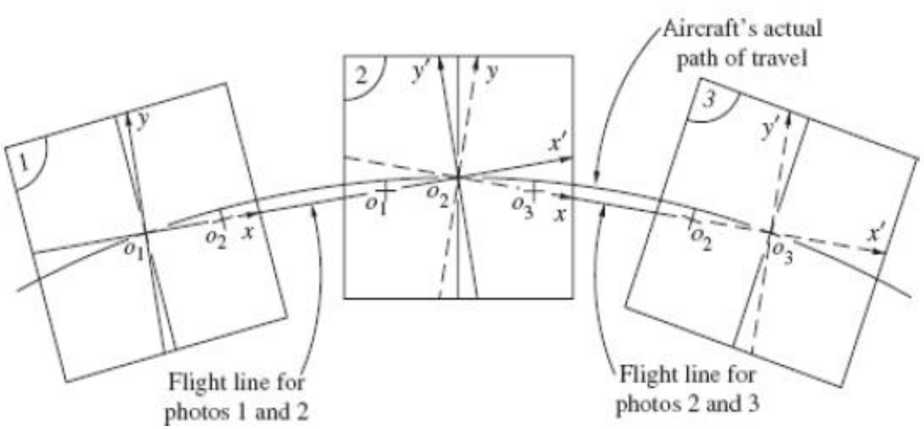


Figure 8-4. Flight-line axes for measurement of stereoscopic parallax.

- An example is shown in Fig. 8-4, where photographs 1 through 3

8-2. Photographic Flight-Line Axes for Parallax Measurement

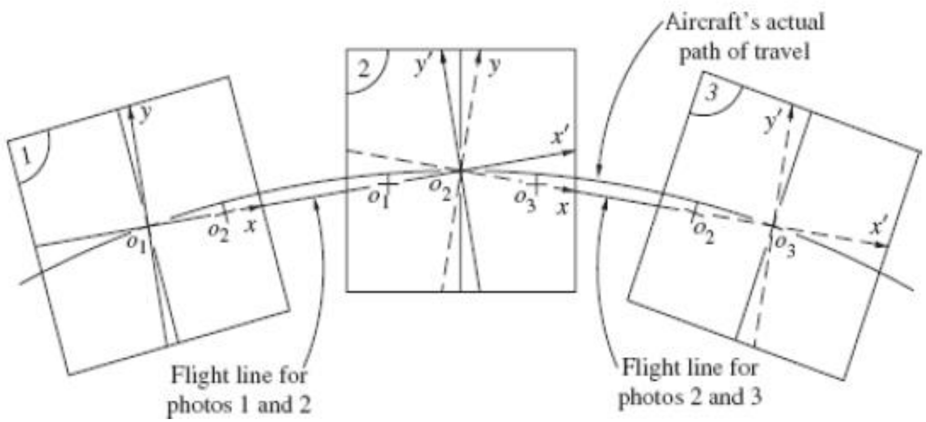


Figure 8–4. Flight–line axes for measurement of stereoscopic parallax.

- **Parallax measurements** in the overlap area of photos 1 and 2 are made with respect to the solid xy axis system of photo 1 and the solid $x'y'$ system of photo 2
- However, due to the aircraft's curved path of travel, the flight line of photos 2 and 3 is not in the same direction as the flight line of photos 1 and 2
- Therefore, **parallax measurements** in the overlap area of photos 2 and 3 must be made with respect to the dashed xy axis system on photo 2 and the dashed $x'y'$ system of photo 3
- It is understood that photographic coordinates for parallax determination are measured with respect to the flight-line axis system

8-3. Monoscopic Methods of Parallax Measurement

❖ Parallaxes of points on a stereopair may be measured either *monoscopically* or *stereoscopically*

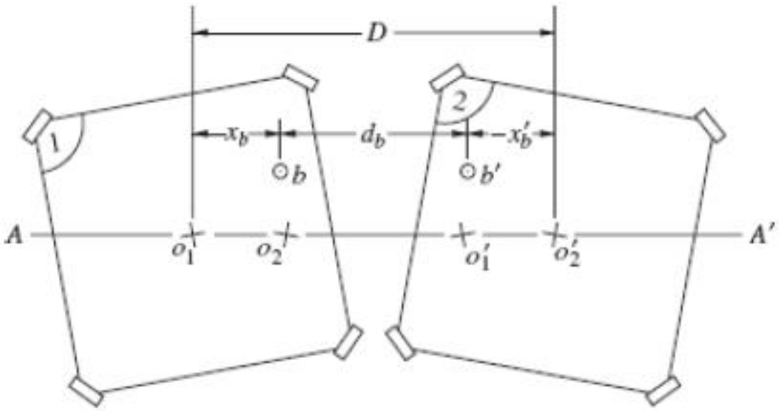


Figure 8-5. Parallax measurement using a simple scale.

$$p_a = x_a - x'_a \qquad (8\text{-}1)$$

❖ The simplest method of parallax measurement is the monoscopic approach, in which Eq. (8-1) is solved after direct measurement of x and x' on the left and right photos, respectively

❖ A disadvantage of this method is that two measurements are required for each point

❖ Another **monoscopic approach** to parallax measurement is to fasten the photographs down on a table or base material, as shown in Fig. 8-5.

❖ In this method the photographic flight lines o_1o_2 and $o'_1o'_2$ are marked as usual

8-3. Monoscopic Methods of Parallax Measurement

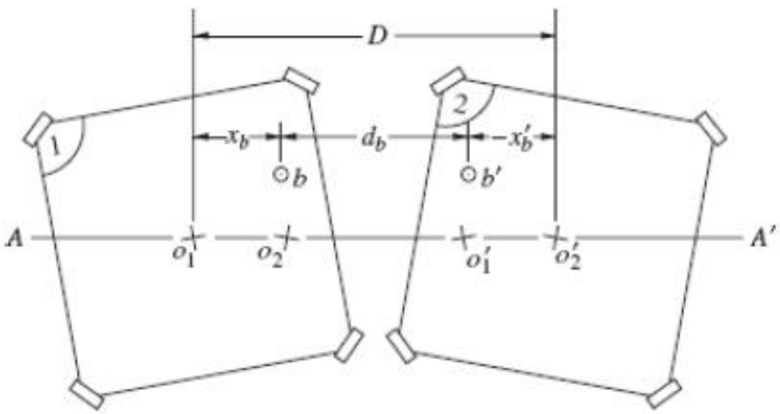


Figure 8-5. Parallax measurement using a simple scale.

- A long straight line AA' is drawn on the base material, and the two photos are carefully mounted as shown so that the photographic flight lines are coincident with this line
- Now that the photos are fastened down, the distance D between the two principal points is a constant which can be measured.
- The parallax of point B is $p = x_b - x_b'$ (note that in Fig. 8-5 the x_b' coordinate is negative)
  - ⇨ However, by examining the figure, it is seen that parallax is also

$$p_b = D - d_b \qquad (8\text{-}2)$$

- With D known in Eq. (8-2), to obtain the parallax of a point it is necessary only to measure the distance d between its images on the left and right photos
- The **advantage** is that <u>for each additional point whose parallax is desired, only a single measurement is required</u>

8-4. Principle of the Floating Mark

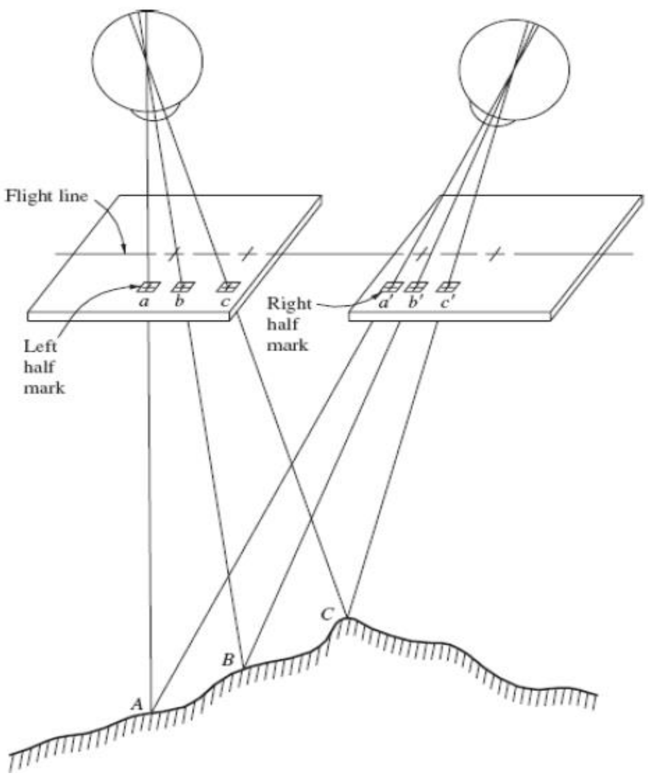


Figure 8-6. The principle of the floating mark.

- **Parallaxes of points** can be measured while viewing stereoscopically with the advantages of speed and accuracy
- Stereoscopic measurement of parallax makes use of the principle of the **floating mark**
- When a stereomodel is viewed through a stereoscope, two small identical marks printed on clear plastic transparencies, called ***half marks***, may be placed over the photographs—one on the left photo and one on the right photo, as illustrated in Fig. 8-6
- The left mark is seen with the left eye and the right mark with the right eye
- The half marks may be shifted in position until they fuse together into a single mark which appears to exist in the stereomodel and to lie at a particular elevation

8-4. Principle of the Floating Mark

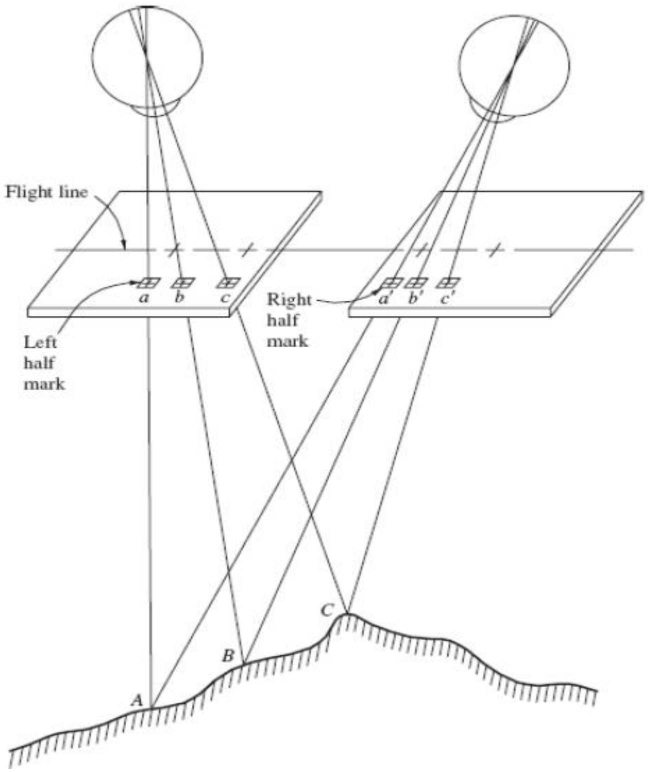


Figure 8-6. The principle of the floating mark.

❖ If the half marks are **moved closer** together, the parallax of the half marks is **increased** and the fused mark will therefore appear to **rise**

❖ Conversely, if the half marks are **moved apart**, parallax is **decreased** and the fused mark appears to **fall**
  - ⇨ This apparent variation in the elevation of the mark as the spacing of half marks is varied is the basis for the term floating mark

❖ The spacing of the half marks, and hence the parallax of the half marks, may be varied so that the floating mark appears to rest exactly on the terrain

❖ This produces the same effect as though an object of the shape of the half marks had existed on the terrain when the photos were originally taken

8-4. Principle of the Floating Mark

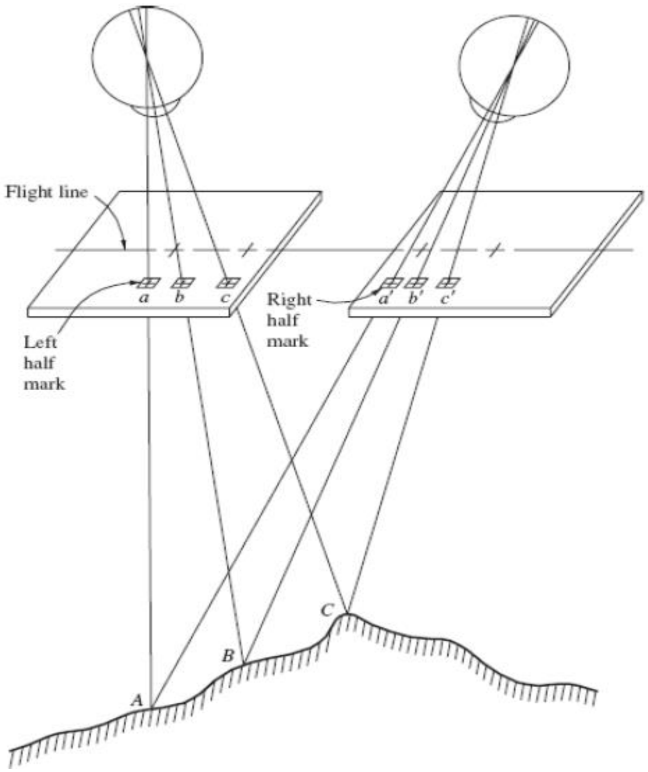


Figure 8–6. The principle of the floating mark.

- The floating mark may be moved about the stereomodel from point to point, and as the terrain varies in elevation, the spacing of the half marks may be varied to make the floating mark rest exactly on the terrain

- Figure 8-6 demonstrates **the principle of the floating mark** and illustrates how the mark may be set exactly on particular points such as $A, B,$ and C by placing the half marks at a and a', b and b', and c and c', respectively

- The **principle of the floating mark** can be used <u>to transfer principal points to their corresponding locations, thereby marking the flight-line axes</u>

8-4. Principle of the Floating Mark

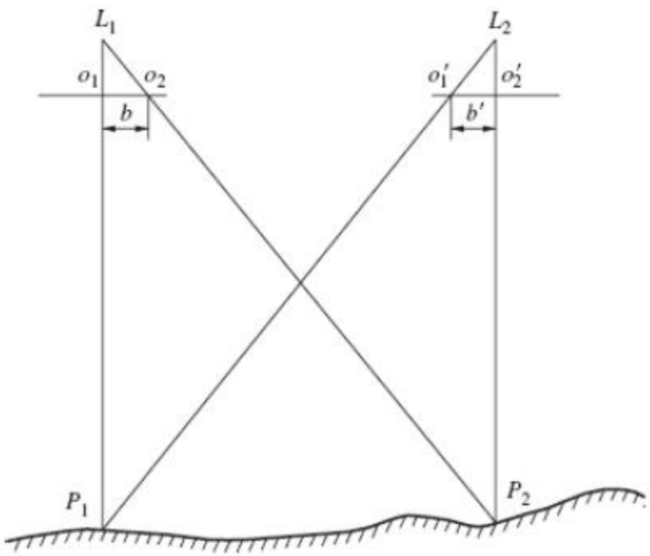


Figure 8–7. Parallax of the principal points.

- The **photo base** is <u>the distance on a photo between the principal point and the corresponding principal point from the overlapping photo</u>
- Figure 8-7 is a vertical section through the exposure stations of a pair of overlapping vertical photos
- By Eq. (8-1), the parallax of the left-photo ground principal point P_1 is $p_{o_1} = x_{o_1} - (x'_{o_1}) = 0 - (-b') = b'$
  (The x coordinate of o on the left photo is zero)
- Also, the parallax of the right-photo ground principal point P_2 is $p_{o_2} = x_{o_2} - (x'_{o_2}) = b - 0 = b$
- From the foregoing, it is seen that the parallax of the left ground principal point is photo base b' measured on the right photo, and the parallax of the right ground principal point is photo base b measured on the left photo.
- In areas of moderate relief, the values of b and b' will be approximately equal, and the photo base for the stereopair can be taken as the average of these two values

8-5. Stereoscopic Methods of Parallax Measurement

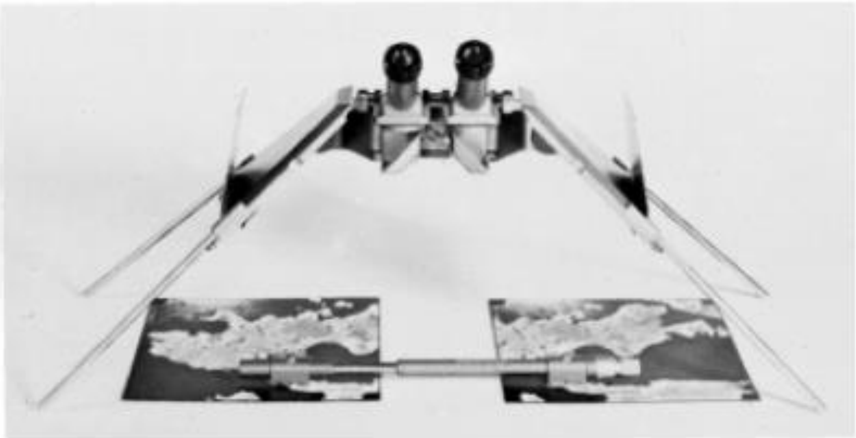
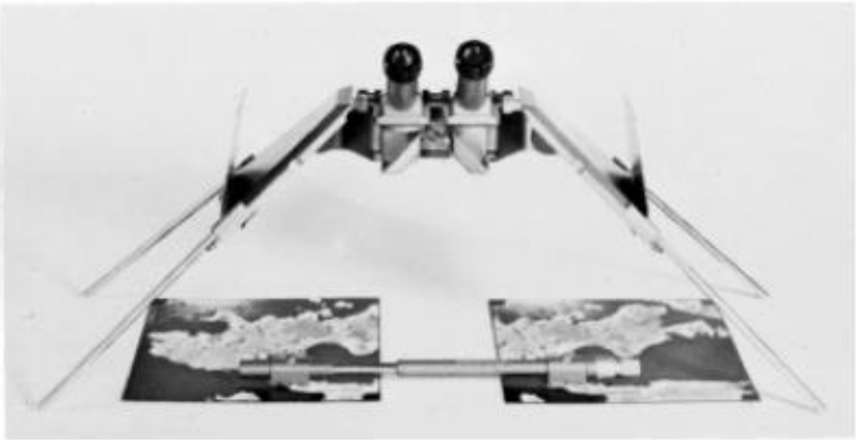


Figure 8-8. Wild ST-4 mirror stereoscope with binocular attachment and parallax bar. (Courtesy LH Systems, LLC.)

- ❖ Through the principle of the floating mark, parallaxes of points may be measured stereoscopically
- ❖ This method employs a stereoscope in conjunction with an instrument called a parallax bar, also frequently called a stereometer
- ❖ A parallax bar consists of a metal rod to which are fastened two half marks
- ❖ The right half mark may be moved with respect to the left mark by turning a micrometer screw
- ❖ Readings from the micrometer are taken with the floating mark set exactly on points whose parallaxes are desired
- ❖ From the micrometer readings, parallaxes or differences in parallax are obtained
- ❖ A parallax bar is shown lying on the photos beneath a mirror stereoscope in Fig. 8-8

8-5. Stereoscopic Methods of Parallax Measurement



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- This method employs a stereoscope in conjunction with an instrument called a parallax bar, also frequently called a stereometer
- A parallax bar consists of a metal rod to which are fastened two half marks
- When a parallax bar is used, the two photos of a stereopair are first carefully oriented for comfortable stereoscopic viewing, in such a way that the flight line of each photo lies precisely along a common straight line
- The photos are then fastened securely, and the parallax bar is placed on the photos
- The left half mark, called the fixed mark, is unclamped and moved so that when the floating mark is fused on a terrain point of average elevation, the parallax bar reading is approximately in the middle of the run of the graduations

8-5. Stereoscopic Methods of Parallax Measurement

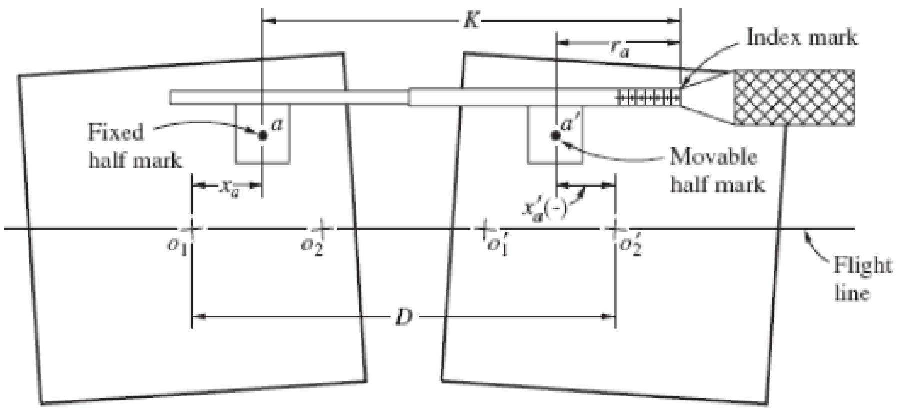

Figure 8-9. Schematic diagram of the parallax bar.

- Figure 8-9 is a **schematic diagram** illustrating the operating principle of the parallax bar
- After the photos have been oriented and the left half mark is fixed in position as just described, the parallax bar constant C for the setup is determined
- For the setup, the spacing between principal points is a constant, denoted by D
- Once the fixed mark is clamped, the distance from the fixed mark to the index mark of the parallax bar is also a constant, denoted by K
- From Fig. 8-9, the parallax of point A is $p_a = x_a - x'_a = D - (\mathrm{K} - r_a) = (D - K) + r_a$
- The term $(D - K)$ is C, the parallax bar constant for the setup, and r is the micrometer reading
- By substituting C into the above equation, the expression becomes

$p_a = C + r_a$ (8-3)

8-5. Stereoscopic Methods of Parallax Measurement

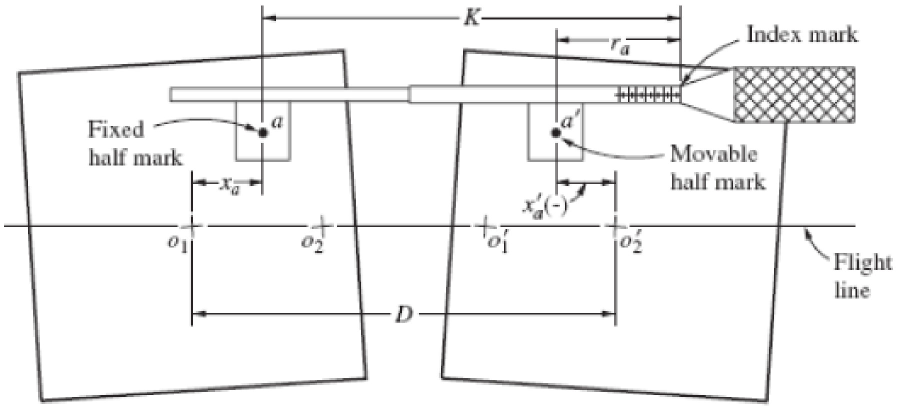

Figure 8–9. Schematic diagram of the parallax bar.

❖ To calculate the parallax bar constant, a micrometer reading is taken with the floating mark set on a selected point

$$p_a = x_a - x'_a \qquad (8\text{-}1)$$

$$p_a = C + r_a \qquad (8\text{-}3)$$

$$C = p - r \qquad (8\text{-}4)$$

❖ The parallax of that point is also directly measured monoscopically and calculated using Eq. (8-1)

❖ Then with p and r for that point known, the value of C is calculated by using Eq. (8-3), as Eq. (8-4)

❖ One of the advantages of measuring parallax stereoscopically is increased speed, for once the parallax bar constant is determined, the parallaxes of all other points are quickly obtained with a single micrometer reading for each point

❖ Another advantage is increased accuracy

❖ An experienced person using quality equipment and clear photos is generally able to obtain parallaxes to within approximately 0.03 mm of their correct values

8-6. Parallax Equations

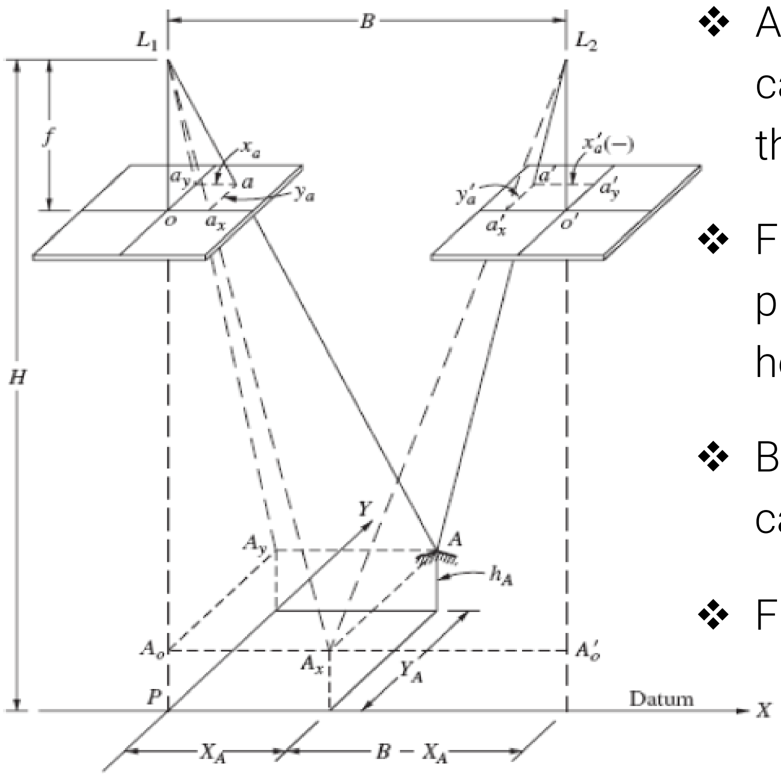

Figure 8−10. Geometry of an overlapping pair of vertical photographs.

- As noted earlier, X, Y, and Z ground coordinates can be calculated for points based upon the measurements of their parallaxes
- Figure 8-10 illustrates an overlapping pair of vertical photographs which have been exposed at equal flying heights above datum
- By equating similar triangles of Fig. 8-10, formulas for calculating h_A, X_A, and Y_A may be derived
- From similar triangles $L_1 o a_y$ and $L_1 A_o A_y$,

$$\frac{Y_A}{H - h_A} = \frac{y_a}{f} \quad \text{from which} \quad Y_A = \frac{y_a}{f}(H - h_A) \qquad \text{(a)}$$

- Equating similar triangles, $L_1 o a_x$ and $L_1 A_o A_x$, we have

$$\frac{X_A}{H - h_A} = \frac{x_a}{f} \quad \text{from which} \quad X_A = \frac{x_a}{f}(H - h_A) \qquad \text{(b)}$$

8-6. Parallax Equations

Also from similar triangles $L_2 o' a_x'$ and $L_2 A_o' A_x'$,

$$\frac{B - X_A}{H - h_A} = \frac{-x'_a}{f}$$

from which

$$X_A = B + \frac{x'_a}{f}(H - h_A) \qquad \text{(c)}$$

Equating Eqs. (b) and (c) and reducing gives

$$h_A = H - \frac{Bf}{x_a - x'_a} \qquad \text{(d)}$$

Substituting p_a for $x_a - x_a'$ into Eq. (d) yields

$$h_A = H - \frac{Bf}{p_a} \qquad \text{(8-5)}$$

Substituting Eq. (8-5) into each of Eqs. (b) and reducing gives

$$X_A = B\frac{x_a}{p_a} \qquad \text{(8-6)}$$

$$Y_A = B\frac{y_a}{p_a} \qquad \text{(8-7)}$$

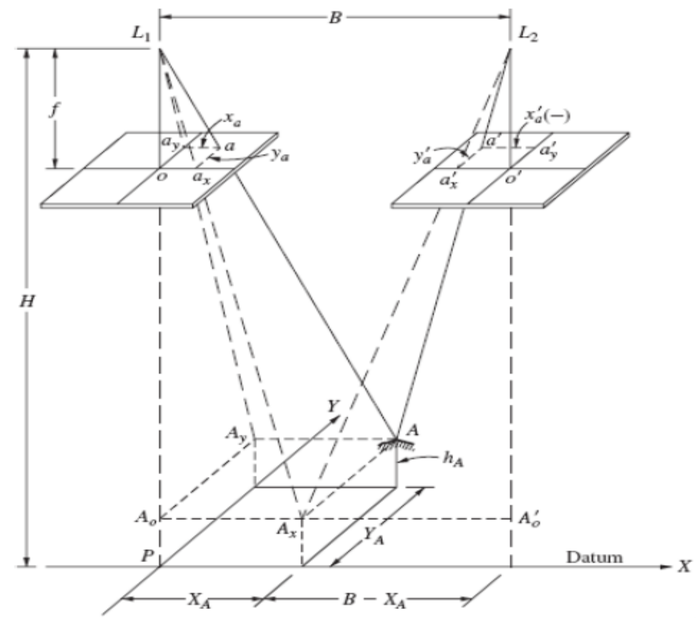

Figure 8-10. Geometry of an overlapping pair of vertical photographs.

- In Eqs. (8-5), (8-6), and (8-7), h is the elevation of point A above datum, H is the flying height above datum, B is the air base, f is the focal length of the camera, p is the parallax of point A, X_A and Y_A are the ground coordinates of point A in the previously defined unique arbitrary coordinate system, and x_a and y_a the photo coordinates of point a measured with respect to the flight-line axes on the left photo

8-6. Parallax Equations

Also from similar triangles $L_2o'a_x'$ and $L_2A_o'A_x'$,

$$\frac{B - X_A}{H - h_A} = \frac{-x'_a}{f}$$

from which

$$X_A = B + \frac{x'_a}{f}(H - h_A) \qquad \text{(c)}$$

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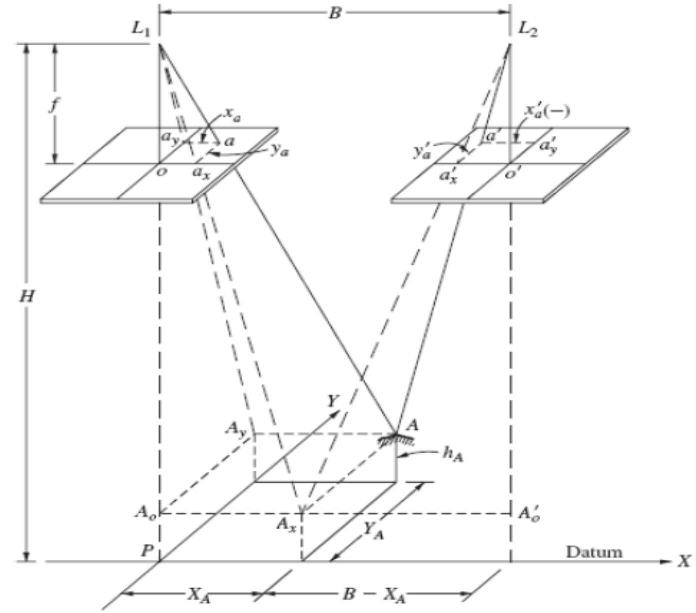

Figure 8-10. Geometry of an overlapping pair of vertical photographs.

- Equations (8-5), (8-6), and (8-7) are commonly called the parallax equations
- These equations enable a moderate accuracy survey of the overlap area of a stereopair to be made, provided the focal length is known and sufficient ground control is available so the air base B and flying height H can be calculated

8-6. Parallax Equations

Also from similar triangles $L_2o'a_x'$ and $L_2A_o'A_x'$,

$$\frac{B - X_A}{H - h_A} = \frac{-x'_a}{f}$$

from which

$$X_A = B + \frac{x'_a}{f}(H - h_A) \qquad \text{(c)}$$

Equating Eqs. (b) and (c) and reducing gives

$$h_A = H - \frac{Bf}{x_a - x'_a} \qquad \text{(d)}$$

Substituting p_a for $x_a - x_a'$ into Eq. (d) yields

$$h_A = H - \frac{Bf}{p_a} \qquad \text{(8-5)}$$

Substituting Eq. (8-5) into each of Eqs. (b) and reducing gives

$$X_A = B\frac{x_a}{p_a} \qquad \text{(8-6)}$$

$$Y_A = B\frac{y_a}{p_a} \qquad \text{(8-7)}$$

- Equations (8-6) and (8-7) yield X and Y ground coordinates in the unique arbitrary coordinate system of the stereopair, which is not related to any standard twodimensional ground coordinate system
- However, if arbitrary XY coordinates are determined using these equations for at least two points whose ground coordinates are also known in a standard two-dimensional coordinate system(e.g., state plane coordinates), then the arbitrary XY coordinates of all other points can be transformed into that ground system through a two-dimensional coordinate transformation

8-7. Elevations by Parallax Differences

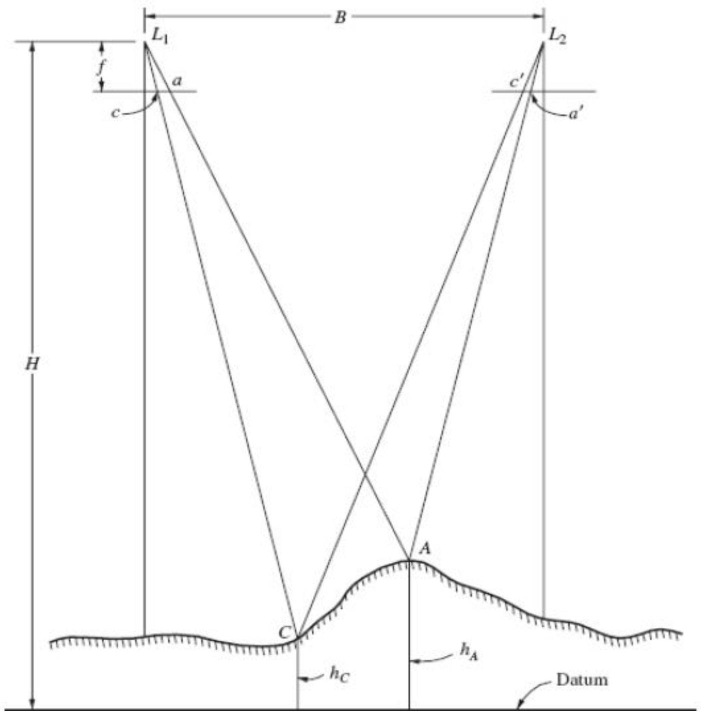

Figure 8−11. Elevations by parallax differences.

$$h_A = H - \frac{Bf}{p_a} \qquad \text{(8-5)}$$

- **Parallax differences** between one point and another are <u>caused by different elevations of the two points</u>
- While parallax Eq. (8-5) serves to define the relationship of stereoscopic parallax to flying height, elevation, air base, and camera focal length, parallax differences are more convenient for determining elevations
- In Fig. 8-11, object point C is a control point whose elevation h_c above datum is known
- The elevation of object point A is desired
- By rearranging Eq. (8-5), parallaxes of both points can be expressed as

$$p_c = \frac{fB}{H - h_C} \qquad \text{(e)}$$

$$p_a = \frac{fB}{H - h_A} \qquad \text{(f)}$$

8-7. Elevations by Parallax Differences

- The difference in parallax $p_a - p_c$, obtained by subtracting Eq. (e) from Eq. (f) and rearranging, is

$$p_a - p_c = \frac{fB(h_A - h_C)}{(H - h_A)(H - h_C)} \qquad \text{(g)}$$

- Let $p_a - p_c$ equal Δp, the difference in parallax
- By substituting $H - h_A$ from Eq. (f), and Δp into (g) and reducing, the following expression for elevation h_A is obtained:

$$h_A - h_C = \frac{\Delta p(H - h_C)}{p_a} \qquad \text{(8-8)}$$

- If a number of control points are located throughout the overlap area, use of Eq. (8-8) permits elevations of unknown points to be most accurately determined from the parallax difference of the nearest control point
- This minimizes the effects of two primary errors—photographic tilt and imperfect alignment of the photos for parallax measurement

8-8. Simplified Equation for Heights of Objects from Parallax Differences

- In many applications it is necessary to estimate heights of objects to a moderate level of accuracy
- Utilizing parallax differences for height determination is particularly useful when application of relief displacement is not possible because either the feature is not vertical (e.g., a construction crane) or the base of the feature is obscured (e.g., trees in a forest)
- In a situation like this, a **parallax difference** can be determined between a point on the ground and the top of the feature
- A fundamental **assumption** is, of course, that the point on the ground is at the same elevation as the base of the feature ⇨ In a large number of cases, this assumption is valid as long as only moderate accuracy is required
- A simplified equation for height determination can be obtained from Eq. (8-8) by choosing the vertical datum to be the elevation of the point on the ground that is used as the basis for the parallax difference
- This makes h zero, and Eq. (8-8) simplifies to

$$h_A - h_C = \frac{\Delta p(H - h_C)}{p_a} \quad \text{(8-8)} \qquad ⇨ \qquad h_A = \frac{\Delta p H}{p_a} \quad \text{(8-9)}$$

8-8. Simplified Equation for Heights of Objects from Parallax Differences

$$h_A = \frac{\Delta p H}{p_a} \quad (8\text{-}9)$$

- In Eq. (8-9), h is the height of point A above ground, $\Delta p = p_a - p_c$ is the difference in parallax between the top of the feature and the ground (p_c is the parallax of the ground), and H is the flying height above ground, since datum is at ground

- If the heights of many features are needed in an area where the ground is approximately level, the photo base b can be utilized as the parallax of the ground point

- In this case, Eq. (8-9) can be modified to

$$h_A = \frac{\Delta p H}{b + \Delta p} \quad (8\text{-}10)$$

- In Eq. (8-10), b is the photo base for the stereopair, $\Delta p = p_a - b$, and the other terms are as previously defined

- For very low flying heights or in areas of significant relief, or both, the assumptions of Eq. (8-10) are not met ⇨ in these cases, Eq. (8-8) should be used

- Equation (8-10) is especially convenient in photo interpretation where rough elevations, building and tree heights, etc. are often needed

8-9. Measurement of Parallax Differences

- Parallax differences may be determined in any of the following ways:
  1. By monoscopic measurement of parallaxes followed by subtraction
  2. By taking differences in parallax bar readings
  3. By parallax wedge

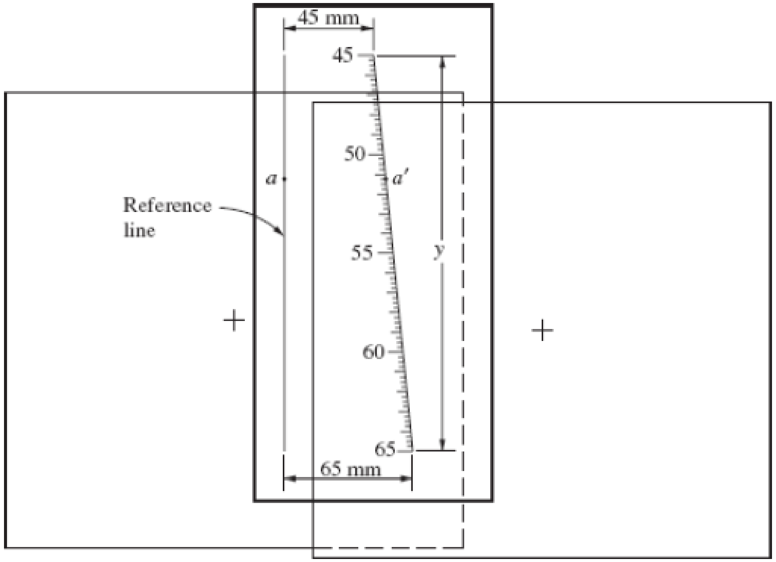

Figure 8-12. Parallax wedge.

- A parallax wedge, as illustrated in Fig. 8-12, consists of a piece of transparent film upon which are drawn two converging lines
- The left line is a reference line while the line on the right contains graduations from which readings can be made
- The spacing of the two lines depends on whether the parallax wedge will be used with a mirror stereoscope or a pocket stereoscope

8-9. Measurement of Parallax Differences

- When a **parallax wedge** is used, the photos are first carefully oriented for viewing with a pocket stereoscope and secured

- The parallax wedge is placed in the overlap area and viewed stereoscopically
  - ⇨ The two lines of the parallax wedge will fuse and appear as a single floating line in areas where the spacing of the lines is slightly less than the spacing of corresponding photo images

- The floating line will appear to split where the parallax of the lines is equal to that of the photo images

- The position of the parallax wedge can be adjusted so that the floating line splits forming a wedge exactly at a point whose parallax is desired, and at that point a reading is taken from the scale

- **Parallax differences** are obtained by simply taking differences in parallax wedge readings for different points

8-10. Computing Flying Height and Air Base

- **To use parallax equations**, it is generally necessary <u>to compute the flying height and air base</u>
- For best results, the average of flying heights for the two photos of a stereopair should be used
- If the air base is known and if one vertical control point is available in the overlap area, flying height for the stereopair may be calculated by using Eq. (8-5)

$$h_A = H - \frac{Bf}{p_a} \qquad (8\text{-}5)$$

- If the flying height above datum is known and if one vertical control point is available in the overlap area, the air base for the stereopair may be calculated by using Eq. (8-5)

8-10. Computing Flying Height and Air Base

❖ If a line of known horizontal length appears in the overlap area, then the air base can be readily calculated

❖ The horizontal length of the line may be expressed in terms of rectangular coordinates, according to the pythagorean theorem, as $AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$

❖ Substituting Eqs. (8-6) and (8-7) into the above for the rectangular coordinates gives

$$AB = \sqrt{\left(\frac{Bx_b}{p_b} - \frac{Bx_a}{p_a}\right)^2 + \left(\frac{By_b}{p_b} - \frac{By_a}{p_a}\right)^2}$$

Solving the above equation for B yields

$$B = \frac{AB}{\sqrt{(x_b/p_b - x_a/p_a)^2 + (y_b/p_b - y_a/p_a)^2}} \qquad \text{(8-11)}$$

8-11. Error Evaluation

Some of the sources of error in computed answers using parallax equations are as follows:

① Locating and marking the flight lines on photos
② Orienting stereopairs for parallax measurement
③ Parallax and photo coordinate measurements
④ Shrinkage or expansion of photographs
⑤ Unequal flying heights for the two photos of stereopairs
⑥ Tilted photographs
⑦ Errors in ground control
⑧ Other errors of lesser consequence such as camera lens distortion and atmospheric refraction distortion

❖ Answers obtained using the various equations presented in the chapter will inevitably contain errors

❖ It is important to be aware of the presence of these errors and to be able to assess their magnitudes

❖ For normal photography intended to be vertical, errors in parallax equation answers due to tilt are compatible with errors from the other sources that have been considered