Chapter 10.
Tilted and Oblique Photographs

10-1. Introduction

- Unavoidable aircraft tilts cause photographs to be exposed with the camera axis tilted slightly from vertical, and the resulting pictures are called *tilted photographs*

- In some cases, aerial photography is purposely angled away from vertical
  - ⇨ These types of images are classified as *high oblique* if the photograph contains the horizon, and *low oblique* otherwise

- Terrestrial photos are almost always taken from an oblique pose
  - ⇨ Horizontal terrestrial photos are obtained if the camera axis is horizontal when the exposure is made

- Six independent parameters called the elements of exterior orientation (sometimes called EOPs) express the spatial position and angular orientation of a photograph
  - ⇨ The spatial position is normally given by $X_L, Y_L,$ and Z_L, the three dimensional coordinates of the exposure station in a ground coordinate system
  - ⇨ Z_L is called H, the height above datum

- Angular orientation is the amount and direction of tilt in the photo

10-1. Introduction

- Unavoidable aircraft tilts cause photographs to be exposed with the camera axis tilted slightly from vertical, and the resulting pictures are called tilted photographs
- In some cases, aerial photography is purposely angled away from vertical
  - ⇨ These types of images are classified as high oblique if the photograph contains the horizon, and low oblique otherwise
- Terrestrial photos are almost always taken from an oblique pose
  - ⇨ Horizontal terrestrial photos are obtained if the camera axis is horizontal when the exposure is made

In this book, two different systems are described:

(1) the tilt-swing-azimuth ($t - s - \alpha$) system
(2) the omega-phi-kappa ($\omega - \phi - \kappa$) system

- The omega-phi-kappa system possesses certain computational advantages over the tilt-swing-azimuth system and is therefore more widely used
- The tilt-swing-azimuth system, however, is more easily understood and shall therefore be considered first

10-2. Point Perspective

- Perspective can be defined as how something is visually perceived under varying circumstances
- Some important properties of a *perspective projection* are that straight lines are preserved and projected lines that are parallel in object space intersect when projected onto the image plane
- More specifically, horizontal parallel lines intersect at the horizon, and vertical lines intersect at either the zenith or nadir in an image
  ⇨ The locations at which these lines intersect on images are called *vanishing points*

10-2. Point Perspective

Figure 10-1. One-point perspective image

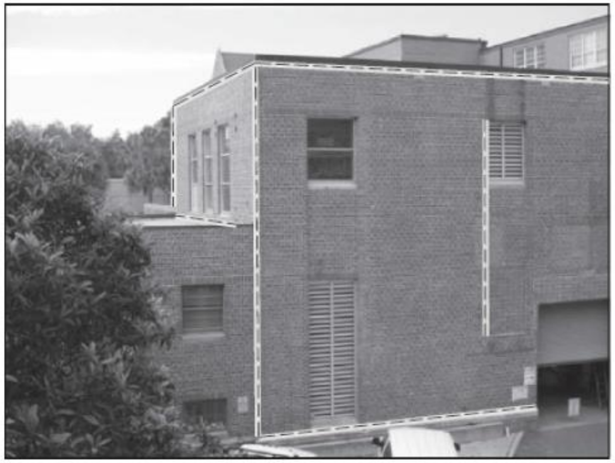


Figure 10-2. Two-point perspective image.

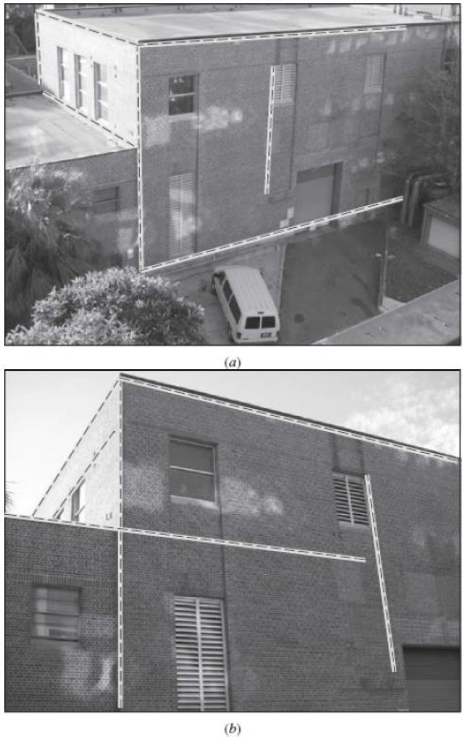


Figure 10-3. (a) Three-point perspective image where vertical lines intersect at nadir. (b) Three-point perspective image where vertical lines intersect at zenith.

❖ Figures 10-1, 10-2, and 10-3 illustrate one-, two-, and three-point perspective images of a building, respectively

❖ Notice that in Fig. 10-1, the horizontal lines at the left appear to converge toward a vanishing point in the distance, while the vertical lines appear parallel and the horizontal lines to the right appear parallel

⇨ For this figure, the optical axis of the camera was oriented perpendicular to the side of the building which resulted in one point perspective

10-2. Point Perspective

Figure 10–1. One–point perspective image.

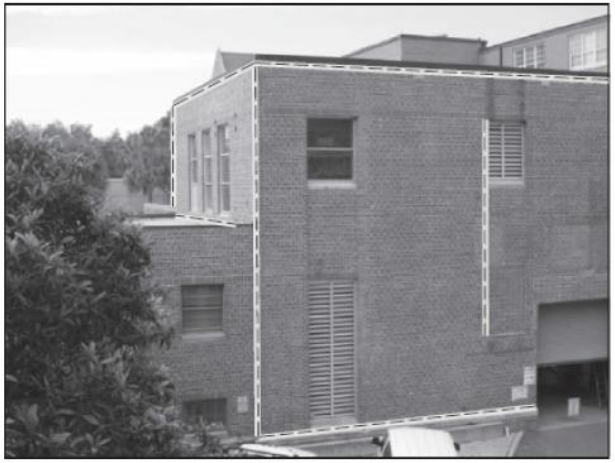
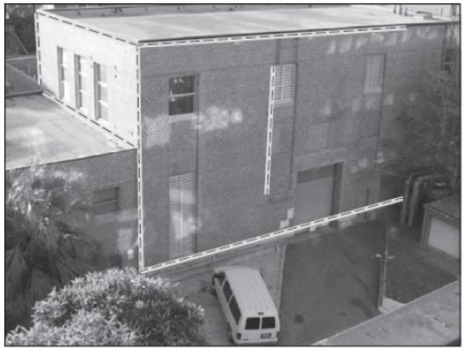
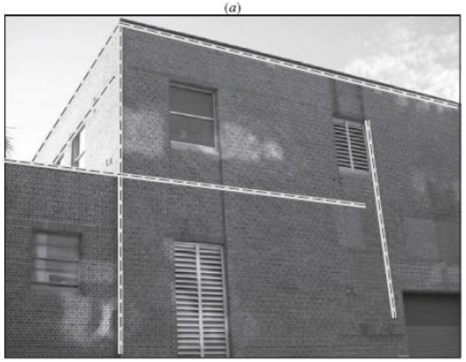


Figure 10–2. Two–point perspective image.



(a)



(b)

Figure 10–3. (a) Three–point perspective image where vertical lines intersect at nadir. (b) Three–point perspective image where vertical lines intersect at zenith.

- In Fig. 10-2, there are two sets of horizontal parallel lines shown along two sides of the building
- Both sets of lines, if extended, intersect at two points on the horizon to the left and right of the photo
- The vertical lines shown in the photo appear parallel because the optical axis of the camera was horizontal (perpendicular to the vertical)
- It was not perpendicular to either side of the building however, which resulted in two point perspective

10-2. Point Perspective

Figure 10–1. One–point perspective image.

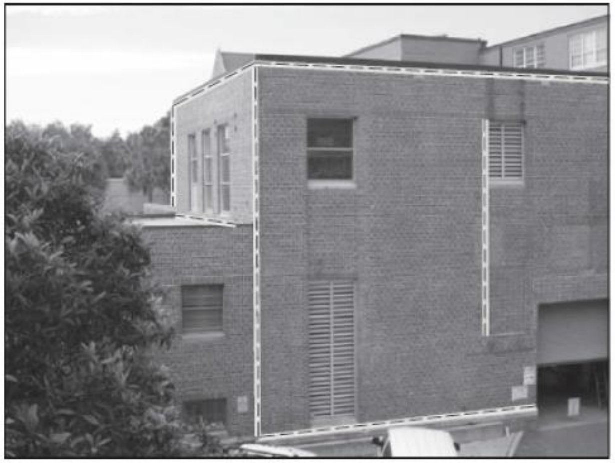


Figure 10–2. Two–point perspective image.

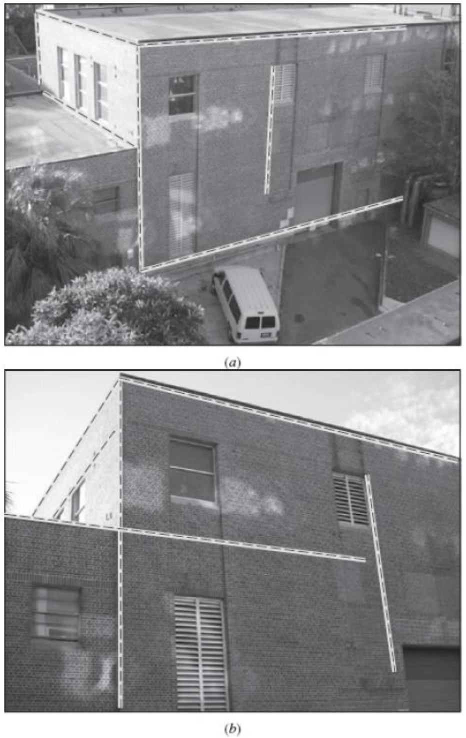


Figure 10–3. (a) Three–point perspective image where vertical lines intersect at nadir. (b) Three–point perspective image where vertical lines intersect at zenith.

- Fig. 10-3 shows three point perspective of the building from two different poses
- In Fig. 10-3a, the horizontal lines also converge toward two points on the horizon, and since the camera is inclined down from horizontal, the vertical lines converge toward the nadir
- Conversely, in Fig. 10-3b, since the camera is inclined up from horizontal, the vertical lines converge toward the zenith

10-3. Angular Orientation in Tilt, Swing, and Azimuth

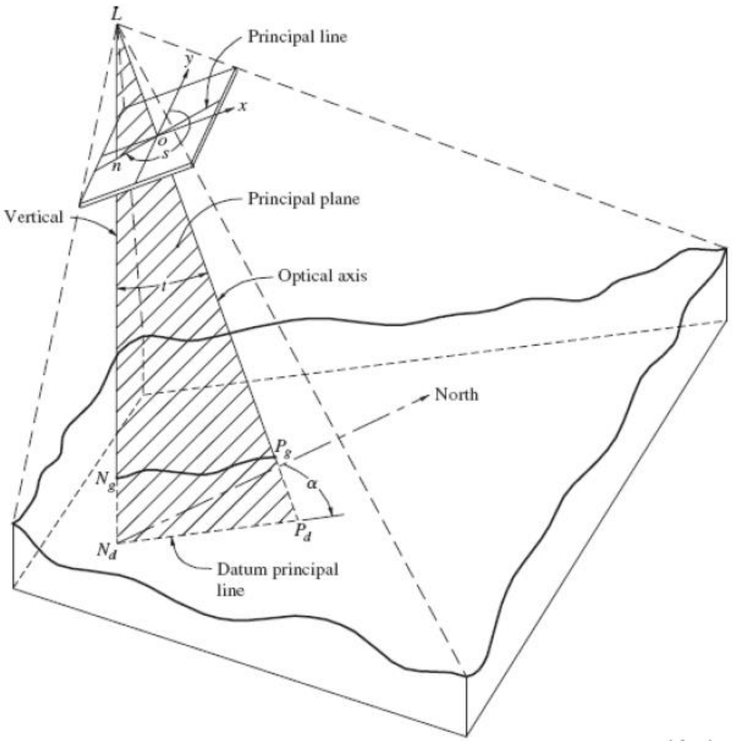


Figure 10-4. Geometry of a tilted photograph showing tilt-swing-azimuth angular orientation.

- In Fig. 10-4, a tilted aerial photograph is depicted showing the tilt-swing-azimuth angular orientation parameters
- In the figure, L is the exposure station and o is the principal point of the photo positive
- Line Ln is a vertical line, n being the *photographic nadir point*, which occurs where the vertical line intersects the plane of the photograph
- The extension of Ln intersects the ground surface at N_g, the *ground nadir point*, and it intersects the datum surface at N_d, the *datum nadir point*
- Line Lo is the camera optical axis; its extension intersects the ground at P_g, the *ground principal point*, and it intersects the datum plane at P_d, the *datum principal point*

10-3. Angular Orientation in Tilt, Swing, and Azimuth

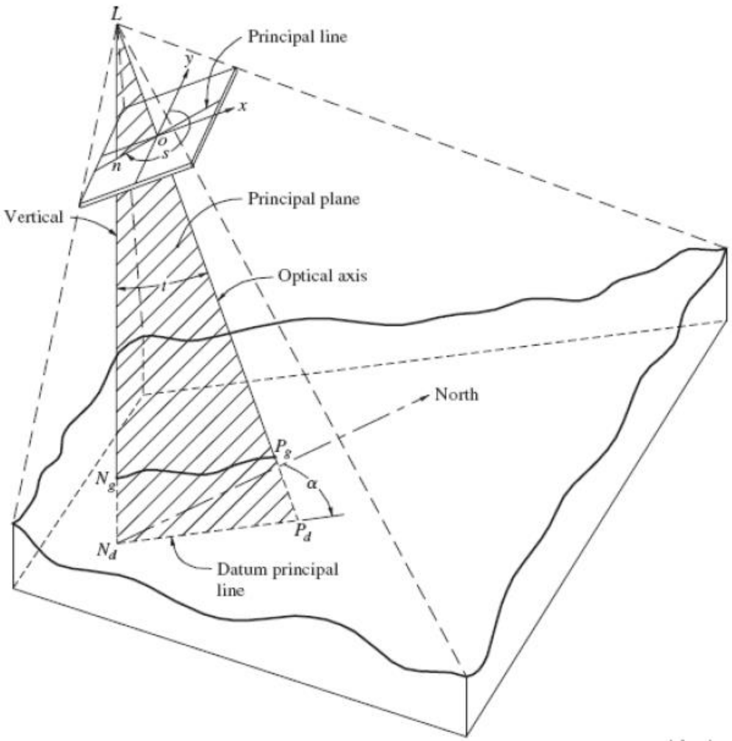


Figure 10-4. Geometry of a tilted photograph showing tilt-swing-azimuth angular orientation.

- In Fig. 10-4, a tilted aerial photograph is depicted showing the tilt-swing-azimuth angular orientation parameters
- In the figure, L is the exposure station and o is the principal point of the photo positive
- Line Ln is a vertical line, n being the *photographic nadir point*, which occurs where the vertical line intersects the plane of the photograph

- One of the orientation angles, *tilt*, is the angle t or nLo between the optical axis and the vertical line Ln
- The tilt angle gives the magnitude of tilt of a photo

10-3. Angular Orientation in Tilt, Swing, and Azimuth

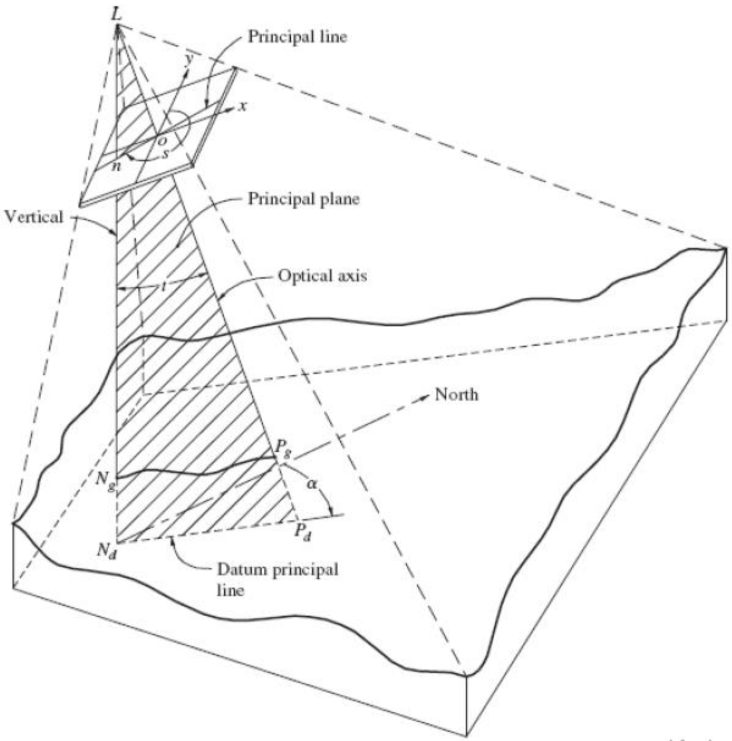


Figure 10-4. Geometry of a tilted photograph showing tilt-swing-azimuth angular orientation.

- Vertical plane Lno is called the *principal plane*
- Its line of intersection with the plane of the photograph occurs along line no, which is called the *principal line*
- The position of the principal line on the photo with respect to the reference fiducial axis system is given by s, the *swing* angle

- *Swing* is defined as <u>the clockwise angle measured in the plane of the photograph from the positive y axis to the downward or nadir end of the principal line</u>, as shown in Fig. 10-4
- The swing angle gives the direction of tilt on the photo. Its value can be anywhere between 0° and 360°, or alternatively, between -180° and 180°

10-3. Angular Orientation in Tilt, Swing, and Azimuth

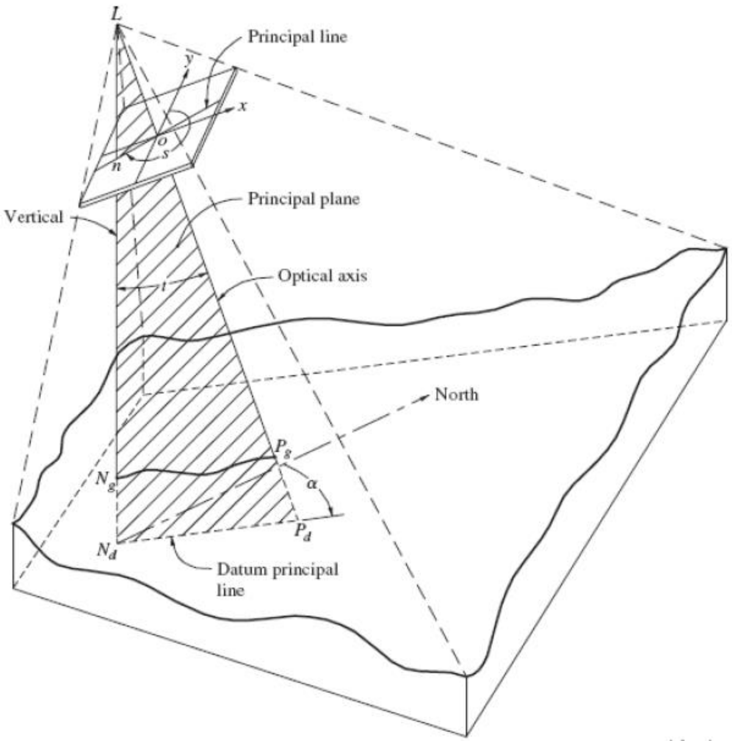


Figure 10-4. Geometry of a tilted photograph showing tilt-swing-azimuth angular orientation.

- The third angular orientation parameter, α or *azimuth*, gives the orientation of the principal plane with respect to the ground reference axis system
- Azimuth is the clockwise angle measured from the ground Y axis (usually north) to the *datum principal line*, $N_d P_d$
- It is measured in the datum plane or in a plane parallel to the datum plane, and its value can be anywhere between 0° and 360°, or alternatively, between -180° and 180°

- The three angles of tilt, swing, and azimuth completely define the angular orientation of a tilted photograph in space
- If the tilt angle is zero, the photo is vertical, thus a vertical photograph is simply a special case of the general tilted photograph

10-4. Auxiliary Tilted Photo Coordinate System

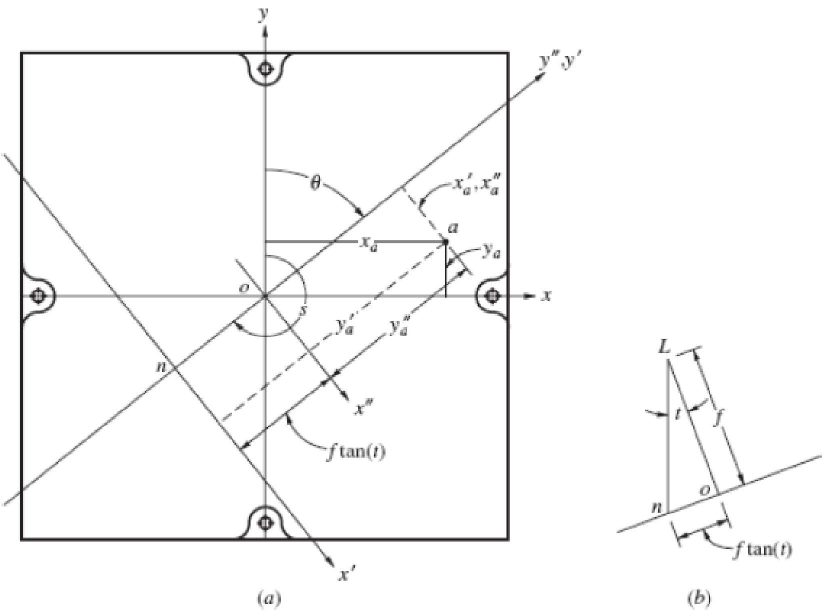


Figure 10–5. (a) Auxiliary $x'y'$image coordinate system for a tilted photograph. (b) Principal plane of a tilted photo.

- In the tilt-swing-azimuth system, certain computations require the use of an auxiliary $x'y'$ rectangular photographic coordinate system, as shown in Fig. 10-5a
- This system, as shown in Fig. 10-5a, has its origin at the photographic nadir point n and its y' axis coincides with the principal line (positive in the direction from n to o)
- Positive $x'$is 90° clockwise from positive y'

- In solving tilted photo problems in the tilt-swing-azimuth system, photo coordinates are usually first measured in the fiducial coordinate system and then converted to the auxiliary system numerically

10-4. Auxiliary Tilted Photo Coordinate System

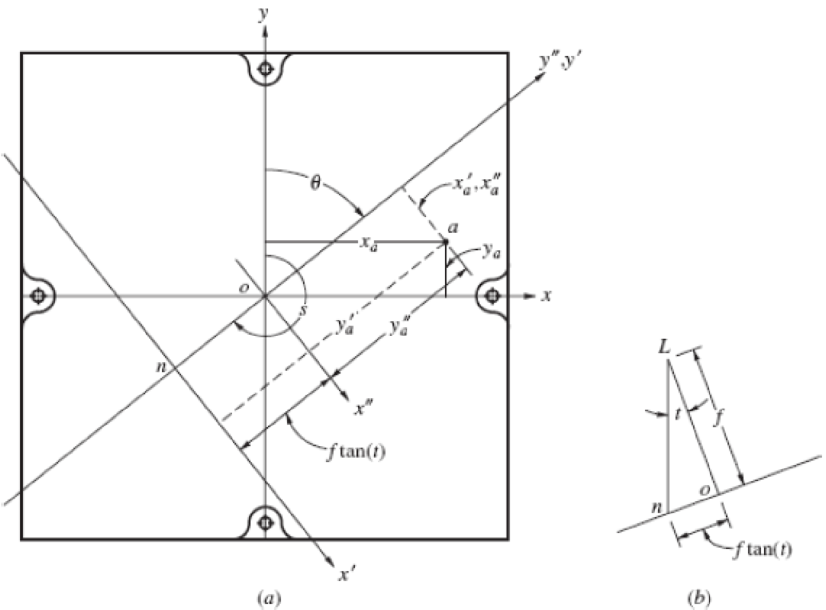


Figure 10−5. (a) Auxiliary $x'y'$image coordinate system for a tilted photograph. (b) Principal plane of a tilted photo.

- For any point in a tilted photo, the conversion from the $\boldsymbol{xy}$ fiducial system to the $\boldsymbol{x'y'}$-tilted system requires
  - (1) a rotation about the principal point through the angle $\boldsymbol{\theta}$
  - (2) a translation of origin from $\boldsymbol{o}$ to $\boldsymbol{n}$

- The rotation angle $\boldsymbol{\theta}$ is defined as

$$\theta = s - 180° \qquad (10\text{-}1)$$

- The coordinates of image point $\boldsymbol{a}$ after rotation are $\boldsymbol{x''_a}$ and $\boldsymbol{y'_a}$, as shown in Fig. 10-5a
- These coordinates, calculated in the same fashion as the $\boldsymbol{E'_c}$ and $\boldsymbol{N'_c}$ coordinates of Eqs. (C-2), are specified by

$$x'' = x_a \cos\theta - y_a \sin\theta$$

$$y'' = x_a \sin\theta + y_a \cos\theta$$

10-4. Auxiliary Tilted Photo Coordinate System

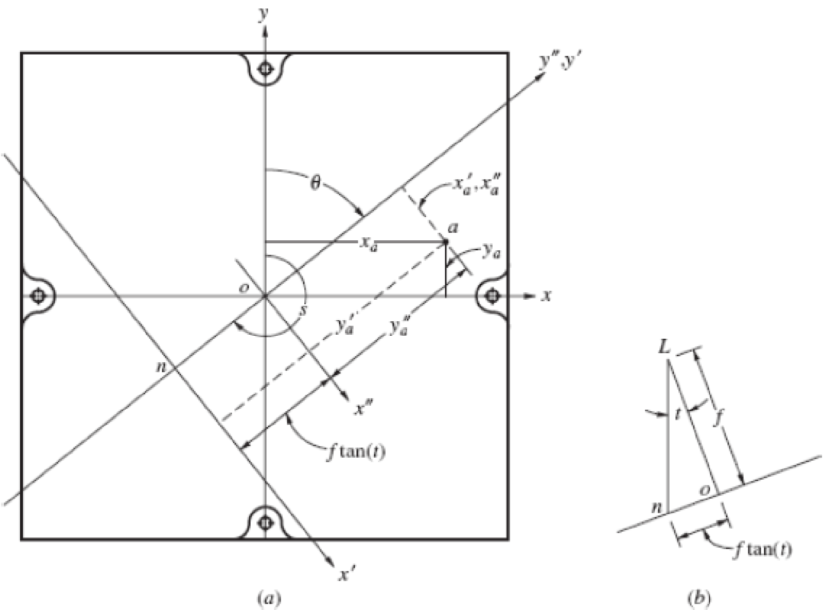


Figure 10−5. (a) Auxiliary $x'y'$image coordinate system for a tilted photograph. (b) Principal plane of a tilted photo.

- Auxiliary coordinate y'_a is obtained by adding the translation distance on to y''_a
- From Fig. 10-5b, which is a side view of the principal plane, on is $f$tant

$\theta = s - 180°$ (10-1)

- Since there is no x translation, $x'_a = x''_a$; and from the relationship between s and θ in Eq. (10-1), $\sin\theta = -\sin s$ and $\cos\theta = -\cos s$

- Therefore the coordinates x'_a and y'_a of a point a in the required auxiliary coordinate system are

$$x'_a = -x_a \cos s + y_a \sin s$$
$$y'_a = -x_a \sin s - y_a \cos s + f \tan t \quad (10\text{-}2)$$

- In Eqs. (10-2), x_a and $y_a$are the fiducial coordinates of point a, f is the camera focal length, and t and s are the tilt and swing angles, respectively

10-5. Scale of a Tilted Photograph

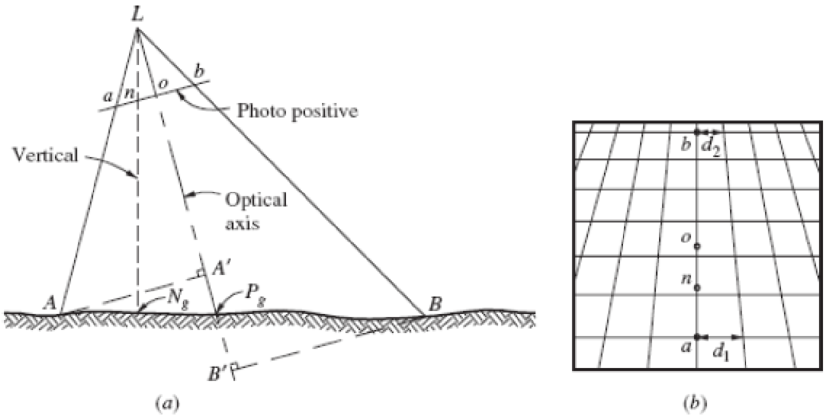


Figure 10–6. (a) Principal plane of a tilted photograph taken over approximately flat ground. (b) Image on the titled photo of a square grid.

- ❖ The shorter the object distance, the larger the scale, and vice versa
- ❖ For vertical photos, variations in object distances were caused only by topographic relief

- ❖ In a tilted photograph, relief variations also cause changes in scale, but scale in various parts of the photo is further affected by the magnitude and angular orientation of the tilt
- ❖ Figure 10-6a portrays the principal plane of a tilted photograph taken over a square grid on approximately flat ground
- ❖ Figure 10-6b illustrates the appearance of the grid on the resulting tilted photograph

10-5. Scale of a Tilted Photograph

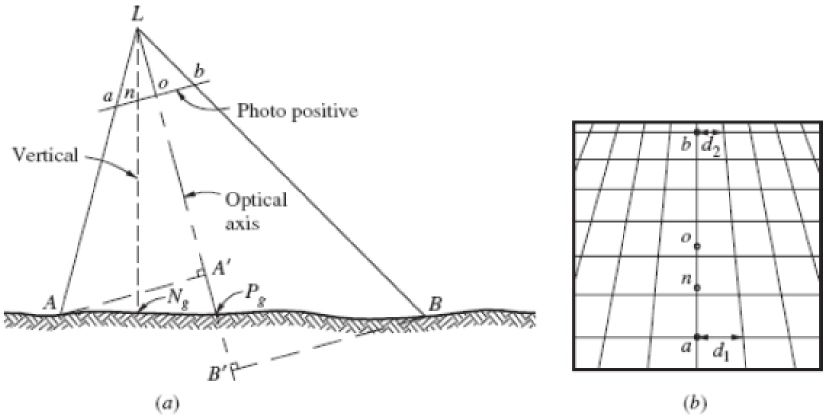


Figure 10–6. (a) Principal plane of a tilted photograph taken over approximately flat ground. (b) Image on the titled photo of a square grid.

❖ Due to tilt, object distance LA' in Fig. 10-6a is less than object distance LB', and hence a grid line near A would appear larger (at a greater scale) than a grid line near B

❖ This is illustrated in Fig. 10-6b, where photo distance d_1 appears longer than photo distance d_2 , yet both are the same length on the ground

❖ The scale at any point on a tilted photograph is readily calculated if tilt and swing for the photograph and the flying height of the photo and elevation of the point above datum are known

10-5. Scale of a Tilted Photograph

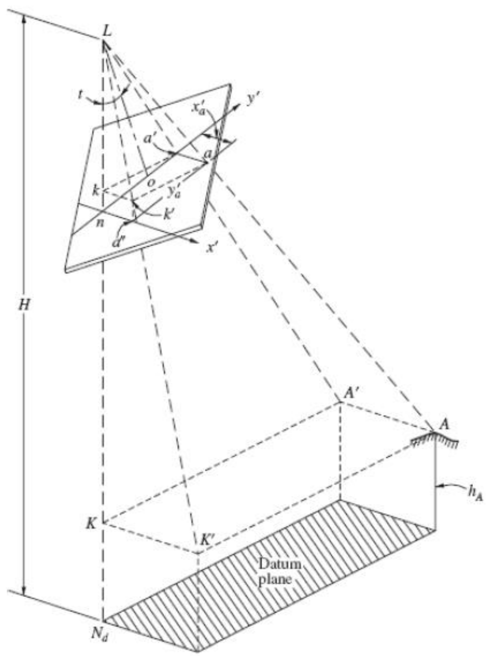


Figure 10-7. Scale of a tilted photograph.

- Figure 10-7 illustrates a tilted photo taken from a flying height H above datum; Lo is the camera focal length
- The image of object point A appears at a on the tilted photo, and its coordinates in the auxiliary tilted photo coordinate system are x'_a and y'_a
- The elevation of object point A above datum is h_A
- Object plane $AA'KK'$ is a horizontal plane constructed at a distance h_A above datum
- Image plane $aa'kk'$ is also constructed horizontally
- The scale relationship between the two parallel planes is the scale of the tilted photograph at point a because the image plane contains image point a and the object plane contains object point A

10-5. Scale of a Tilted Photograph

- The scale relationship is the ratio of photo distance aa' to ground distance AA' and may be derived from similar triangles $La'a$ and $LA'A$, and Lka' and LKA' as follows:

$$S_a = \frac{aa'}{AA'} = \frac{La'}{LA'} = \frac{Lk}{LK} \qquad \text{(a)}$$

but

$$Lk = Ln - kn = \frac{f}{\cos t} - y'_a \sin t \qquad \text{also} \qquad LK = H - h_A$$

- Substituting Lk and LK into Eq. (a) and dropping subscripts yields

$$S = \frac{f/\cos t - y'\sin t}{H - h} \qquad \text{(10-3)}$$

- In Eq. (10-3), S is the scale on a tilted photograph for any point whose elevation is h above datum
- Flying height above datum for the photo is H; f is the camera focal length; and y' is the coordinate of the point in the auxiliary system calculated by Eqs. (10-2)

$$\begin{aligned} x'_a &= -x_a \cos s + y_a \sin s \\ y'_a &= -x_a \sin s - y_a \cos s + f \tan t \end{aligned} \qquad \text{(10-2)}$$

10-5. Scale of a Tilted Photograph

$$S = \frac{f/\cos t - y'\sin t}{H - h} \qquad (10\text{-}3)$$

- If the units of f and y' are millimeters and if H and h are meters, then the scale ratio is obtained in millimeters per meter
- To obtain a dimensionless ratio, the right side of Eq. (10-3) must be multiplied by 1 m/1000 mm in that case
- Examination of Eq. (10-3) shows that scale increases with increasing terrain elevation
- If the photo is taken over level ground, then h is constant but scale still varies throughout the photograph with variations in y'

10-6. Relief Displacement on a Tilted Photograph

- Image displacements on tilted photographs caused by topographic relief occur much the same as they do on vertical photos, except that relief displacements on tilted photographs occur along radial lines from the nadir point
- Relief displacements on a truly vertical photograph are also radial from the nadir point, but in that special case the nadir point coincides with the principal point
- On a tilted photograph, image displacements due to relief vary in magnitude depending upon flying height, height of object, amount of tilt, and location of the image in the photograph
- Relief displacement is zero for images at the nadir point and increases with increased radial distances from the nadir
- Tilted photos are those that were intended to be vertical, but contain small unavoidable amounts of tilt
- In practice, tilted photos are therefore so nearly vertical that their nadir points are generally very close to their principal points

10-6. Relief Displacement on a Tilted Photograph

$$d = \frac{rh}{H} \qquad (6\text{-}11)$$

where d=relief displacement
h = height above datum of object point whose image is displaced
r = radial distance on photograph from principal point to displaced image
(The units of d and r must be the same.)
H = flying height above same datum selected for measurement of h

- ❖ Even for a photograph containing 3° of tilt taken with a 152-mmfocal-length camera, distance on is only about 8 mm
- ❖ Relief displacements on tilted photos may therefore be calculated with satisfactory accuracy using Eq. (6-11), which applies to a vertical photograph
- ❖ When this equation is used, radial distances r are measured from the principal point, even though theoretically, they should be measured from the nadir point

10-7. Determining the Angle of Inclination of the Camera Axis in Oblique Photography

- Oblique photos, in which the camera axis is inclined either up or down from horizontal at the time of exposure, represent the typical case in terrestrial photogrammetry
- Since the base configuration for terrestrial photos is a horizontal pose, it is useful to characterize its orientation by the angular deviation from horizontal

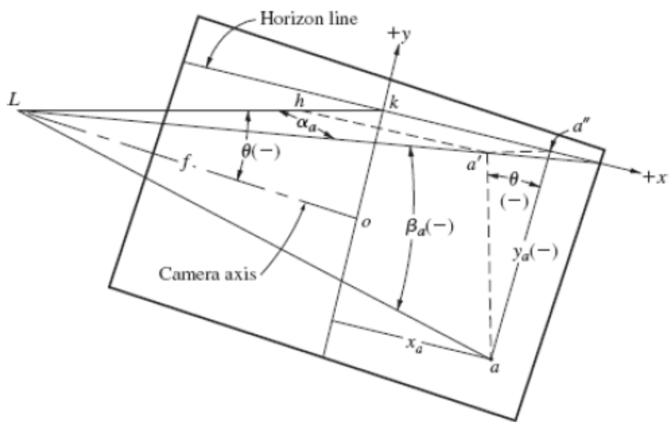


Figure 10–8. Horizontal and vertical angles from measurements on an oblique terrestrial photo.

- Figure 10-8 illustrates an oblique terrestrial photo taken with the camera axis inclined at an angle θ (not to be confused with θ from horizontal
- In this case the axis is inclined downward, and θ is called a *depression* angle
- If the camera axis were inclined upward, θ would be an *elevation* angle

10-7. Determining the Angle of Inclination of the Camera Axis in Oblique Photography

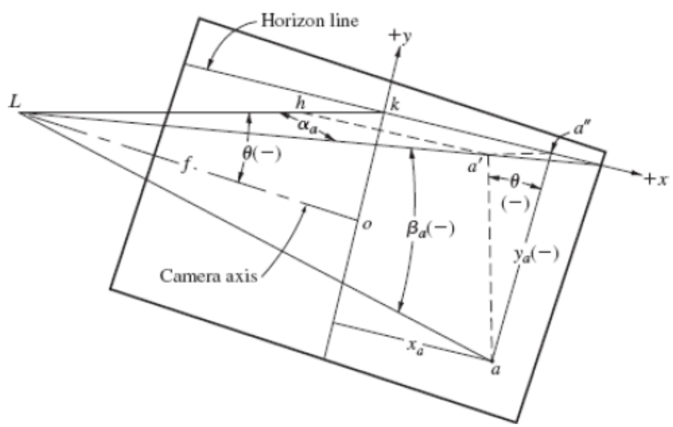


Figure 10−8. Horizontal and vertical angles from measurements on an oblique terrestrial photo.

- The angle of inclination of the camera axis is an important variable for certain elementary methods of determining object space positions of points whose images appear on overlapping terrestrial photos
- With some terrestrial cameras the angle of inclination can be set or measured so that it becomes a known quantity
- If it is unknown for a particular photo or photos, methods are available for computing its value
- One elementary method of determining the angle of inclination of the camera axis of a terrestrial photo relies on the fundamental principles of perspective geometry
- If a photograph contains images of linear features which are horizontal or vertical, the horizon or nadir can be located through graphical construction

10-7. Determining the Angle of Inclination of the Camera Axis in Oblique Photography

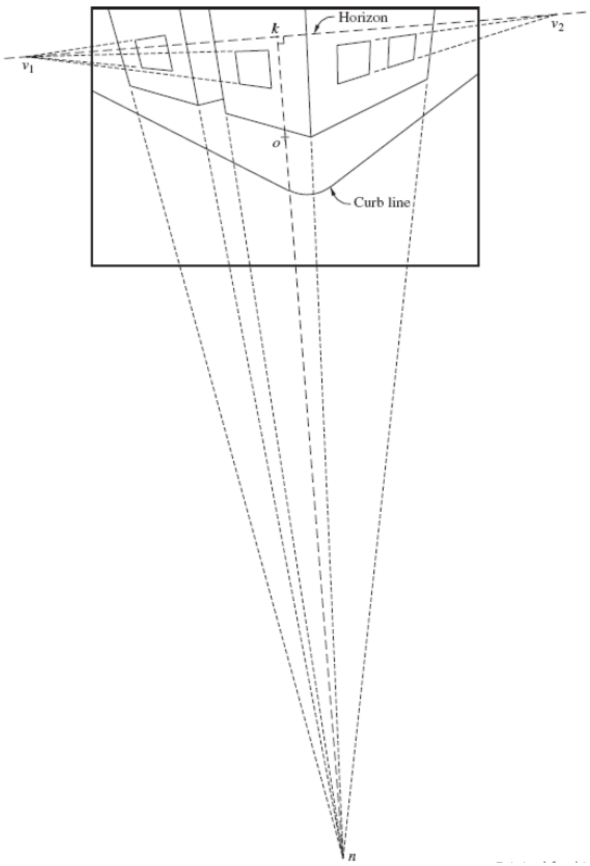


Figure 10–9. Location of horizon and nadir on an oblique photograph.

- Figure 10-9 illustrates an oblique terrestrial photo of two buildings at a street corner
- In this figure, dashed lines are extended from the tops and bottoms of the windows (horizontal parallel lines) to their intersections at vanishing points $v1$ and $v2$, which when connected define the horizon
- Also shown are dashed lines extending from the vertical building edges to their intersection at the nadir point n
- The line from n through the principal point o intersects the horizon at a right angle at point k
- A reference photo coordinate system can be established for the oblique photo of Fig. 10-9 from the preceding graphical construction

10-7. Determining the Angle of Inclination of the Camera Axis in Oblique Photography

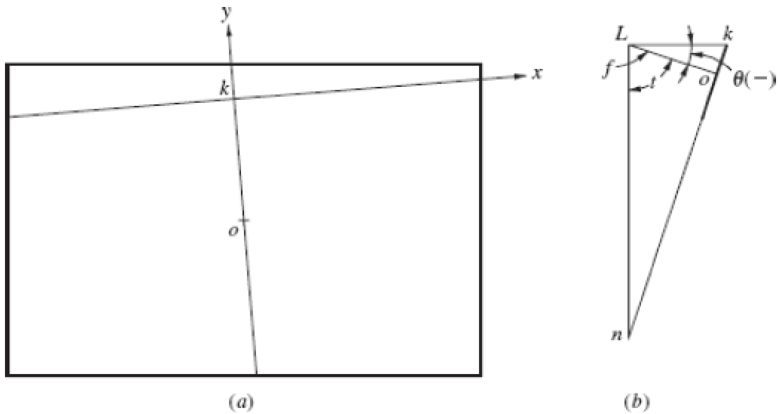


Figure 10–10. (a) Oblique photo coordinate axis system. (b) Side view of principal plane showing depression angle θ and tilt angle t.

- Figure 10-10a shows the x and y axes of the oblique photo coordinate system with its origin at point k
- Note that the x axis coincides with the horizon
  - Figure 10-10b shows a profile view of the principal plane Lkn
- The optical axis Lo is inclined downward at a depression angle θ
- To conform with usual sign convention, depression angles are considered negative in algebraic sign
- If the camera axis is inclined upward, θ is considered positive

10-7. Determining the Angle of Inclination of the Camera Axis in Oblique Photography

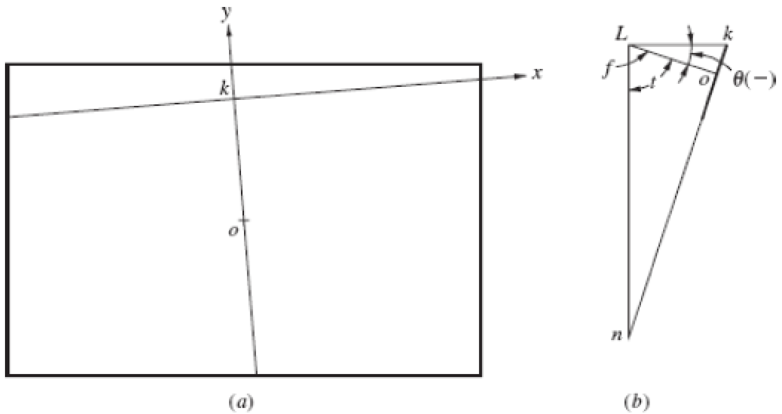


Figure 10-10. (a) Oblique photo coordinate axis system. (b) Side view of principal plane showing depression angle θ and tilt angle t.

❖ Angle θ can be determined by either of two approaches

① The first approach requires that the horizon be located by vanishing points

⇨ Then line ko is drawn at a right angle to the horizon through point o (the principal point of the photo) to define the y axis

⇨ Distance ko is then measured

⇨ Angle θ is computed from the following (see Fig. 10-10b): $\theta = \tan^{-1}\left(\frac{y_0}{f}\right)$ (10-4)

⇨ In Eq. (10-4), θ is the depression (or elevation) angle, y_0 is the y coordinate of the principal point in the oblique photo coordinate system, and f is the camera focal length

⇨ The correct algebraic sign must be applied to y_0 so that angle θ will have the correct algebraic sign, and it is also necessary to use an appropriate value for the focal length f

⇨ If the graphical analysis is being made on an enlarged photographic print, the focal length of the camera must be correspondingly enlarged

10-7. Determining the Angle of Inclination of the Camera Axis in Oblique Photography

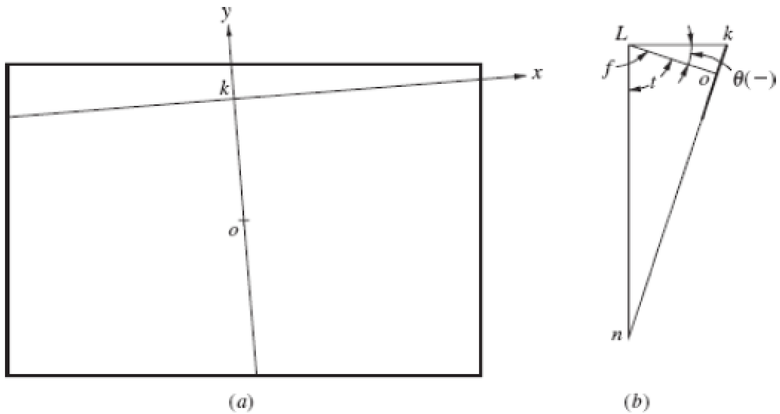


Figure 10-10. (a) Oblique photo coordinate axis system. (b) Side view of principal plane showing depression angle θ and tilt angle t.

❖ Angle θ can be determined by either of two approaches

② The second approach to determining angle θ is based on the location of the nadir

⇨ After the nadir has been determined, distance on is measured and the tilt angle t computed by

$$\therefore t = \tan^{-1}\left(\frac{on}{f}\right) \quad (10\text{-}5)$$

⇨ Once the tilt angle has been determined, angle θ can be computed from $\theta = t - 90°$ (10-6)

⇨ If a photograph is taken in which the optical axis is pointed upward from the horizon, angle θ is an elevation angle, and vertical parallel lines will intersect at the zenith

⇨ In this case, the distance oz from the principal point to the zenith point is measured, and angle θ can be computed by $\theta = 90° - \tan^{-1}\left(\frac{oz}{f}\right)$ (10-7)

⇨ Analytical methods can be used for determining the angle of inclination of the camera axes for terrestrial photos, and these methods provide the highest levels of accuracy

10-8. Computing Horizontal and Vertical Angles from Oblique Photos

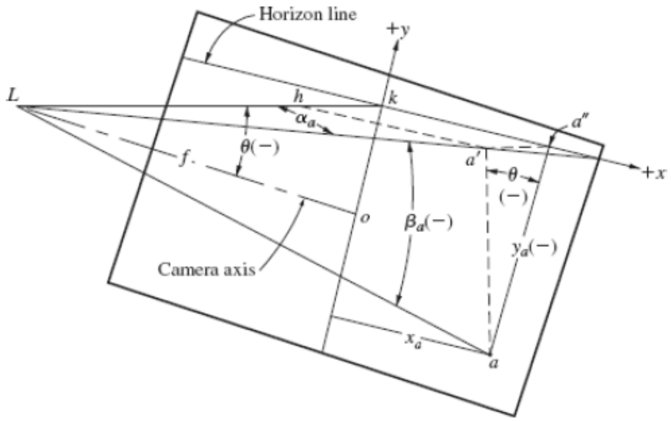


Figure 10-8. Horizontal and vertical angles from measurements on an oblique terrestrial photo.

- ❖ Once angle θ has been determined, horizontal and vertical angles can be computed for points whose images appear in the photograph
- ❖ Refer to Fig. 10-8 which shows the geometry of an oblique terrestrial photograph

- ❖ In this figure, L is the exposure station, and Lo is f , the camera focal length; Lk is a horizontal line intersecting the photo at k
- ❖ The x axis coincides with the horizon line, and the y axis passes through the principal point o, perpendicular to the x axis at point k
- ❖ Point a is an image point, and aa' is a vertical line, with a' being located in the horizontal plane of the exposure station and horizon line

10-8. Computing Horizontal and Vertical Angles from Oblique Photos

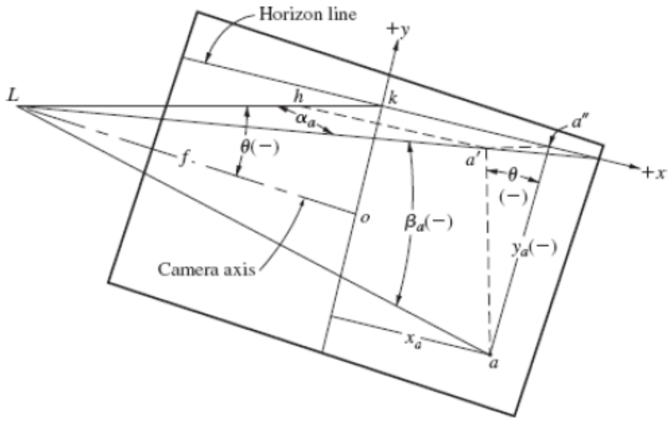


Figure 10−8. Horizontal and vertical angles from measurements on an oblique terrestrial photo.

❖ Horizontal angle α_a between the vertical plane, $(L\ a'\ a)$, containing image point a and the vertical plane, Lko, containing the camera axis is

$$\alpha_a = \tan^{-1}\left(\frac{ha'}{Lk - hk}\right) = \tan^{-1}\left(\frac{x_a}{f\sec\theta - y_a\sin\theta}\right) \qquad (10\text{-}8)$$

❖ In Eq. (10-8) note that correct algebraic signs must be applied to x_a, y_a and θ

❖ Algebraic signs of α angles are positive if they are clockwise from the optical axis and negative if they are counterclockwise

❖ After the horizontal angle α_a has been determined, vertical angle β_a to image point a can be calculated from the following equation:

$$\beta_a = \tan^{-1}\left(\frac{aa'}{La'}\right) = \tan^{-1}\left(\frac{aa'}{(Lk - hk)\sec\alpha_a}\right)$$
$$= \tan^{-1}\left(\frac{y_a\cos\theta}{(f\sec\theta - y_a\sin\theta)\sec\alpha_a}\right) \qquad (10\text{-}9)$$

❖ The algebraic signs of β angles are automatically obtained from the signs of the y coordinates used in Eq. (10-9)

10-9. Angular Orientation in Omega-Phi-Kappa

- As previously stated, besides the tilt-swing-azimuth system, angular orientation of a photograph can be expressed in terms of three rotation angles, $omega$, phi, and $kappa$
- These three angles uniquely define the angular relationships between the three axes of the photo (image) coordinate system and the three axes of the ground (object) coordinate system

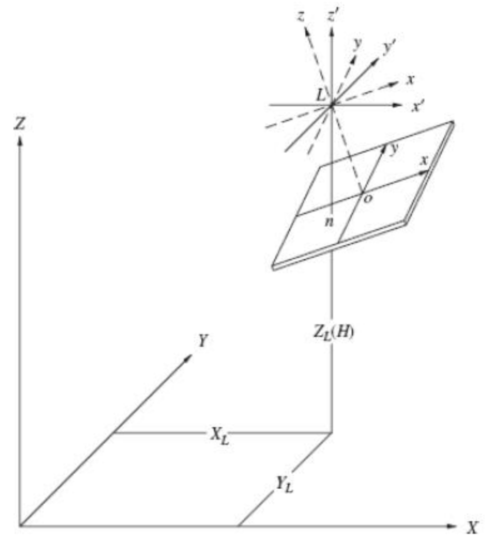


Figure 10-11. Orientation of a photograph in the omega-phi-kappa system.

- Figure 10-11 illustrates a tilted photo in space, and the ground coordinate system is XYZ
- The tilted photo image coordinate system is xyz (shown dashed), and its origin is at exposure station L
- Consider an image coordinate system $x'y'z'$ with origin also at L and with its respective axes mutually parallel to the axes of the ground coordinate system, as shown in Fig. 10-11

10-9. Angular Orientation in Omega-Phi-Kappa

❖ As previously stated, besides the tilt-swing-azimuth system, angular orientation of a photograph can be expressed in terms of three rotation angles, $omega$, phi, and $kappa$

❖ These three angles uniquely define the angular relationships between the three axes of the photo (image) coordinate system and the three axes of the ground (object) coordinate system

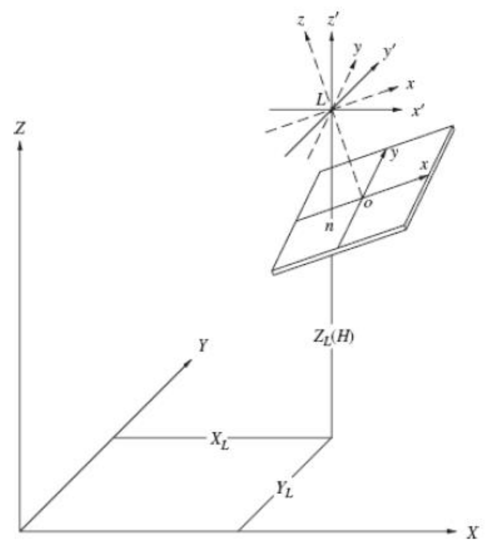


Figure 10-11. Orientation of a photograph in the omega-phi-kappa system.

❖ As a result of three sequential rotations through the angles of omega, phi, and kappa, the $x'y'z'$ axis system can be made to coincide with the photographic xyz system

❖ Each of the rotation angles omega, phi, and kappa is considered positive if counterclockwise when viewed from the positive end of the rotation axis

10-9. Angular Orientation in Omega-Phi-Kappa

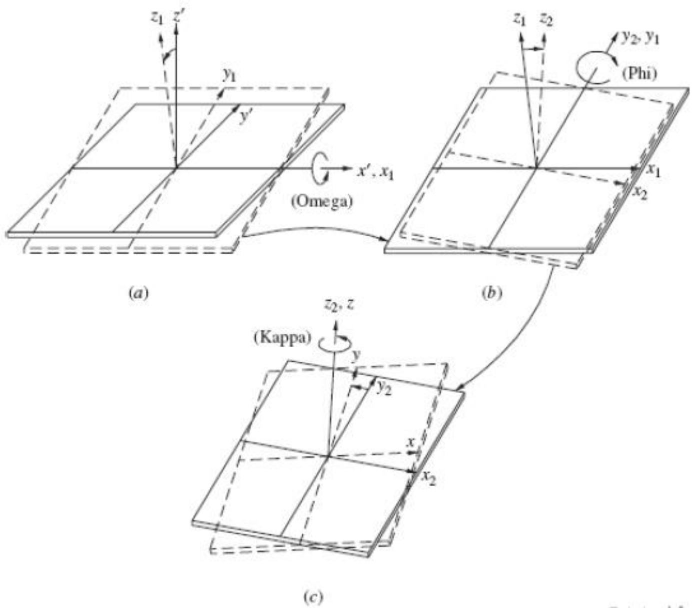


Figure 10–12. (a) Rotation about the x' axis through angle omega. (b) Rotation about the $y_1$axis through angle phi. (c) Rotation about the z_2 axis through angle kappa.

- ❖ The sequence of the three rotations is illustrated in Fig. 10-12
- ❖ The first rotation, as illustrated in Fig. 10-12a, is about the x' axis through an angle omega
- ❖ This first rotation creates a new axis system $x_1 y_1 z_1$
- ❖ The second rotation phi is about the y_1 axis, as illustrated in Fig. 10-12b
- ❖ As a result of the phi rotation a new axis system $x_1 y_1 z_1$ is created
- ❖ The third and final rotation is about the y_1 axis through the angle kappa(Fig. 10-12c)
  ⇨ This third rotation creates the xyz coordinate system which is the photographic image system

10-9. Angular Orientation in Omega-Phi-Kappa

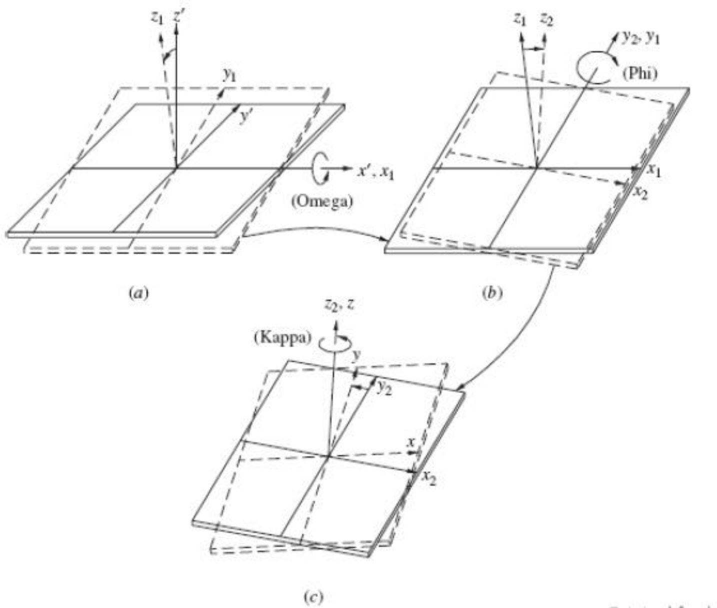


Figure 10−12. (a) Rotation about the x’ axis through angle omega. (b) Rotation about the y_1 axis through angle phi. (c) Rotation about the z_2 axis through angle kappa.

- ❖ For any photo there exist a unique set of angles omega, phi, and kappa which explicitly define the angular orientation of the photograph with respect to the reference ground coordinate system, provided appropriate ranges are maintained

- ❖ These three angles are related to the previously described tilt, swing, and azimuth angles; and if either set of three orientation angles is known for any photo, the other three can be determined

- ❖ In the omega-phi-kappa system, as with the tilt-swing-azimuth system, the space position of any photo is given by the exposure station coordinates X_L, Y_L and Z_L (or H)

10-10. Determining the Elements of Exterior Orientation

- Many different methods, both graphical and numerical, have been devised to determine the six elements of exterior orientation of a single photograph
- In general all methods require photographic images of at least three control points whose X, Y, and Z ground coordinates are known, and the calibrated focal length of the camera must be known
- Although the most commonly used method to find these elements is space resection by *collinearity*
- The method of space resection by collinearity is a purely numerical method which simultaneously yields all six elements of exterior orientation
- Normally the angular values of omega, phi, and kappa are obtained in the solution, although the method is versatile and tilt, swing, and azimuth could also be obtained

10-10. Determining the Elements of Exterior Orientation

- Space resection by collinearity permits the use of redundant amounts of ground control; hence least squares computational techniques can be used to determine most probable values for the six elements
- This method is a rather lengthy procedure requiring an iterative solution of nonlinear equations, but when programmed for a computer, a solution is easily obtained
- Space resection by collinearity involves formulating the so-called *collinearity equations* for a number of control points whose X, Y, and Z ground coordinates are known and whose images appear in the photo
- The equations are then solved for the six unknown elements of exterior orientation which appear in them
- The collinearity equations express the condition that for any given photograph the exposure station, any object point and its corresponding image all lie on a straight line

10-11. Rectification of Tilted Photographs

- ❖ Rectification is the process of making equivalent vertical photographs from tilted photo negatives
- ❖ The resulting equivalent vertical photos are called *rectified photographs*
- ❖ Rectified photos theoretically are truly vertical photos, and as such they are free from displacements of images due to tilt
  - ⇨ They do, however, still contain image displacements and scale variations due to topographic relief
  - ⇨ These relief displacements and scale variations can also be removed in a process called *differential rectification* or *orthorectification*, and the resulting products are then called *orthophotos*
- ❖ Orthophotos are often preferred over rectified photos because of their superior geometric quality
  - ⇨ Nevertheless, rectified photos are still quite popular because they do make very good map substitutes where terrain variations are moderate

10-11. Rectification of Tilted Photographs

❖ Rectification is generally performed by any of three methods: analytically, optically-mechanically, and digital

1) Analytical rectification has the disadvantage that it can be applied only to individual discrete points

   - The resulting rectified photos produced by analytical methods are not really photos at all since they are not composed of photo images
   - Rather, they are plots of individual points in their rectified locations

2) The optical-mechanical and digital methods produce an actual picture in which the images of the tilted photo have been transformed to their rectified locations

   - The products of these two methods can be used in the production of photomaps and mosaics. In any of these rectification procedures, the rectified photos can be simultaneously ratioed
   - This is particularly advantageous if rectified photos are being made for the purpose of constructing a controlled mosaic, since all photos in the strip of block can be brought to a common scale
   - Thus, the resulting mosaic will have a more uniform scale throughout

10-11. Rectification of Tilted Photographs

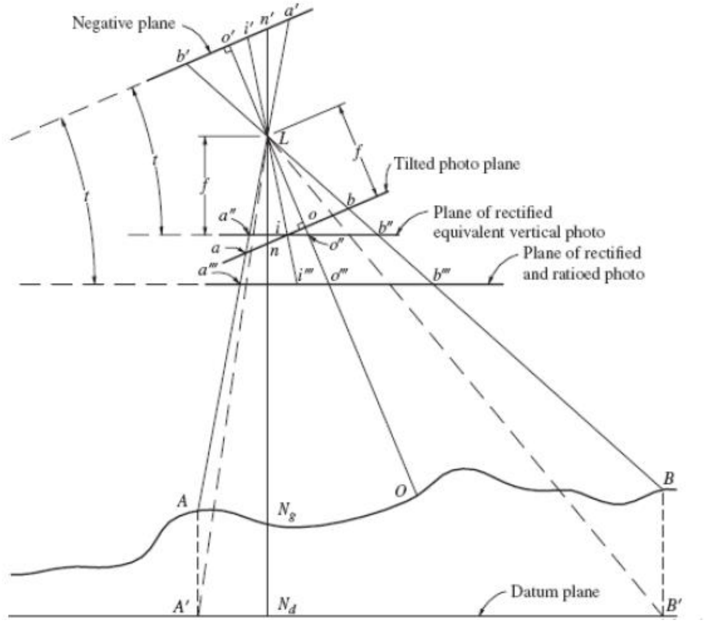


Figure 10–13. Principal plane of a tilted photograph showing the basic geometry of rectification.

- The fundamental geometry of rectification is illustrated in Fig. 10-13
  ⇨ This figure shows a side view of the principal plane of a tilted photo
- When the exposure was made, the negative plane made an angle t with the datum plane
- Rays from A and B were imaged at $\boldsymbol{a'}$ and $\boldsymbol{b'}$, respectively, on the negative, and their corresponding locations on the tilted photo are at $\boldsymbol{a}$ and $\boldsymbol{b}$
- The plane of an *equivalent vertical photo* is shown parallel to the datum plane and passing through $\boldsymbol{i}$, the *isocenter* of the tilted photo and the plane of a ratioed rectified photo is also shown
- It is likewise parallel to the datum plane but exists at a level other than that of the equivalent vertical photo plane
- Figure 10-13 illustrates, by virtue of lines LA' and LB', that although tilt displacements are removed in rectification, displacements due to relief are still present

10-12. Correction for Relief of Ground Control Points Used in Rectification

- Since rectified and ratioed aerial photos retain the effects of relief, ground control points used in rectification must be adjusted slightly to accommodate their relief displacements
- Conceptually, the process involves plotting ground control points at the locations that they will occupy in the rectified and ratioed photo
- To this end, the positional displacements due to relief of the control points must be computed and applied to their horizontal positions in a radial direction from the exposure station so that they will line up with points in the rectified photo
- This procedure requires that the coordinates X_L, Y_L, and Z_L (or H) of the exposure station (which can be computed by space resection) and the $X, Y,$ and Z (or h) coordinates for each ground control point be known
- In addition, when the rectified photos are to be ratioed as well, it is convenient to select a plane in object space, at a specified elevation, to which the scale of the ratioed photo will be related
- Generally, the elevation of this plane will be chosen as the elevation of average terrain, h_{avg}

10-12. Correction for Relief of Ground Control Points Used in Rectification

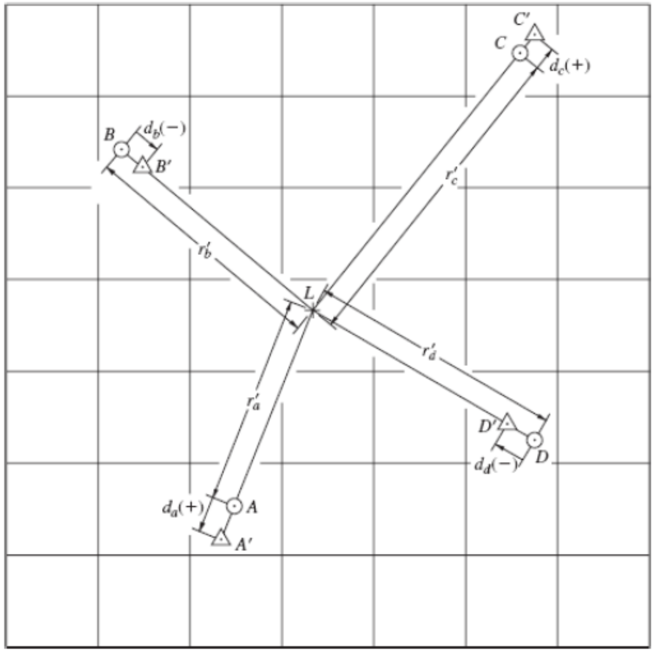


Figure 10–14. Plot of control points for rectification showing corrections made for relief displacements.

- ❖ The procedure is illustrated in Fig. 10-14, which shows the horizontal position of the exposure station, L, denoted as a cross and the unadjusted horizontal positions of four ground control points, A, B, C, and D, denoted as circles
- ❖ Also illustrated are radial distances r' from L to each of the points, as well as the relief displacements, d, from the location in the plane of average terrain to the photo locations, A', B', C', and D', of the control points, denoted as triangles
- ❖ Since the elevations of these control points may be higher or lower than the average terrain, the relief displacements may be either outward (control point higher than average terrain) or inward (control point lower than average terrain)
- ❖ Note that in Fig. 10-14, the elevations of points B and D are less than average terrain elevation and the elevations of points A and C are greater

10-12. Correction for Relief of Ground Control Points Used in Rectification

❖ Determination of the coordinates of a displaced point (triangle) involves several steps

❖ Initially, the value of r' is computed by Eq. (10-10) using the point's horizontal coordinates X and Y, and the horizontal coordinates of the exposure station, X_L and Y_L

$$r' = \sqrt{(X - X_L)^2 + (Y - Y_L)^2} \qquad (10\text{-}10)$$

❖ The relief displacement, d, is calculated by Eq. (10-11), which is a variation of Eq. (6-11):

$$d = \frac{r'(h - h_{avg})}{H - h} \qquad (10\text{-}11)$$

$$d = \frac{rh}{H} \qquad (6\text{-}11)$$

where d=relief displacement
h = height above datum of object point whose image is displaced
r = radial distance on photograph from principal point to displaced image
(The units of d and r must be the same.)
H = flying height above same datum selected for measurement of h

10-12. Correction for Relief of Ground Control Points Used in Rectification

$$d = \frac{r'(h - h_{avg})}{H - h} \quad (10\text{-}11) \qquad r' = \sqrt{(X - X_L)^2 + (Y - Y_L)^2} \quad (10\text{-}10)$$

- In Eq. (10-11), d is the relief displacement, r' is the radial distance [computed by Eq. (10-10)], h_{avg} the height of the control point above datum, h the average terrain height in the tilted photo (also the height above datum of the ratioed photo plane), and H the flying height above datum for the tilted photo
- The units of all terms in the equation are those of the object space coordinate system
- Once the displacement d has been computed, the radial distance r to the displaced (image) location of the point can be computed by the Eq. (10-12) (be careful to use the proper algebraic sign of d)

$$r = r' + d \quad (10\text{-}12)$$

10-12. Correction for Relief of Ground Control Points Used in Rectification

❖ The azimuth, α , from the exposure location to the control point can be computed by

$$\alpha = \tan^{-1}\left(\frac{X - X_L}{Y - Y_L}\right) \qquad (10\text{-}13)$$

❖ In Eq. (10-13), it is necessary to use the full circle inverse tangent function so that the entire range of azimuth can be determined

❖ Finally, the X' and Y' coordinates of the displaced (image) point are computed by

$$X' = X_L + r\sin\alpha \qquad (10\text{-}14)$$

$$Y' = Y_L + r\cos\alpha \qquad (10\text{-}15)$$

❖ The resulting coordinates, X' and Y', are appropriate for use in any of the methods of rectification

10-13. Analytical Rectification

- There are several methods available for performing analytical (numerical) rectification
- Each of the analytical methods performs rectification point by point, and each requires that sufficient ground control appear in the tilted photo
- Basic input required for the numerical methods, in addition to ground coordinates of control points, is x and y photo coordinates of all control points plus those of the points to be rectified ⇨ These are normally measured on a comparator
- Due to the lengthy calculations required, numerical rectification is generally performed through the use of a computer program
- Of the available methods of analytical rectification, the one that uses the two-dimensional projective transformation is the most convenient and is the only method that will be discussed here. The transformation equations are as Eqs. (10-16), for convenience:

$$X = \frac{a_1 x + b_1 y + c_1}{a_3 x + b_3 y + 1}$$

$$Y = \frac{a_2 x + b_2 y + c_2}{a_3 x + b_3 y + 1} \qquad (10\text{-}16)$$

10-13. Analytical Rectification

$$X = \frac{a_1 x + b_1 y + c_1}{a_3 x + b_3 y + 1} \qquad Y = \frac{a_2 x + b_2 y + c_2}{a_3 x + b_3 y + 1} \qquad (10\text{-}16)$$

❖ In Eqs. (10-16), X and Y are ground coordinates, x and y are photo coordinates (in the fiducial axis system), and the $a's$, $b's$, and $c's$ are the eight parameters of the transformation

❖ The use of these equations to perform analytical rectification is a two-step process

① First, a pair of Eq. (10-16) is written for each ground control point

- Four control points will produce eight equations, so that a unique solution can be made for the eight unknown parameters
- It is strongly recommended that more than four control points be used so that an improved solution can be arrived at by using least squares
- An added benefit is that redundant measurements may provide the ability of detecting mistakes in the coordinates, something which is not afforded by the unique solution using four control points

10-13. Analytical Rectification

$$X = \frac{a_1 x + b_1 y + c_1}{a_3 x + b_3 y + 1} \qquad Y = \frac{a_2 x + b_2 y + c_2}{a_3 x + b_3 y + 1} \qquad (10\text{-}16)$$

❖ In Eqs. (10-16), X and Y are ground coordinates, x and y are photo coordinates (in the fiducial axis system), and the $a's$, $b's$, and $c's$ are the eight parameters of the transformation

❖ The use of these equations to perform analytical rectification is a two-step process

② Once the eight parameters have been determined, the second step of the solution can be performed

- Solving Eqs. (10-16) for each point whose X and Y rectified coordinates are desired
- After rectified coordinates have been computed in the ground coordinate system, they can be plotted at the scale desired for the rectified and ratioed photo
- This analytical method is only rigorous if the ground coordinates, X and Y, of Eqs. (10-16) have been modified for relief displacements
- If this is not done, a quasi-rectification results; although if the terrain is relatively flat and level, the errors will be minimal

10-14. Optical-Mechanical Rectification

- In practice, the optical-mechanical method is still being used, although digital methods have rapidly surpassed this approach
- The optical-mechanical method relies on instruments called ***rectifiers***
- They produce rectified and ratioed photos through the photographic process of projection printing ⇨ thus they must be operated in a darkroom

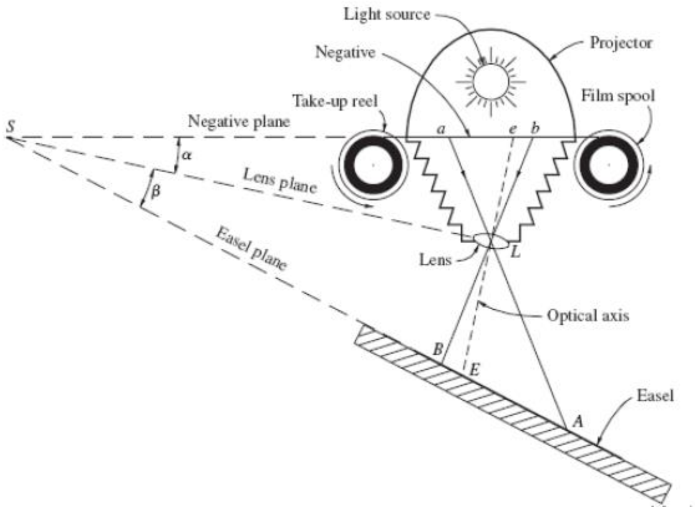


Figure 10-15. Schematic diagram of a tilting-lens optical-mechanical rectifier showing a side view of the principal plane.

- As illustrated in Fig. 10-15, the basic components of a rectifier consist of a lens, a light source with reflector, a stage for mounting the tilted photo negative, and an easel which holds the photographic emulsion upon which the rectified photo is exposed

10-14. Optical-Mechanical Rectification

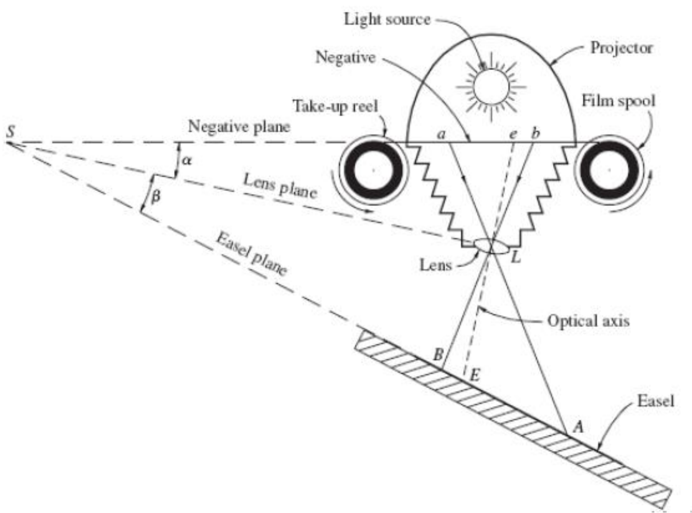


Figure 10-15. Schematic diagram of a tilting-lens optical-mechanical rectifier showing a side view of the principal plane.

❖ The instrument is constructed with controls so that the *easel plane*, *lens plane* (plane perpendicular to the optical axis of the rectifier lens), and *negative plane* can be tilted with respect to each other, and controls that allow a shift to be applied to the negative plane

⇨ Thus provision is made for varying angles α and β of Fig. 10-15

❖ So that rectified photos can be ratioed to varying scales, the rectifier must also have a magnification capability, and this is achieved by varying the projection distance (distance LE of Fig. 10-15 from the lens to the easel plane)

❖ To do this, however, and still maintain proper focus in accordance with Eq. (2-4), it is necessary to simultaneously vary the image distance (distance Le of Fig. 10-15 from the lens to the negative plane)

10-14. Optical-Mechanical Rectification

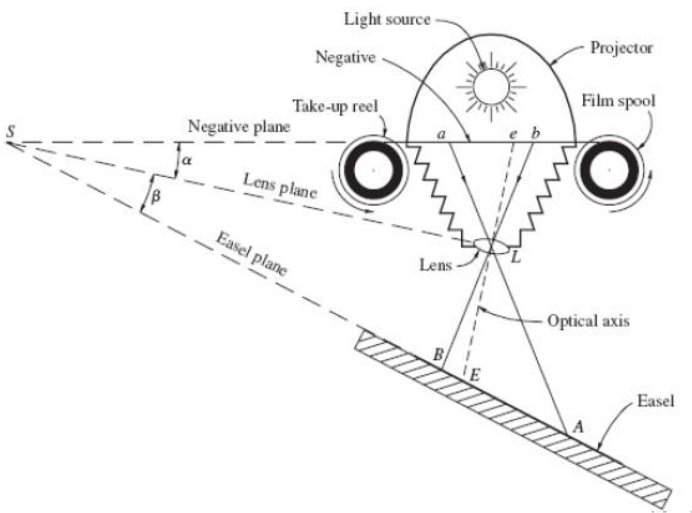


Figure 10-15. Schematic diagram of a tilting-lens optical-mechanical rectifier showing a side view of the principal plane.

- ❖ The actual magnification that results along the axis of the rectifier lens is the ratio $LE/(Le)$, but it varies elsewhere in the photo due to the variable scale of the tilted photograph
- ❖ From Fig. 10-15 it can be seen that in rectification, projection distances vary depending upon the locations of points on the photo
- ❖ To achieve sharp focus for all images in spite of this, the *Scheimpflug condition* must be satisfied
- ❖ The Scheimpflug condition states that, in projecting images through a lens, if the negative and easel planes are not parallel, the negative plane, lens plane, and easel plane must all intersect along a common line to satisfy the lens formula and achieve sharp focus for all images
- ❖ Note that this condition is satisfied in Fig. 10-15 where these planes intersect at S

10-15. Digital Rectification

❖ Rectified photos can be produced by digital techniques that incorporate a photogrammetric scanner and computer processing
⇨ This procedure is a special case of the more general concept of georeferencing

❖ Rectification requires that a projective transformation be used to relate the image to the ground coordinate system whereas georeferencing often uses simpler transformations such as the two-dimensional conformal or the two-dimensional affine transformation

(a) (b)

Figure 10–16. (a) Digital image of an oblique, nonrectified photograph. (b) Image after digital rectification.

❖ Figure 10-16a shows a portion of a scanned aerial oblique

❖ Note that images in the foreground (near the bottom) are at a larger scale than those near the top

10-15. Digital Rectification

Figure 10-16. (a) Digital image of an oblique, nonrectified photograph. (b) Image after digital rectification.

- ❖ This photograph contains a generally rectangular street pattern, where the streets appear nonparallel due to the large amount of tilt
- ❖ Figure 10-16b shows a digitally rectified version of the oblique photograph shown in Fig. 10-16a
- ❖ Note that in this rectified image, the general street pattern is approximately rectangular, although displacements remain due to relief in the terrain as well as buildings

10-15. Digital Rectification

- Three primary pieces of equipment needed for digital rectification are a digital image (either taken from a digital camera or a scanned photo), computer, and plotting device capable of producing digital image output

- While a high-quality photogrammetric scanner is an expensive device, it is also highly versatile and can be used for many other digital-photogrammetric operations

- Low-accuracy desktop scanners can also be used for digital rectification
  ⇨ However, the geometric accuracy of the product will be substantially lower

- Current plotting devices, generally large-format ink jet printers, produce image output of good quality, although the geometric accuracy and image resolution may be slightly less than that of photographic rectifiers

10-16. Atmospheric Rectification in Tilted Aerial Photographs

❖ Atmospheric refraction in aerial photographs occurs radially from the nadir point

❖ Now that parameters of tilted photographs have been described, a technique that corrects image coordinates in a tilted photo for the effects of atmospheric refraction is presented

❖ In most practical situations, the assumption of vertical photography is sufficient when calculating atmospheric refraction

❖ However for highest accuracy, when dealing with high altitude photography (e.g., flying height greater than 5000 m) and photographs with excessive tilts (e.g., tilts greater than 5°) should be considered when atmospheric refraction corrections are made

10-16. Atmospheric Rectification in Tilted Aerial Photographs

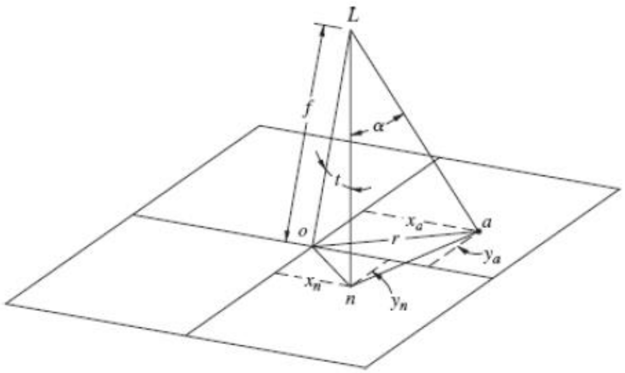


Figure 10–17. Diagram of a tilted photo with image point a whose coordinates will be corrected for atmospheric refraction.

- Figure 10-17 illustrates a tilted photo containing image point a at coordinates x_a and y_a
- The photographic nadir point n exists at coordinates x_n and y_n (in the fiducial axes coordinate system) which can be computed by the following equations

$$x_n = on \sin s = f \tan t \sin s$$

$$y_n = on \cos s = f \tan t \cos s \qquad (10\text{-}17)$$

10-16. Atmospheric Rectification in Tilted Aerial Photographs

- The angle between the vertical line Ln and the incoming light ray through point a is designated as α
- Angle α can be computed by the application of the law of cosines to triangle Lna
- After angle α has been determined, the refraction angle $\Delta\alpha$ can be calculated by Eq. (4-18) using a K value computed by Eq. (4-19)

$$\Delta\alpha = K \tan\alpha \qquad (4\text{–}18)$$

$$K = (7.4 \times 10^{-4})(H - h)[1 - 0.02(2H - h)] \qquad (4\text{–}19)$$

$$(na)^2 = (Ln)^2 + (La)^2 - 2(Ln)(La)\cos\alpha$$

Rearranging gives

$$\alpha = \cos^{-1}\left[\frac{(Ln)^2 + (La)^2 - (na)^2}{2(Ln)(La)}\right] \qquad (10\text{-}18)$$

where

$$na = \sqrt{(x_a - x_n)^2 + (y_a - y_n)^2} \qquad (10\text{-}19)$$

$$Ln = \frac{f}{\cos t} \qquad (10\text{-}20)$$

$$La = \sqrt{f^2 + r^2} = \sqrt{f^2 + x_a^2 + y_a^2} \qquad (10\text{-}21)$$

10-16. Atmospheric Rectification in Tilted Aerial Photographs

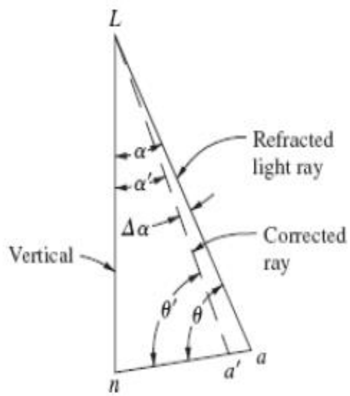


Figure 10−18. Vertical plane through tilted photo of Fig. 10−17 showing refracted light ray and corrected ray.

❖ Figure 10-18 shows triangle $\boldsymbol{Lna}$ salong with the light ray $\boldsymbol{La'}$ which indicates the direction from the ground point that has been corrected for atmospheric refraction

❖ From the figure, angle $\boldsymbol{\theta}$ can be computed by the law of cosines as

$$\theta = \cos^{-1}\left(\frac{(La)^2 + (na)^2 - (Ln)^2}{2(La)(na)}\right) \qquad (10\text{-}22)$$

❖ Angle $\boldsymbol{\theta'}$ can then be computed by $\theta' = \theta + \Delta\alpha$ (10-23)

❖ Application of the law of sines to triangle $\boldsymbol{Lna'}$ gives $\dfrac{\sin\theta'}{Ln} = \dfrac{\sin\alpha'}{na'}$

❖ Solving for $\boldsymbol{na'}$ gives $na' = Ln\dfrac{\sin\alpha'}{\sin\theta'}$ (10-24) where $\alpha' = \alpha - \Delta\alpha$ (10-25)

❖ Ln and θ' are computed by Eqs. (10-20) and (10-23), respectively

$$Ln = \frac{f}{\cos t} \qquad (10\text{-}20) \qquad\qquad \theta' = \theta + \Delta\alpha \qquad (10\text{-}23)$$

❖ The displacement, $\boldsymbol{aa'}$ due to atmospheric refraction is then

$$aa' = na - na' \qquad (10\text{-}26)$$

10-16. Atmospheric Rectification in Tilted Aerial Photographs

- To compute the coordinate corrections δx and δy, displacement aa' must be applied in the direction of line an, as shown in Fig. 10-19

- The direction of line an is specified by angle β as shown in the figure and can be computed by

$$\beta = \tan^{-1}\left(\frac{x_a - x_n}{y_a - y_n}\right) \qquad (10\text{-}27)$$

- The corrections are then given by

$$\delta x = aa' \sin\beta$$
$$\delta y = aa' \cos\beta \qquad (10\text{-}28)$$

- Finally, the coordinates x'_a and y'_a for point a corrected for atmospheric refraction are computed by

$$x'_a = x_a - \delta x$$
$$y'_a = y_a - \delta y \qquad (10\text{-}29)$$

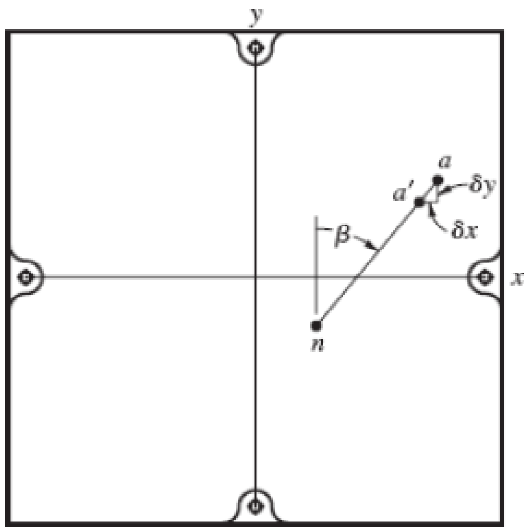


Figure 10−19. Photograph positions of image point a and its new location a’ after correction for atmospheric refraction.

10-16. Atmospheric Rectification in Tilted Aerial Photographs

❖ Two final comments regarding **refraction in a tilted photo** are in order

1) The values of tilt and swing are not known until after an analytical solution is performed
   ⇨ However, **photo coordinates** are necessary in order to compute the analytical solution
   ⇨ Therefore **refinement** for atmospheric refraction in a tilted photograph must be performed in an iterative fashion
   ⇨ Since analytical photogrammetric solutions are generally iterative due to the nonlinear equations involved, **tilted photo refraction corrections** can be conveniently inserted into the iterative loop

2) The foregoing discussion of atmospheric refraction in a tilted photograph assumes that the tilt angle is the angle between **the optical axis** and **a vertical line**
   ⇨ In analytical photogrammetry, **a local vertical coordinate system** is generally used for object space
   ⇨ As a result, tilt angles will be related to the direction of the local vertical Z axis which is generally different from the true vertical direction
   ⇨ This effect is negligible for practical situations
   ⇨ Unless a photograph is more than about 300 km from the local vertical origin, the effect from ignoring this difference in vertical direction will generally be less than 0.001 mm