

Chapter 9.

Elementary Methods of Planimetric Mapping for GIS

9-1. Introduction

- ❖ This chapter describes elementary methods that can be used for compiling planimetric maps from vertical photographs and satellite images
 - ⇒ These include :
 - (1) tracing with the use of reflection instruments
 - (2) *georeferencing* of digital imagery
 - (3) performing heads-up digitizing
 - (4) preparing *photomaps*
 - (5) constructing *mosaics*
- ❖ For map revision over limited areas, and in many applications of geographic information systems (GISs), particularly those involving natural resource mapping, the high accuracy afforded by rigorous photogrammetric methods is often not necessary
- ❖ By using appropriate elementary methods, substantial cost savings can be realized while obtaining planimetric information at acceptable accuracy

9-2. Planimetric Mapping With Reflection Instruments

- ❖ A number of instruments have been devised that enable an operator to view an image superimposed with a map manuscript
- ❖ The basic operation of these instruments is fundamentally the same
 - ⇒ Photo control points which have been plotted to scale on the map manuscript are used as a basis for orienting the image
 - ⇒ By adjusting image scale, rotation, translation, and in some cases tilt or nonuniform image stretch, the images of the photo control points can be made to approximately coincide with the plotted control points on the manuscript
 - ⇒ The degree of coincidence depends on a number of factors including the amount of terrain relief
 - ⇒ Once the device is oriented, additional image features can be directly traced onto the manuscript by virtue of the operator's simultaneous view of the oriented image and the map

9-2. Planimetric Mapping With Reflection Instruments

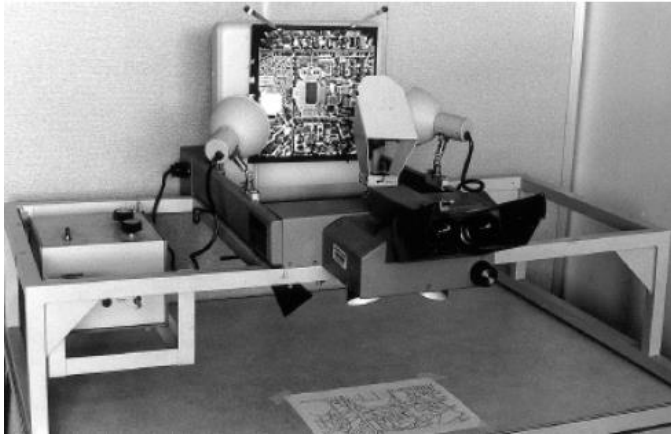


Figure 9-1. Zoom transfer scope.
(Courtesy University of Florida.)

- ❖ The zoom transfer scope (ZTS) of Fig. 9-1 is a versatile reflection instrument which is used primarily for planimetric map revision
- ❖ With the ZTS, an aerial photograph or other hard-copy image is placed on a viewing stage and secured with clamps
- ❖ The map manuscript is placed on a horizontal working surface which can be viewed simultaneously with the image
- ❖ The instrument can accommodate large differences in scale from photo to map by means of zoom optics which provide continuous variations in magnification from 1 to 7X
- ❖ In addition, the anamorphic optical system of the ZTS enables different magnification ratios to be applied in the x and y directions

9-2. Planimetric Mapping With Reflection Instruments

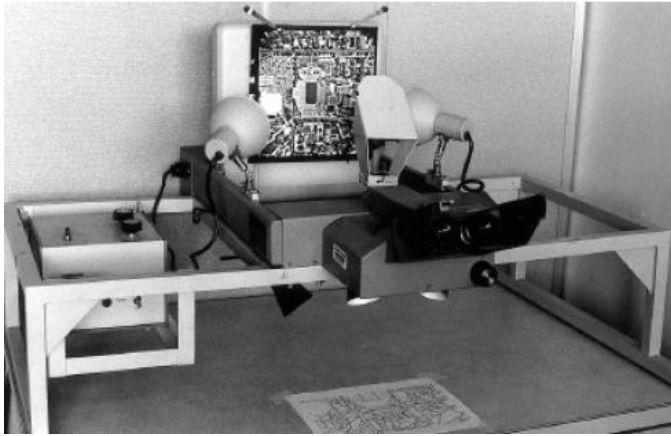


Figure 9-1. Zoom transfer scope.
(Courtesy University of Florida.)

- ❖ The ZTS also has the capability of removing the effects of nonorthogonal *affine distortion* in the image by means of rotating prisms
 - ❖ These features facilitate adjusting photo images to coincide with map points or control points
 - ❖ Once the image and map are adjusted, the manuscript is taped down and the operator then traces the desired features
-
- ❖ Of course, photographs are perspective projections, they contain scale variations due to tilt and relief
 - ⇒ Therefore, in compiling maps using reflection instruments, traced features will contain any scale variations that exist in the photo images
 - ⇒ However, if relief is moderate, and relatively small areas are traced that lie within a perimeter of control points, the resulting map will be sufficiently accurate to satisfy many purposes

9-3. Georeferencing of Digital Imagery

- ❖ *Georeferencing*, sometimes called *ground registration*, is a technique whereby a digital image is processed so that the columns and rows of the resulting product are aligned with north and east in a ground coordinate system
- ❖ In the remote-sensing field, georeferencing is referred as *rectification*
 - ⇒ However, in the context of photogrammetry, the term rectification is reserved for the process of removing the effects of tilt from an aerial photograph
- ❖ The process of georeferencing an image involves two fundamental steps:
 - (1) computing the parameters of a two-dimensional coordinate transformation which relates the digital image to the ground system and
 - (2) filling an array, which is aligned with the ground coordinate system, with the appropriate digital numbers that quantify the brightness of the ground at the corresponding locations

9-3. Georeferencing of Digital Imagery

- ❖ In the first step, a number of ground control points (GCPs) are selected which can be identified in the image and for which accurate ground coordinates are available
- ❖ The column and row image coordinates of each GCP are obtained and subsequently related to the ground coordinates
- ❖ The simplest method for obtaining image coordinates of a GCP is to display the image on a computer screen
- ❖ Then by using a mouse, a cursor is guided to the location of the image point and recorded by clicking on it
- ❖ The coordinate transformation in this first step of the georeferencing process converts from ground coordinates (x and y) to image coordinates (X and Y)
- ❖ At first, the direction of this conversion (from ground to image) may seem backward
 - ⇒ However, for reasons which will become clear in this section, the transformation must be performed in this manner
 - ⇒ Furthermore, the use of lowercase x, y for ground coordinates and uppercase X, Y for image coordinates seems to be a reversal from the usual convention
 - ⇒ Actually, this is appropriate when one looks at the uppercase/lowercase convention from a different point of view

9-3. Georeferencing of Digital Imagery

- ❖ For **georeferencing**, since the ground system is the "from" system, lowercase x and y are used for its coordinates \Rightarrow Since the image system is the "to" system, uppercase X and Y are used for its coordinates
- ❖ Image coordinates of the common points will serve as control for the transformation, and the resulting parameters will give the relationship from ground to image coordinates
- ❖ Any two-dimensional coordinate transformation could be used, but the conformal and affine are most often employed because of their convenience and suitability

$$\begin{aligned} X &= ax - by + T_X \\ Y &= ay + bx + T_Y \end{aligned} \quad (9-1)$$

❖ Equations 9-1 express this transformation relationship

- ❖ In Eq. (9-1), x and y are coordinates of points in the ground system
 - $\Rightarrow X$ and Y are coordinates of points in the image that have been obtained by converting from their column and row values
 - $\Rightarrow a, b, T_X$, and T_Y are parameters which are determined during this first step
 - \Rightarrow Once computed, these parameters are used to transform coordinates of additional points from the ground system to the image system
 - \Rightarrow The reason for this arrangement of the transformation will become apparent in the description of the second step

9-3. Georeferencing of Digital Imagery

- ❖ During the second step of georeferencing, an image that is aligned with the ground coordinate system is produced
- ❖ To understand this step, it is useful to visualize the ground as being divided into a rectangular grid of individual elements called *groundels* (ground elements), analogous to pixels (picture elements) of a digital image
- ❖ The difference is that while pixels have no specific relationship to the ground, groundels are arranged at a nominal spacing in a grid which is parallel to the ground coordinate system
- ❖ For each groundel, the x and y coordinates of its center point are transformed, based on the parameters computed in the first step, into corresponding image coordinates
- ❖ The brightness value from the image at this corresponding location is then inserted into the groundel array
 - ⇒ This involves the process of *resampling*
 - ⇒ After this process has been applied to each groundel, the georeferenced image is contained in the array of groundels

9-3. Georeferencing of Digital Imagery

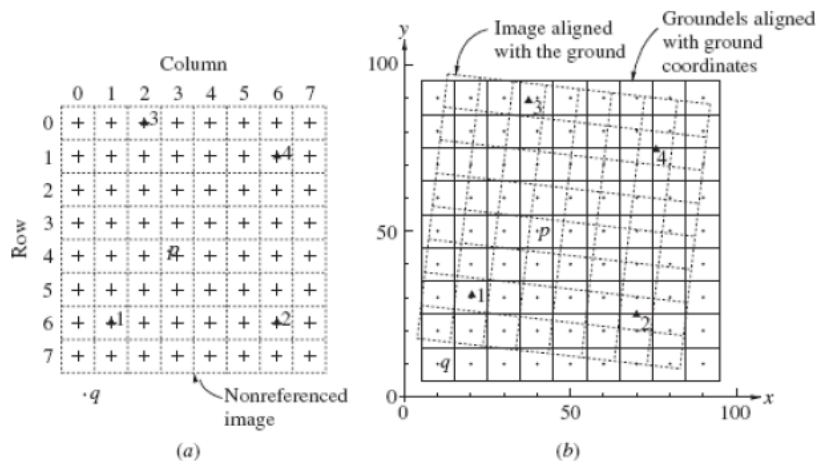


Figure 9-2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

- ❖ To illustrate the above-described process of georeferencing, refer to Figs. 9-2a and b
- ❖ Figure 9-2a represents an 8-pixel by 8-pixel digital image prior to georeferencing
- ❖ The solidline grid of Fig. 9-2b represents an array of groundels which are nominally aligned with a ground coordinate system xy
- ❖ Nominal ground coordinates are associated with the centers of the groundels, shown as small dots in Fig. 9-2b
- ❖ The dashed lines represent the digital image as it would appear if properly aligned with the ground system

9-3. Georeferencing of Digital Imagery

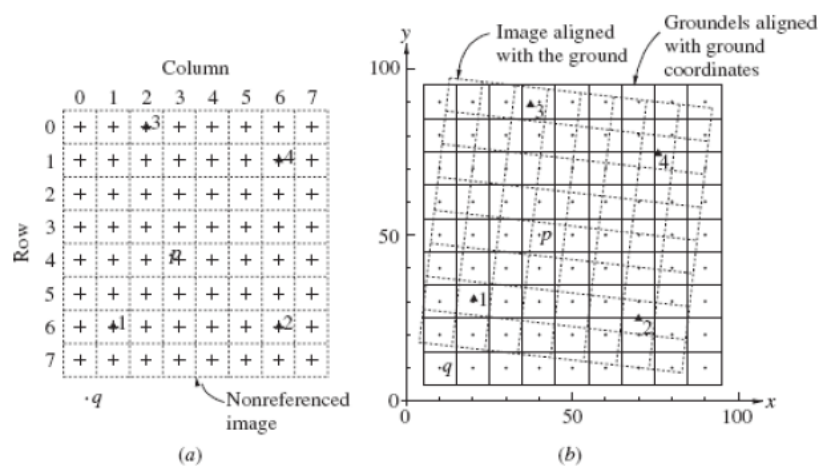


Figure 9-2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

Table 9-1. Ground and Image Coordinates for Control Points of Fig. 9-2

Point	(Ground Coordinates) x, m	(Ground Coordinates) y, m	(Image Coordinates) Column	(Image Coordinates) Row
1	20.4	30.6	1	6
2	70.1	24.9	6	6
3	37.1	89.3	2	0
4	75.8	74.4	6	1

- ❖ Four control points (1, 2, 3, and 4) having coordinates in the ground xy system appear in the image
- ❖ Coordinates of these control points in both the image system of Fig. 9-2a and the ground system are listed in Table 9-1
- ❖ Note that image coordinates, which correspond to pixel centers [indicated by crosses in Fig. 9-2a], are specified to the nearest whole row and column, which is typical of most georeferencing applications

9-3. Georeferencing of Digital Imagery

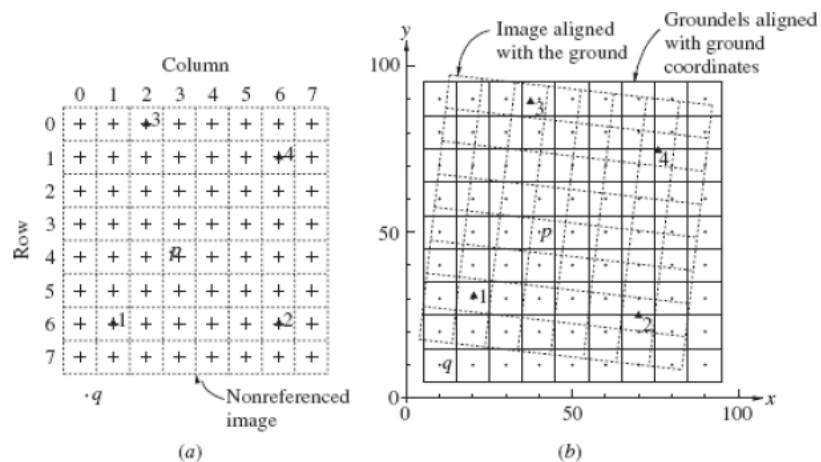


Figure 9-2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

- ❖ Note that in Fig. 9-2a the column coordinates increase from left to right, similar to conventional X coordinates
 - ⇒ However the row coordinates increase from top to bottom, which is in the opposite direction from conventional Y coordinates
 - ⇒ Because of this configuration, using the column coordinate as the abscissa and the row coordinate as the ordinate results in a "left-handed" coordinate system, as opposed to the more commonly used "right-handed" system, which is typical of ground coordinate systems

- ❖ To prepare for the coordinate transformation which will yield the mathematical relationship between the image and ground coordinates, it is useful to convert the column and row coordinates of image pixels to the more conventional XY system

9-3. Georeferencing of Digital Imagery

- ❖ Transformation of a left-handed system to a right-handed system can cause mathematical inconsistencies in certain coordinate transformations, such as the conformal transformation

⇒ This inconsistency can be remedied by a number of approaches.

Approach 1

- To use the row coordinate as the abscissa and the column coordinate as the ordinate
- This results in a right-handed coordinate system which is rotated 90° clockwise

Approach 2

- To simply use a transformation that is able to account for the difference between left- and right-handed coordinate systems
- The affine transformation has this capability by virtue of its separate x and y scale factors, one of which will be positive and the other, negative

Approach 3

- To convert the image coordinates from a left-handed system with the origin in the upper left corner to a right-handed system with its origin in the lower left corner
- This can be accomplished by selecting the column coordinate as the abscissa and calculating ordinate values by subtracting row coordinates from the maximum row number

9-3. Georeferencing of Digital Imagery

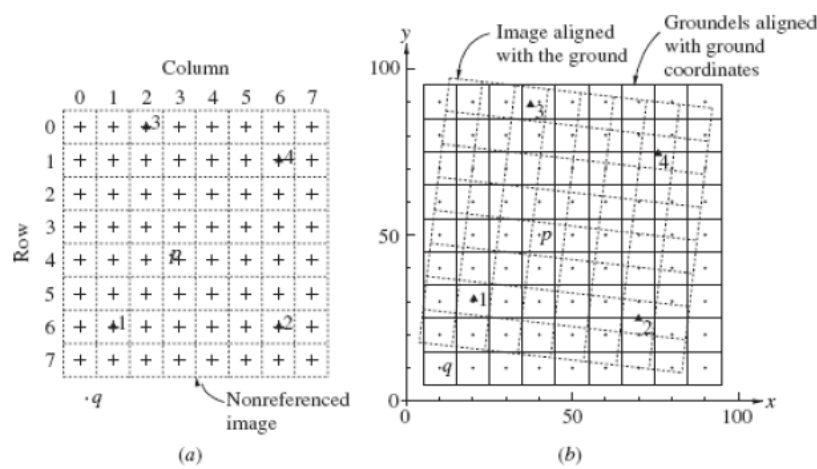


Figure 9–2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

Table 9–2. Column and Row Image Coordinates of Table 9–1 Converted to XY Coordinates

Point	(Original Image Coordinates) Column	(Original Image Coordinates) Row	(Converted Image Coordinates) X = Column	(Converted Image Coordinates) Y = 7 - Row
1	1	6	1	1
2	6	6	6	1
3	2	0	2	7
4	6	1	6	6

- ❖ Assume that the two-dimensional conformal transformation will be used to relate the ground coordinate system of Fig. 9-2b to the image coordinate system of Fig.9-2a
- ❖ Using image coordinates from Table 9-1, a right-handed XY coordinate system will be created using the third method
- ❖ The resulting XY image coordinates are listed in Table 9-2

9-3. Georeferencing of Digital Imagery

- ❖ To transform from the xy ground coordinate system to the XY converted image coordinate system, the following equations [of the form of Eq. (9-1)] are written:

$$\begin{array}{ll}
 \begin{array}{l} X = ax - by + T_X \\ Y = ay + bx + T_Y \end{array} & (9-1) \quad \Rightarrow \quad \begin{array}{l} 20.4a - 30.6b + T_X = 1 + V_{X_1} \\ 30.6a + 20.4b + T_Y = 1 + V_{Y_1} \\ 70.1a - 24.9b + T_X = 6 + V_{X_2} \\ 24.9a + 70.1b + T_Y = 1 + V_{Y_2} \\ 37.1a - 89.3b + T_X = 2 + V_{X_3} \\ 89.3a + 37.1b + T_Y = 7 + V_{Y_3} \\ 75.8a - 74.4b + T_X = 6 + V_{X_4} \\ 74.4a + 75.8b + T_Y = 6 + V_{Y_4} \end{array} \\
 & (9-2)
 \end{array}$$

- ❖ Using the least squares techniques, the equations are solved for the most probable values of a , b , T_X , and T_Y , which are

$$\begin{array}{ll}
 a = 0.09921 & T_X = -0.67 \\
 b = 0.01148 & T_Y = -2.27
 \end{array}$$

- ❖ Once these transformation parameters have been computed, the georeferencing is completed by filling the ground array with brightness values from the digital image

9-3. Georeferencing of Digital Imagery

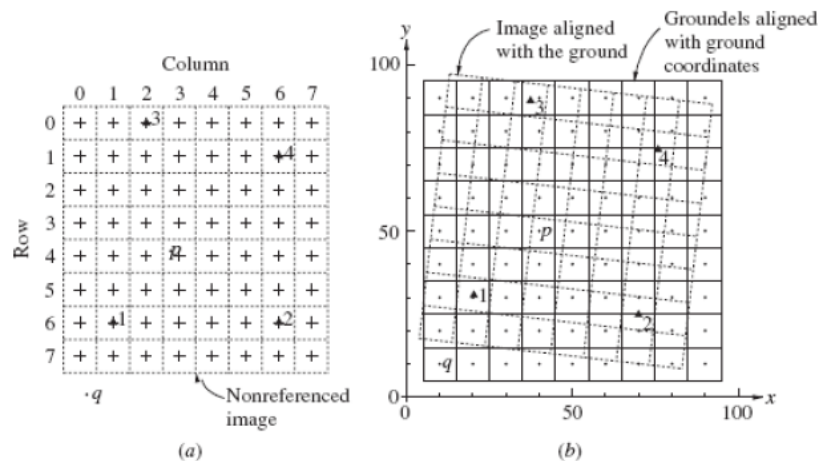


Figure 9-2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

- ❖ For example, point p in Fig. 9-2b has ground coordinates of $x = 40$ and $y = 50$, and its image coordinates X and Y , by substitution into Eqs. (9-1), are

$$X = 40a - 50b + T_x = 40(0.09921) - 50(0.01148) + (-0.67) = 2.7$$

$$Y = 50a + 40b + T_y = 50(0.09921) + 40(0.01148) + (-2.27) = 3.1$$

- ❖ Expressing the location in column and row coordinates gives

$$\text{Column} = X = 2.7$$

$$\text{Row} = 7 - Y = 3.9$$

⇒ This column and row location is indicated by the position of point p in Fig. 9-2a

- ❖ Referring to Fig. 9-2b, which contains a 9×9 groundel grid, this involves 81 separate applications of the above transformation, each followed by access of the brightness value from the image through resampling

9-3. Georeferencing of Digital Imagery

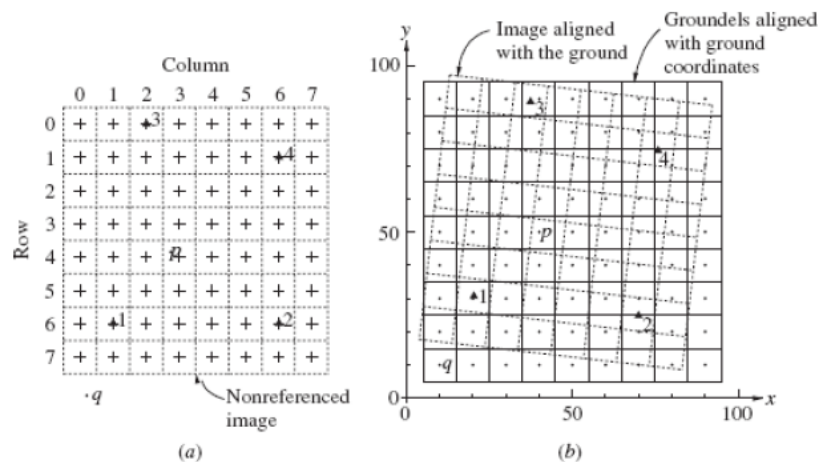


Figure 9-2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

- ❖ Expressing the location in column and row coordinates gives

$$\text{Column} = X = 2.7$$

$$\text{Row} = 7 - Y = 3.9$$

⇒ This column and row location is indicated by the position of point **p** in Fig. 9-2a

- ❖ This position falls within the area of the pixel at column = 3 and row = 4, so the brightness associated with that particular image pixel could be copied into the corresponding groundel at $x = 40$ and $y = 50$

⇒ This is the *nearest-neighbor* method of resampling

⇒ Alternatively, *bilinear* or *bicubic interpolation* could be used for the resampling

- ❖ Referring to Fig. 9-2b, which contains a 9×9 groundel grid, this involves 81 separate applications of the above transformation, each followed by access of the brightness value from the image through resampling

9-3. Georeferencing of Digital Imagery

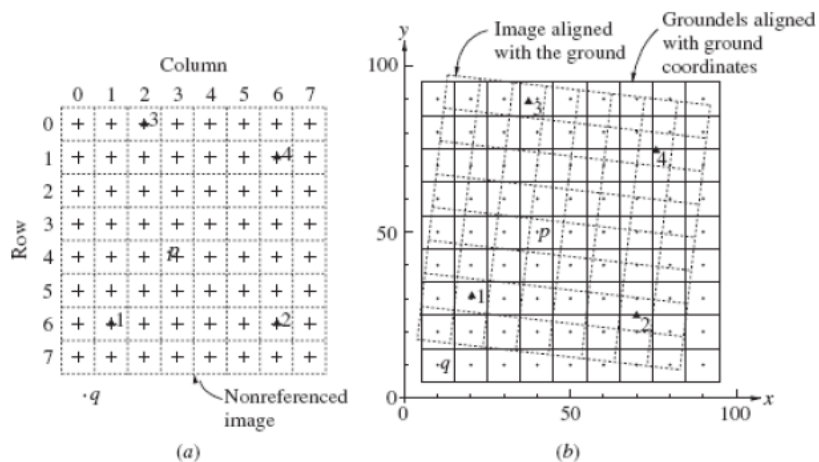


Figure 9-2. (a) Schematic representation of nonreferenced digital image. (b) Schematic representation of georeferenced image (solid lines) and orientation of image aligned with the ground (dashed lines).

- ❖ Transforming these coordinates into the XY image system gives

$$X = 10a - 10b + T_X = 10(0.09921) - 10(0.01148) + (-0.67) = 0.2$$

$$Y = 10a + 10b + T_Y = 10(0.09921) + 10(0.01148) + (-2.27) = -1.2$$

- ❖ Expressing the location in column and row coordinates gives

$$\text{Column} = X = 0.2$$

$$\text{Row} = 7 - Y = 8.2$$

- ❖ The row coordinate of 8.2 is beyond the maximum row number of 7, there is no brightness value from the image corresponding to this particular groundel
 \Rightarrow It is appropriate to use a default value of 0 for the groundel brightness value

9-3. Georeferencing of Digital Imagery

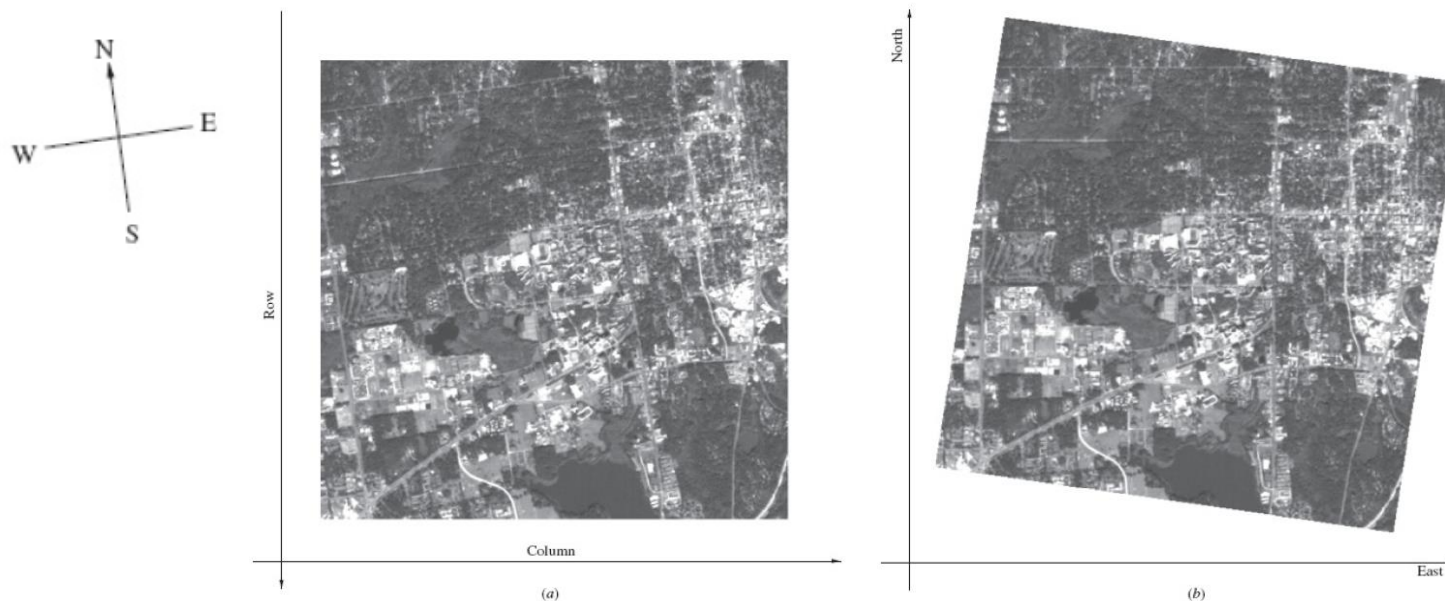


Figure 9-3. (a) Nonreferenced satellite. (b) Georeferenced image. (Courtesy University of Florida.)

- ❖ Figure 9-3 shows the result of georeferencing as applied to a satellite image
- ❖ Figure 9-3a shows the nonreferenced image
- ❖ Notice that even though the true directions of streets in the area correspond to cardinal directions (north, south, east, and west), the general directions of streets appear to be rotated approximately 10° in the image
- ❖ Figure 9-3b shows the resulting georeferenced image where the directions of streets now properly correspond to cardinal directions

9-3. Georeferencing of Digital Imagery

- ❖ When **georeferencing digital imagery**, it is beneficial to choose groundel dimensions that are consistent with the pixel dimensions of the image
- ❖ When choosing ground control points for georeferencing, it is important to select a well-distributed pattern of points, preferably with a point in each of the four corners as well as additional points spaced uniformly throughout the interior
- ❖ It is particularly important to avoid having all control points in a small cluster in one area of the image
- ❖ In addition, if the affine or projective transformation is used, it is critical that not all the control points lie along a straight line; otherwise, the transformation becomes ill-conditioned and yields poor results

9-3. Georeferencing of Digital Imagery

- ❖ Depending on the transformation used and the required geometric accuracy, the number of control points per image should be roughly 8 to 15
- ❖ With a smaller number of control points, the accuracy will generally be lower and it will be more difficult to detect errors
- ❖ For **the highest accuracy**, a large number of control points are always desirable
 - ⇒ However, beyond about 15 points, the marginal benefit of additional points seldom outweighs the extra effort involved
- ❖ The geometric accuracy of a georeferenced image can be assessed through **evaluation of the residuals from the transformation**, which should always be computed and checked
 - ⇒ Large residuals may indicate errors in measured image or ground coordinates
- ❖ Many computer software packages calculate the *root mean square* (RMS) of the residuals, which gives a nominal indication of accuracy
 - ⇒ This value can be misleading, however, since the probability of an actual point's position being within the two-dimensional RMS error region is only about 40%
 - ⇒ To increase the level of confidence to about 95 percent, the RMS value should be multiplied by 2.5

9-3. Georeferencing of Digital Imagery

- ❖ It is possible to compute a transformation with small residuals but still have low accuracy in terms of absolute position
 - ⇒ This would occur if, for example, the ground coordinates were scaled from a map that contains a significant systematic error
- ❖ To disclose systematic errors of this type, it is advisable to perform a field test on a randomly selected sample of image points (checkpoints) using an independent source of higher accuracy, such as a **global positioning system (GPS) survey**
- ❖ The magnitudes of discrepancies between coordinates as obtained from the georeferenced image and those obtained by the higher-accuracy method can be used to assess accuracy

9-4. Heads-Up Digitizing

- ❖ *Heads-up digitizing* is a term used to describe the process where features, which are visible on a georeferenced digital image displayed on a computer screen, are digitized directly with a mouse
- ❖ The human operator views the digital image simultaneously with the digitized features on the computer screen
 - ⇒ Since the operator's gaze remains on the computer screen throughout, the operation is referred to as *heads-up*
- ❖ Heads-up digitizing requires software that is capable of displaying a georeferenced image in the proper coordinate reference frame and enables simultaneous on-screen digitizing
- ❖ A **key advantage of heads-up digitizing** is that the points and lines associated with the digitized features are displayed as an overlay on the digital image, and if **mistakes** are made while pointing on the features, they can be readily detected and subsequently remeasured

9-5. Photomaps

- ❖ Photomaps are simply aerial photos that are used directly as planimetric map substitutes
 - ⇒ Title information, place names, and other data may be superimposed on the photos in the same way that it is done on maps
- ❖ Photomaps may be prepared from single aerial photos, or they may be made by piecing together two or more individual overlapping photos to form a single continuous composite picture
 - ⇒ These composites are commonly referred to as *mosaics*
- ❖ Photomaps show relative planimetric locations of a virtually unlimited number of objects, whereas features on maps—which are shown with lines and symbols—are limited by what was produced by the mapmaker
- ❖ Photomaps of large areas can be prepared in much less time and at considerably lower cost than maps
- ❖ Photomaps are easily understood and interpreted by people without photogrammetry backgrounds because objects are shown by their images
 - ⇒ For this reason they are very useful in describing proposed construction or existing conditions to members of the general public, who may be confused by the same representations on a map

9-5. Photomaps

- ❖ Photomaps have one serious **disadvantage**: they are not true planimetric representations
 - ⇒ Rather, they are constructed from perspective photographs, which are subject to image displacements and scale variations
- ❖ The most serious image displacements and scale variations are caused by variations in the terrain elevation and tilting of the camera axis
 - ⇒ These displacements are generally most severe at the edges of the photographs
 - ⇒ some small errors result from printer distortion of digital images (or shrinkage or expansion of the photo papers), camera lens imperfections, and atmospheric refraction, but these are generally negligible
- ❖ The effects of tilt can be eliminated by rectifying the photograph
 - ⇒ Rectification does not remove the effects of topographic relief, therefore the scale of the photomap is never constant throughout unless the terrain is perfectly level
 - ⇒ Relief displacements can be minimized by increasing the flying height, while at the same time using a longer-focal-length camera to compensate for the decrease in scale

9-5. Photomaps

- ❖ In measuring distances or directions from a photomap, due to image displacements, the scaled values will not be true
 - ⇒ They are often used for qualitative studies only, and in that case slight planimetric inaccuracies caused by image displacements are of little consequence
- ❖ Because of their many **advantages**, photomaps are quite widely used
 - ⇒ Their value is perhaps most appreciated in the field of planning, both in land-use planning and in planning for engineering projects
 - ⇒ A photomap that shows an area completely and comprehensively can be rapidly and economically prepared
- ❖ **Photomaps** are valuable in numerous other miscellaneous areas besides planning
 - ⇒ They are used to study geologic features, to inventory natural resources, to record the growth of cities and large institutions, to monitor construction activities at intervals of time, to record property boundaries, etc

9-6. Mosaics

- ❖ If a single photo does not contain extensive enough coverage to serve as a photomap of an area, an aerial mosaic may be prepared
- ❖ Traditionally, mosaics were constructed manually from hard-copy paper prints, but in this day and age digital mosaics are more common
- ❖ Whether accomplished manually or digitally, mosaics are constructed from a block of overlapping photographs which are trimmed and joined, much as cloth patches are stitched together to form a quilt
- ❖ A special type of digital mosaic, known as a *composite orthophoto*, provides the most geometrically accurate image product available
 - ⇒ However, its production is far more complex than that of a simple mosaic
- ❖ Aerial mosaics generally fall into three classes:

(1) *controlled*

(2) *semicontrolled*

(3) *uncontrolled*

9-6. Mosaics

- ❖ A **controlled mosaic** is the most accurate of the three classes
- ❖ In the manual process, this type of mosaic is prepared from photographs that have been rectified and ratioed; i.e., all prints are made into equivalent vertical photographs that have the same nominal scale
- ❖ In assembling the mosaic, image positions of common features on adjacent photos are matched as closely as possible
- ❖ To increase the overall accuracy of the assembly, a plot of control points is prepared at the same scale as the ratioed photos
- ❖ Then in piecing the photos together to form the mosaic, the control point images are also matched to their corresponding plotted control points to constrain the positions of the photos
- ❖ Controlled digital mosaics have similar characteristics, but they are prepared in an analytical process similar to georeferencing
- ❖ Along the edges between adjacent photos, images of features are aligned to the extent possible, although they will seldom line up exactly
 - ⇒ These residual misalignments exist primarily because of relief displacements.

9-6. Mosaics

- ❖ An **uncontrolled mosaic** is prepared by simply matching the image details of adjacent photos
- ❖ There is no ground control, and aerial photographs that have not been rectified or ratioed are used
- ❖ Uncontrolled mosaics are more easily and quickly prepared than controlled mosaics
⇒ They are not as accurate as controlled mosaics, but for many qualitative uses they are completely satisfactory
- ❖ **Semiconrolled mosaics** are assembled by utilizing some combinations of the specifications for controlled and uncontrolled mosaics
- ❖ A semiconrolled mosaic may be prepared, for example, by using ground control but employing photos that have not been rectified or ratioed
- ❖ The other combination would be to use rectified and ratioed photos but no ground control
- ❖ Semiconrolled mosaics are a compromise between economy and accuracy

9-6. Mosaics

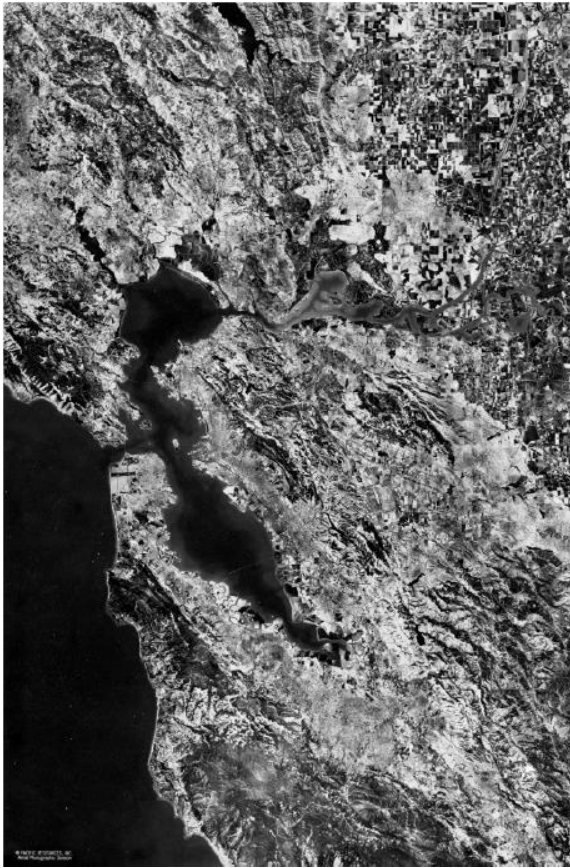


Figure 9–4. Semicontrolled mosaic showing the entire San Francisco Bay area. This mosaic consists of more than 2000 individual aerial photographs.
(Copyright photo, courtesy Pacific Resources, Inc.)

- ❖ The mosaic of Fig. 9-4 is a semicontrolled mosaic prepared from nonrectified photos but assembled to fit U.S. Geological Survey quadrangle maps

9-7. Uncontrolled Digital Mosaics

- ❖ As noted above, a fundamental operation performed in the production of mosaics is the matching of images from adjacent photographs
- ❖ With digital images, the matching procedure involves identifying *tie points* in the overlap areas between adjacent photos
- ❖ To match a pair of photos, coordinates of the common tie points are measured in both photos, and a two-dimensional coordinate transformation is performed to define the geometric relationship between the photographs

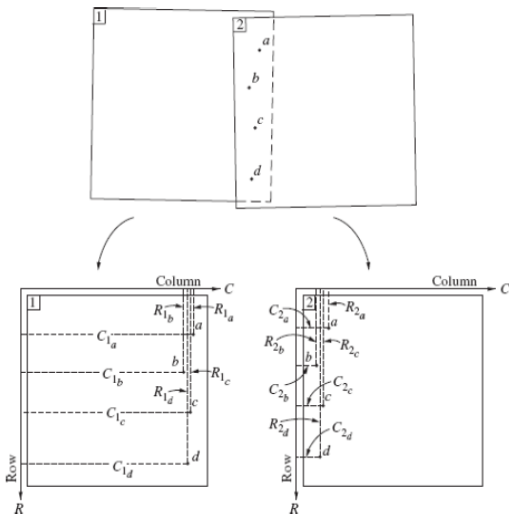


Figure 9-5. Use of tie points to join adjacent digital photographs.

- ❖ This process is illustrated in Fig. 9-5, which shows two adjacent photographs in superposition
- ❖ In this figure, the overlap area between photos 1 and 2 contains four tie points, *a, b, c, and d*
- ❖ In the separated photographs shown at the bottom of the figure, each of these tie points has row and column coordinates in the digital image coordinate system of each photo
 \Rightarrow For instance, point *a* has coordinates C_{1a}, R_{1a} in photo 1 and C_{2a}, R_{2a} in photo 2

9-7. Uncontrolled Digital Mosaics

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- ❖ With digital images, the matching procedure involves identifying *tie points* in the overlap areas between adjacent photos
- ❖ To match a pair of photos, coordinates of the common tie points are measured in both photos, and a two-dimensional coordinate transformation is performed to define the geometric relationship between the photographs

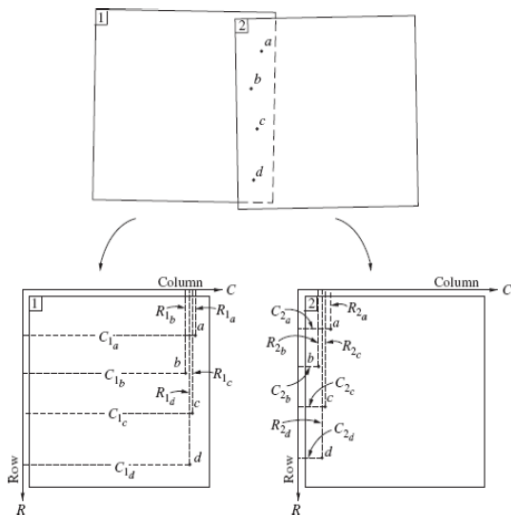


Figure 9-5. Use of tie points to join adjacent digital photographs.

- ❖ By selecting the coordinates of the tie points from photo 2 as control, a two-dimensional coordinate transformation can be computed that determines the parameters for transforming photo 1 coordinates to photo 2 coordinates
- ❖ In the example of Fig. 9-5, since the tie points lie nearly in a straight line, a conformal transformation should be used because more complex transformations, such as affine or projective, become mathematically unstable when the control configuration is linear or nearly so

9-7. Uncontrolled Digital Mosaics

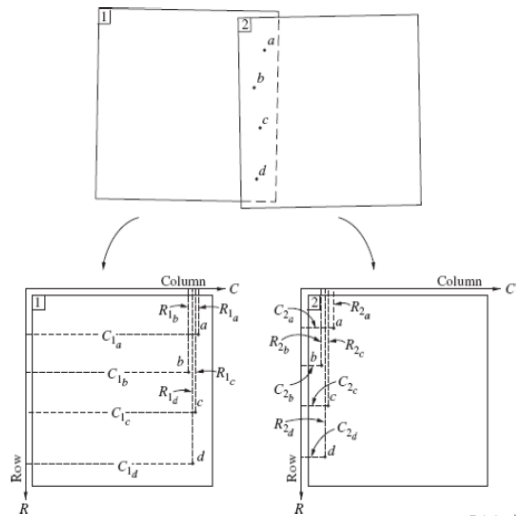


Figure 9-5. Use of tie points to join adjacent digital photographs.

- ❖ After the transformation, the digital coverage of photo 1 can be extended to include the additional area covered by photo 2
- ❖ This is done by increasing the number of columns in the photo 1 array to create sufficient storage space to accommodate additional pixels from the photo 2 array
- ❖ The brightness values associated with these additional columns are initially undefined

- ❖ Then, in a process similar to that of georeferencing, the coordinates of each pixel in the extended area of photo 1 are transformed to the coordinate system of photo 2, using the previously determined parameters
- ❖ Brightness values are then resampled from photo 2 and stored in the extended portion of the photo 1 array
- ❖ The result is a single image which incorporates the coverage of both photos

9-7. Uncontrolled Digital Mosaics

- ❖ When two photographs are attached by the foregoing method, a choice can be made as to which one is used to provide the brightness values in the overlap area
- ❖ The simplest approach is to retain the values from the photo 1 array, however, this is generally not the best approach for two reasons
 - (1) The extreme right edge of photo 1 typically contains the image of a side fiducial mark, which obscures the underlying terrain
 - (2) Since relief displacements are more severe at greater distances from the principal point of a vertical photo, images of features along the seam between photos are more likely to be disjointed

⇒ To minimize these problems, the common area should be divided approximately into two halves, with one-half of the brightness values coming from each photograph
- ❖ Large, uncontrolled digital mosaics can be constructed by repeating this attachment process for each photograph in a strip or block
- ❖ Successive attachments of photos create an accumulation of errors and can cause the digital mosaic to become significantly warped

⇒ This effect can be reduced somewhat by starting in the middle of the block and working outward in all directions

9-8. Semicontrolled Digital Mosaics

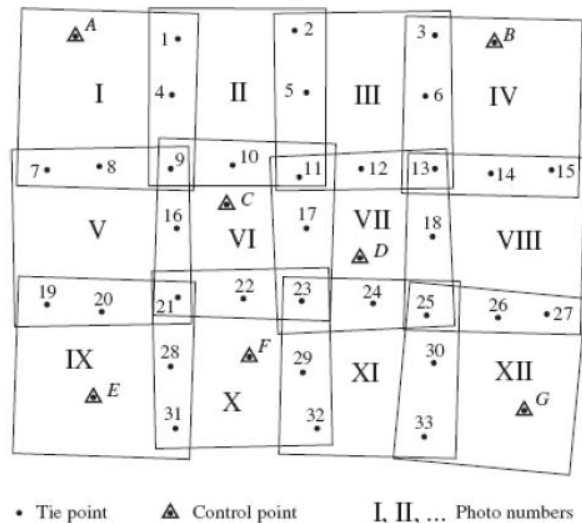


Figure 9-6. Control and tie point configuration for a semicontrolled digital mosaic.

- ❖ The more likely situation associated with generation of a **semicontrolled digital mosaic** is to use nonrectified photographs in conjunction with a distributed configuration of control points
- ❖ This situation is illustrated in Fig. 9-6
- ❖ The figure shows 12 photographs (I~XII) arranged in a block formation with tie points (1~33) in the overlap areas between adjacent photos

- ❖ Seven control points (**A** through **G**) are distributed about the block
 - ⇒ Note that there is no single photograph with more than one control point
 - ⇒ Therefore, individual rectification of photographs is not possible
 - ⇒ It is possible, however, to simultaneously transform points from all 12 photographs in a single least squares adjustment

9-8. Semicontrolled Digital Mosaics

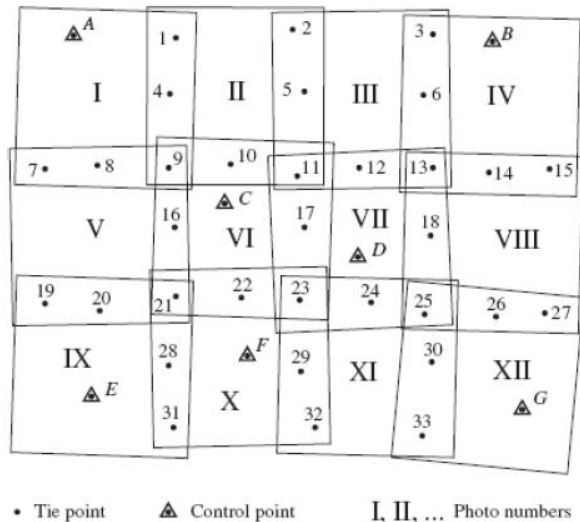


Figure 9-6. Control and tie point configuration for a semicontrolled digital mosaic.

- ❖ Since the block shown in Fig. 9-6 contains 12 individual photographs, 12 sets of four parameters (total of 48) are required to transform the individual photographs to the ground coordinate system
- ❖ Calculation of the 48 unknown parameters requires a least squares adjustment involving two forms of observation equation pairs

$$\begin{aligned} X &= ax - by + T_x \\ Y &= ay + bx + T_y \end{aligned} \quad (9-1)$$

- ❖ The first type of observation equation pair is used for control points, and it has the same form as Eqs. (9-1) except that appropriate subscripts are included for the parameters and coordinates

9-8. Semicontrolled Digital Mosaics

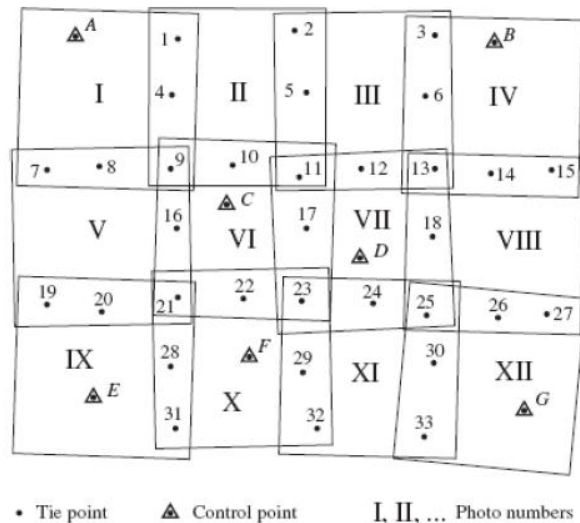


Figure 9-6. Control and tie point configuration for a semicontrolled digital mosaic.

$$X_c = a_{VI}x_{c_{VI}} - b_{VI}y_{c_{VI}} + T_{x_{VI}} \quad (9-3)$$

$$Y_c = a_{VI}y_{c_{VI}} + b_{VI}x_{c_{VI}} + T_{y_{VI}}$$

- ❖ Equations (9-3) express the observation equation pair which corresponds to the coordinates of control point **C** on photo **VI**, with appropriate subscripts
- ❖ Equations of this form are written for each control point and for each photo in which it appears

- ❖ As an example, note that in Fig. 9-6 control point **C** appears in photo **VI**. Digital photo coordinates **x** and **y** of the image of **C** in photo **VI**, which can be obtained from the row and column as described in Sec. 9-3, appear on the right-hand side of Eqs. (9-1); and **X** and **Y** ground coordinates of point **C** appear on the left
- ❖ Also included in the right-hand side of Eqs. (9-1) are the parameters **a**, **b**, **T_x**, and **T_y** which correspond to photo **VI**

9-8. Semicontrolled Digital Mosaics

- ❖ The second type of observation pair is for tie points
- ❖ Since tie points have no ground coordinates associated with them, they cannot be used directly in Eqs. (9-1)
- ❖ Instead, the condition that the differences in ground coordinates X and Y for the common tie point based on the photo coordinates x and y in two adjacent photographs must equal zero, is enforced

- ❖ An example of this condition is given by Eqs. (9-4), which express the relationship for tie point 1 between photos **I** and **II**

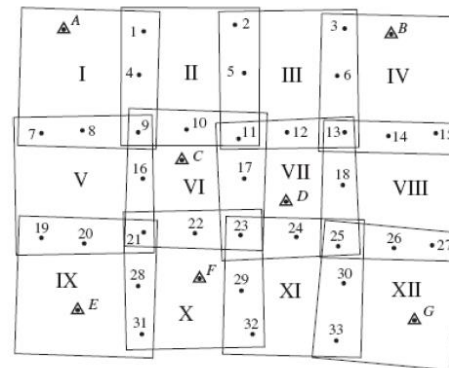
$$\begin{aligned} a_I x_{1_I} - b_I y_{1_I} + T_{x_I} - (a_{II} x_{1_{II}} - b_{II} y_{1_{II}} + T_{x_{II}}) &= 0 \\ a_I y_{1_I} + b_I x_{1_I} + T_{y_I} - (a_{II} y_{1_{II}} + b_{II} x_{1_{II}} + T_{y_{II}}) &= 0 \end{aligned} \quad (9-4)$$

- ❖ A pair of equations of the form of Eqs. (9-4) is written for each occurrence of a tie point between a pair of photographs

9-8. Semicontrolled Digital Mosaics

Table 9-3. Connections Between Photos of Fig. 9-6 Based on Measured Tie Point Coordinates

Point	Photo <i>i</i>	Photo <i>j</i>	Point	Photo <i>i</i>	Photo <i>j</i>	Point	Photo <i>i</i>	Photo <i>j</i>
1	I	II	13	III	VII	13	VII	VIII
4	I	II	13	III	VIII	18	VII	VIII
9	I	II	13	IV	VII	25	VII	VIII
7	I	V	13	IV	VIII	23	VII	X
8	I	V	14	IV	VIII	23	VII	XI
9	I	V	15	IV	VIII	24	VII	XI
9	I	VI	9	V	VI	25	VII	XI
2	II	III	16	V	VI	25	VII	XII
5	II	III	21	V	VI	25	VIII	XI
11	II	III	19	V	IX	25	VIII	XII
9	II	V	20	V	IX	26	VIII	XII
9	II	VI	21	V	IX	27	VIII	XII
10	II	VI	21	V	X	21	IX	X
11	II	VI	11	VI	VII	28	IX	X
11	II	VII	17	VI	VII	31	IX	X
3	III	IV	23	VI	VII	23	X	XI
6	III	IV	21	VI	IX	29	X	XI
13	III	IV	21	VI	X	32	X	XI
11	III	VI	22	VI	X	25	XI	XII
11	III	VII	23	VI	X	30	XI	XII
12	III	VII	23	VI	XI	33	XI	XII



• Tie point ▲ Control point I, II, ... Photo numbers

Figure 9-6. Control and tie point configuration for a semicontrolled digital mosaic.

❖ Table 9-3 lists all the unique occurrences of tie points between adjacent photographs corresponding to Fig. 9-6

Eqs. (9-4)

$$a_1x_1 - b_1y_1 + T_{x_1} - (a_2x_{12} - b_2y_{12} + T_{x_2}) = 0$$

$$a_1y_1 + b_1x_1 + T_{y_1} - (a_2y_{12} + b_2x_{12} + T_{y_2}) = 0$$

- ❖ In this table, there are a total of 63 unique tie point connections between photographs, each of which contributes two equations of the form of Eqs. (9-4)
- ❖ Tie points, such as point 1, which exist only on two photographs contribute only one pair of equations, whereas tie points, such as point 9, which exist on four photographs contribute six pairs of equations to the solution

9-8. Semicontrolled Digital Mosaics

- ❖ After all the equations have been formed, the unknown parameters (a , b , T_X , and T_Y) associated with each photo are computed by least squares
- ❖ These parameters specify the geometric relationships between each photo and the ground system
 - ⇒ However, the direction is the opposite of what is required
- ❖ Regarding georeferencing, coordinates of groundels must be transformed to the image coordinate system so that the brightness of the particular ground element can be determined
 - ⇒ Since parameters a , b , T_X , and T_Y in Eqs. (9-2) transform from image coordinates to ground, the equation must be rearranged
 - ⇒ To do this, equations can be written in matrix form as

$$20.4a - 30.6b + T_X = 1 + V_{X_1}$$

$$30.6a + 20.4b + T_Y = 1 + V_{Y_1}$$

$$70.1a - 24.9b + T_X = 6 + V_{X_2}$$

$$24.9a + 70.1b + T_Y = 1 + V_{Y_2}$$

$$37.1a - 89.3b + T_X = 2 + V_{X_3}$$

$$89.3a + 37.1b + T_Y = 7 + V_{Y_3}$$

$$75.8a - 74.4b + T_X = 6 + V_{X_4}$$

$$74.4a + 75.8b + T_Y = 6 + V_{Y_4}$$

(9-2) 

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \end{bmatrix} \quad (9-5)$$

9-8. Semicontrolled Digital Mosaics

- ❖ Solving Eq. (9-5) for the values of x and y results in Eq. (9-6)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \end{bmatrix} \quad (9-5) \quad \Rightarrow \quad \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}^{-1} \begin{bmatrix} X - T_X \\ Y - T_Y \end{bmatrix} = \frac{1}{a^2 + b^2} \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X - T_X \\ Y - T_Y \end{bmatrix}} \quad (9-6)$$

- ❖ By using the X and Y coordinates for each of the groundels in the area covered by the mosaic and the parameters of the corresponding photograph, Eq. (9-6) can be solved to obtain the position on the photo from which to extract the brightness values
 - ⇒ The values are then resampled from the digital photo and placed in the groundel array
 - ⇒ In areas of overlap between photographs, brightness values can be extracted from either photo

9-9. Controlled Digital Mosaics

- ❖ Controlled mosaics are made from rectified and ratioed photographs in conjunction with ground control ⇒ With digital mosaics, this process begins with photo rectification
⇒ In this process control points are identified in each photograph, and the two-dimensional projective transformation is used to georeference the image, using the process described in Sec. 9-3
- ❖ A minimum of four control points are needed, although more than four points are recommended in order to have redundancy and enable a least squares solution
⇒ Since each rectified digital photograph is georeferenced, the individual groundels will be directly related to ground coordinates
- ❖ All that remains to create the controlled digital mosaic is to assemble the georeferenced images in a single groundel array that encompasses the full ground area of the mosaic
⇒ No further resampling or other significant processing is required, although choices must be made as to which brightness values to choose in overlapping areas
- ❖ An alternative approach can be used where there is insufficient ground control to rectify each digital photograph individually
⇒ It is a slight variation of the simultaneous coordinate transformation approach
⇒ The difference is that instead of using a two-dimensional conformal or affine transformation, the projective transformation is used

9-9. Controlled Digital Mosaics

- ❖ A simultaneous projective transformation is more complicated than either the conformal or the affine because the equations are nonlinear
- ❖ Performing a solution of the nonlinear form of the projective equations requires a **Taylor series expansion** of Eqs.(9-7) as well as initial approximations for the unknown parameters

$$x = \frac{a_1x + b_1y + c_1}{a_3x + b_3y + 1}, \quad y = \frac{a_2x + b_2y + c_2}{a_3x + b_3y + 1} \quad (9-7)$$

- ❖ For near-vertical photographs, a simultaneous affine transformation can be used to obtain initial approximations, because the parameters a_3 , b_3 of Eqs. (9-7) are approximately zero
- ❖ This causes the denominators in these equations to reduce to unity, and thus the projective Eqs. (9-7) are equivalent to the affine equations
- ❖ When the initial approximations have been determined, the simultaneous projective transformation equations (which have been linearized by Taylor's series) can then be formed, and iterated until the solution converges
- ❖ This results in a solution for the projective parameters which specifies the geometric relationship between each photograph and the ground coordinate system

9-9. Controlled Digital Mosaics

- ❖ As was the case with the simultaneous conformal transformation method, the computed parameters describe the relationship in the reverse sense from that required
- ❖ To reverse Eqs. (9-7) so that the x and y photo coordinates are isolated on the left requires substantial algebraic manipulation
- ❖ The algebraic steps required to reverse these equations are left to the reader
- ❖ Equations (9-8) express the necessary relationship in reverse

$$x = \frac{a_1x + b_1y + c_1}{a_3x + b_3y + 1}, \quad y = \frac{a_2x + b_2y + c_2}{a_3x + b_3y + 1} \quad (9-7)$$

$$x = \frac{(b_2 - b_3c_2)X + (b_3c_1 - b_1)Y + b_1c_2 - b_2c_1}{(a_2b_3 - a_3b_2)X + (a_3b_1 - a_1b_3)Y + a_1b_2 - a_2b_1} \quad y = \frac{(a_3c_2 - a_2)X + (a_1 - a_3c_1)Y + a_2c_1 - a_1c_2}{(a_2b_3 - a_3b_2)X + (a_3b_1 - a_1b_3)Y + a_1b_2 - a_2b_1} \quad (9-8)$$

9-9. Controlled Digital Mosaics

$$x = \frac{(b_2 - b_3c_2)X + (b_3c_1 - b_1)Y + b_1c_2 - b_2c_1}{(a_2b_3 - a_3b_2)X + (a_3b_1 - a_1b_3)Y + a_1b_2 - a_2b_1} \quad y = \frac{(a_3c_2 - a_2)X + (a_1 - a_3c_1)Y + a_2c_1 - a_1c_2}{(a_2b_3 - a_3b_2)X + (a_3b_1 - a_1b_3)Y + a_1b_2 - a_2b_1} \quad (9-8)$$

- ❖ The least squares solution is often sensitive to large amounts of relief in the project area
- ❖ Since the solution involves tie points for which there is no known height, corrections cannot be made for their relief displacements
- ❖ If heights in object space of the selected tie points vary substantially from mean terrain, the projective equations can become illconditioned, which can cause the least squares solution to fail
- ❖ In this case, the only recourse is to rectify each digital photograph individually and assemble them as mentioned at the beginning of this section
- ❖ In addition to the difficulties caused by substantial relief, simultaneous projective transformations require more control points throughout the photo block as well as more tie points along the edges than either the simultaneous conformal or affine transformation