Chapter 11. Introduction to Analytical Photogrammetry

11-1. Introduction

- *Analytical photogrammetry* is a term used to describe the rigorous mathematical calculation of coordinates of points in object space based upon camera parameters, measured photo coordinates, and ground control

- Analytical photogrammetry forms the basis of many modern hardware and software systems, including: stereoplotters (analytical and softcopy), digital terrain model generation, orthophoto production, digital photo rectification, and aerotriangulation

This chapter presents an introduction to some fundamental topics and elementary applications in analytical photogrammetry

11-2. Image Measurements

- A fundamental type of measurement used in analytical photogrammetry is **an x and y photo coordinate pair**
  ⇨ These coordinates **must be related to the principal point as the origin**

- Since mathematical relationships in analytical photogrammetry are based on assumptions such as "light rays travel in straight lines" and "the focal plane of a frame camera is flat", various coordinate refinements may be required to correct measured photo coordinates for distortion effects that otherwise cause these assumptions to be violated

- A number of instruments and techniques are available for making photo coordinate measurements

- In many analytical photogrammetry methods, it is necessary to measure image coordinates of common object points that appear in more than one photograph

- In these cases it is essential that the image of each object point be precisely identified between photos so that the measurements will be consistent

11-3. Control Points

- In addition to measurement of image coordinates, a certain number of control points in object space are generally required for analytical photogrammetry

- Object space coordinates of these control points, which may be either imageidentifiable features or exposure stations of the photographs themselves, are generally determined via some type of field survey technique such as GPS

- It is important that the object space coordinates be based on a three-dimensional cartesian system which has straight, mutually perpendicular axes

11-4. Collinearity Condition

❖ The most fundamental and useful relationship in analytical photogrammetry is the *collinearity* condition

❖ Collinearity, is the condition that the exposure station, any object point, and its photo image all lie along a straight line in three-dimensional space

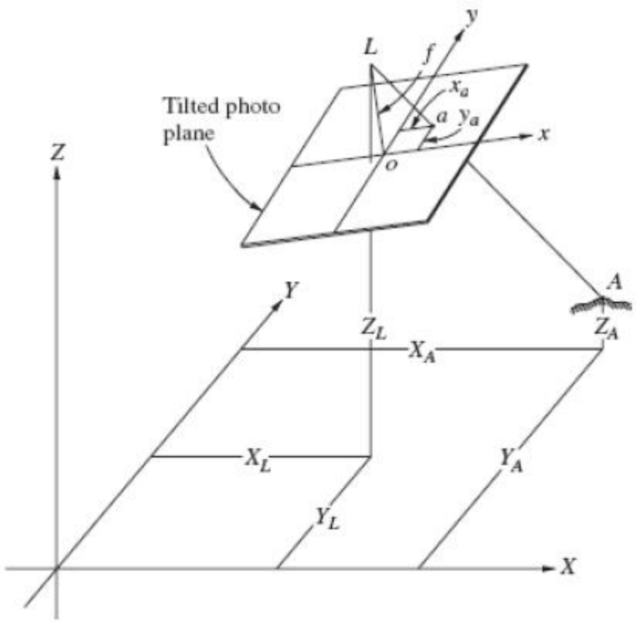


Figure 11-1. The collinearity condition.

❖ The collinearity condition is illustrated in Fig. 11-1, where L, a, and A lie along a straight line

❖ Two equations express the collinearity condition for any point on a photo: one equation for the x photo coordinate and another for the y photo coordinate

11-4. Collinearity Condition

❖ The mathematical relationships are expressed by eqs.11-1, 11-2

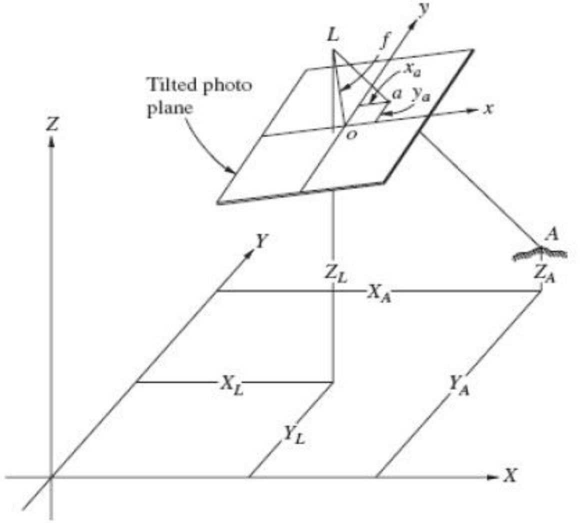


Figure 11–1. The collinearity condition.

$$x_a = x_o - f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \quad (11\text{-}1)$$

$$y_a = y_o - f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \quad (11\text{-}2)$$

❖ In Eqs. (11-1) and (11-2), x and y are the photo coordinates of image point a:

- X_A, Y_A, and Z_A are object space coordinates of point A
- X_L, Y_L, and Z_L are object space coordinates of the exposure station
- f is the camera focal length
- x_0 and y_0 are the coordinates of the principal point (usually known from camera calibration) the $m's$ are functions of three rotation angles, and most often omega, phi, and kappa are the angles employed

11-4. Collinearity Condition

❖ The collinearity equations are nonlinear and can be linearized by using Taylor's theorem

❖ The linearized forms are eqs.11-3, 11-4

$$b_{11}\,d\omega + b_{12}\,d\phi + b_{13}\,d\kappa - b_{14}\,dX_L - b_{15}\,dY_L - b_{16}\,dZ_L + b_{14}\,dX_A + b_{15}\,dY_A + b_{16}\,dZ_A = J + v_{x_a} \quad (11\text{-}3)$$

$$b_{21}\,d\omega + b_{22}\,d\phi + b_{23}\,d\kappa - b_{24}\,dX_L - b_{25}\,dY_L - b_{26}\,dZ_L + b_{24}\,dX_A + b_{25}\,dY_A + b_{26}\,dZ_A = K + v_{y_a} \quad (11\text{-}4)$$

❖ In Eqs. (11-3) and (11-4), V_{x_a} and V_{y_a} are residual errors in measured x_a and y_a image coordinates

- $d\omega, d\varphi,$ and $d\kappa$ are corrections to initial approximations for the orientation angles of the photo
- dX_L, dY_L, and dZ_L are corrections to initial approximations for the exposure station coordinates
- dX_A, dY_A, and dZ_A are corrections to initial values for the object space coordinates of point A

❖ Because higher order terms are ignored in linearization by Taylor's theorem, the linearized forms of the equations are approximations

⇨ They must therefore be solved iteratively, until the magnitudes of corrections to initial approximations become negligible

11-5. Coplanarity Condition

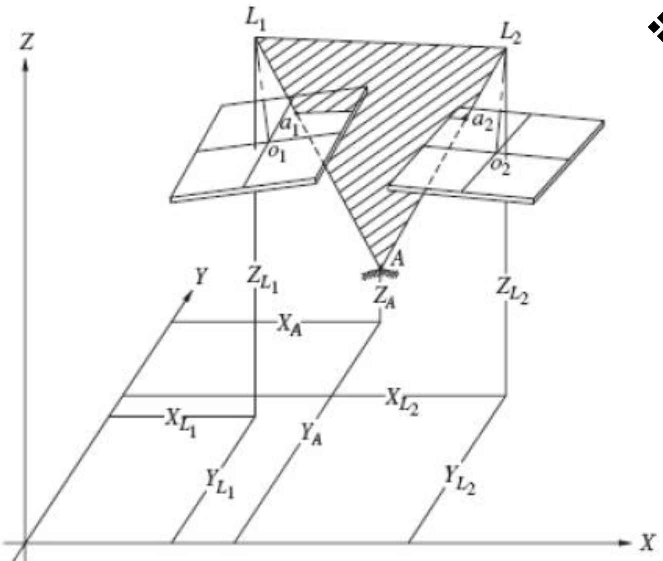


Figure 11−2. The coplanarity condition.

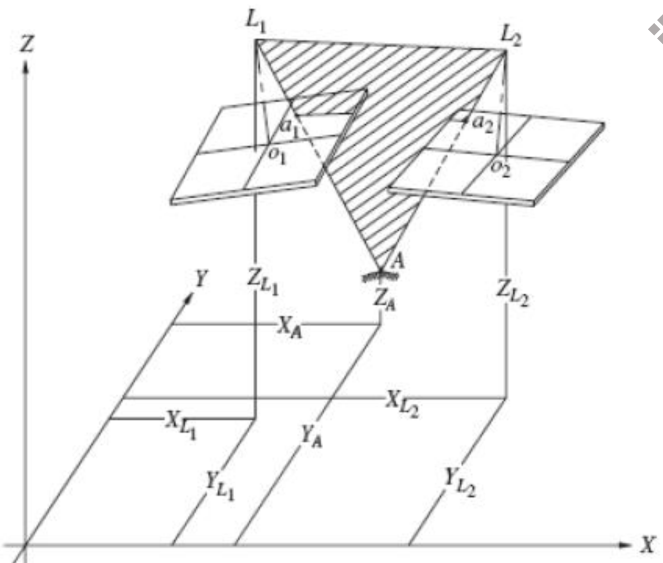
- Coplanarity, as illustrated in Fig. 11-2, is the condition that the two exposure stations of a stereopair, any object point, and its corresponding image points on the two photos all lie in a common plane
  - ⇨ In the figure, for example, points L_1, L_2, a_1, a_2 and A and A all lie in the same plane
  - ⇨ The coplanarity condition is shown in Eq. (11-5).

$$0 = B_X(E_1F_2 - E_2F_1) + B_Y(F_1D_2 - F_2D_1) + B_Z(D_1E_2 - D_2E_1) \qquad (11\text{-}5)$$

- In Eq. (11-5), subscripts 1 and 2 affixed to terms D, E, and F indicate that the terms apply to either photo 1 or photo 2

- The $m's$ again are functions of the three rotation angles omega, phi, and kappa

11-5. Coplanarity Condition



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$$0 = B_X(E_1F_2 - E_2F_1) + B_Y(F_1D_2 - F_2D_1) + B_Z(D_1E_2 - D_2E_1) \qquad (11\text{-}5)$$

- One coplanarity equation may be written for each object point whose images appear on both photos of the stereopair

- The coplanarity equations do not contain object space coordinates as unknowns; rather, they contain only the elements of exterior orientation of the two photos of the stereopair

- Like collinearity equations, the **coplanarity equation** is nonlinear and must be <u>linearized by using Taylor's theorem</u> and <u>solved iteratively for corrections to approximations of the orientation parameters</u>

11-6. Space Resection by Collinearity

- Space resection is a method of determining the six elements of exterior orientation ($\omega, \varphi, \kappa, X_L, Y_L$, and Z_L) of a photograph
- This method requires a minimum of three control points, with known XYZ object space coordinates, to be imaged in the photograph
- If the ground control coordinates are assumed to be known and fixed, then the linearized forms of the space resection collinearity equations for a point A are

$$b_{11}\,d\omega + b_{12}\,d\phi + b_{13}\,d\kappa - b_{14}\,dX_L - b_{15}\,dY_L - b_{16}\,dZ_L = J + v_{x_a} \qquad (11\text{-}6)$$

$$b_{21}\,d\omega + b_{22}\,d\phi + b_{23}\,d\kappa - b_{24}\,dX_L - b_{25}\,dY_L - b_{26}\,dZ_L = K + v_{y_a} \qquad (11\text{-}7)$$

- Two equations are formed for each control point, which gives six equations if the minimum of three control points is used
- In this case a unique solution results for the six unknowns, and the residual terms on the right sides of Eqs. (11-6) and (11-7) will be zero
- If four or more control points are used, more than six equations can be formed, allowing a least squares solution

11-6. Space Resection by Collinearity

- Since the collinearity equations are nonlinear, and have been linearized using Taylor's theorem, initial approximations are required for the unknown orientation parameters
- For the typical case of near-vertical photography, zero values can be used as initial approximations for ω and φ
- The value of Z_L (the height H above datum) can be computed using the method
  ⇨ Since this method requires only two control points, several solutions are possible, using different pairs of control points
- An improved approximation can be made by computing several values for H and taking the average
- After H has been determined, ground coordinates from a vertical photograph can be computed, using the measured x and y photo coordinates, focal length f , flying height H, and elevation of the object point Z

11-6. Space Resection by Collinearity

- A two-dimensional conformal coordinate transformation can then be performed, which relates the ground coordinates as computed from the vertical photo equations to the control values

- The two-dimensional conformal coordinate transformation Eqs.11-8

$$X = ax' - by' + T_x \quad\quad Y = ay' - bx' + T_Y \qquad (11\text{-}8)$$

$$\boxed{X_A = x_a\left(\frac{H - h_A}{f}\right) \quad (6\text{-}5) \qquad Y_A = y_a\left(\frac{H - h_A}{f}\right) \quad (6\text{-}6)}$$

- In Eqs. (11-8), X and Y are ground control coordinates for the point
  - x' and y' are ground coordinates from a vertical photograph as computed by Eqs. (6-5) and (6-6)
  - a, b, T_X, and T_Y are the transformation parameters

- A pair of equations of the type of Eqs. (11-8) can be written for each control point, and the four unknown parameters computed by least squares

- The translation factors T_X, and T_Y determined from this solution can then be used as initial approximations for X_L, and Y_L , respectively. Rotation angle θ can be used as an approximation for κ

11-6. Space Resection by Collinearity

$$b_{11}\,d\omega + b_{12}\,d\phi + b_{13}\,d\kappa - b_{14}\,dX_L - b_{15}\,dY_L - b_{16}\,dZ_L = J + v_{x_a} \qquad (11\text{-}6)$$

$$b_{21}\,d\omega + b_{22}\,d\phi + b_{23}\,d\kappa - b_{24}\,dX_L - b_{25}\,dY_L - b_{26}\,dZ_L = K + v_{y_a} \qquad (11\text{-}7)$$

$$\begin{aligned} X &= ax' - by' + T_X \\ Y &= ay' + bx' + T_Y \end{aligned} \qquad (11\text{-}8)$$

- By using these initial approximations in Eqs. (11-6) and (11-7), a least squares solution can be computed for the unknown corrections to the approximations
- The solution is iterated until the corrections become negligible

11-7. Space Intersection by Collinearity

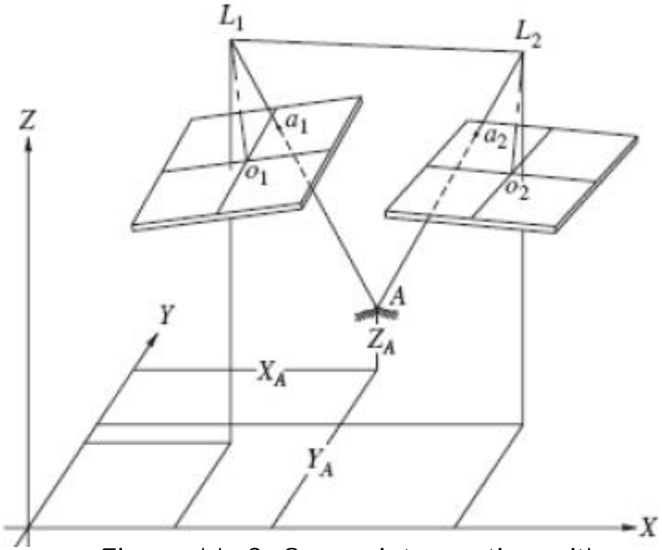

Figure 11-3. Space intersection with a stereopair of aerial photos.

❖ If space resection is used to determine the elements of exterior orientation for both photos of a stereopair, as described in the preceding section, then object point coordinates for points that lie in the stereo overlap area can be calculated

❖ The procedure is known as *space intersection*, so called because corresponding rays to the same object point from the two photos must intersect at the point, as shown in Fig. 11-3

11-7. Space Intersection by Collinearity

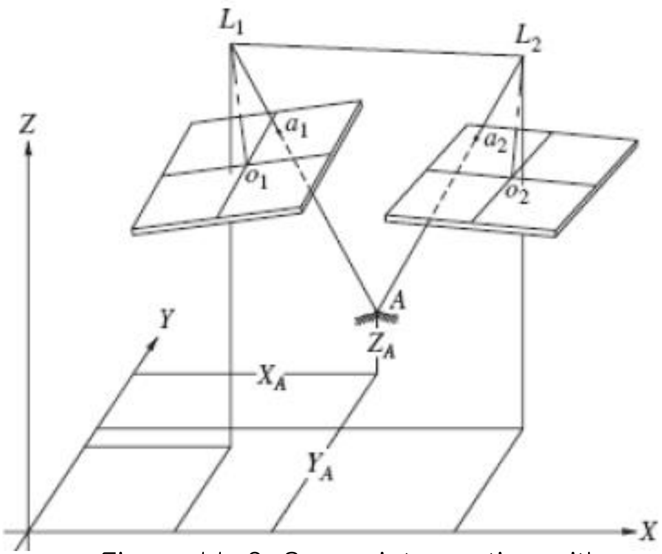

Figure 11-3. Space intersection with a stereopair of aerial photos.

$$b_{11}\,d\omega + b_{12}\,d\phi + b_{13}\,d\kappa - b_{14}\,dX_L - b_{15}\,dY_L - b_{16}\,dZ_L + b_{14}\,dX_A + b_{15}\,dY_A + b_{16}\,dZ_A = J + v_{x_a} \quad (11\text{-}3)$$

$$b_{21}\,d\omega + b_{22}\,d\phi + b_{23}\,d\kappa - b_{24}\,dX_L - b_{25}\,dY_L - b_{26}\,dZ_L + b_{24}\,dX_A + b_{25}\,dY_A + b_{26}\,dZ_A = K + v_{y_a} \quad (11\text{-}4)$$

- To calculate the coordinates of point A by space intersection, collinearity equations of the linearized form given by Eqs. (11-3) and (11-4) can be written for each new point, such as point A of Fig. 11-3

- Note, however, that since the six elements of exterior orientation are known, the only remaining unknowns in these equations are dX_A, dY_A, and dZ_A

- These are corrections to be applied to initial approximations for object space coordinates X_A, Y_A, and Z_A, respectively, for ground point A

- The linearized forms of the space intersection equations for point A are

$$b_{14}\,dX_A + b_{15}\,dY_A + b_{16}\,dZ_A = J + v_{x_a} \quad (11\text{-}9) \qquad b_{24}\,dX_A + b_{25}\,dY_A + b_{26}\,dZ_A = K + v_{y_a} \quad (11\text{-}10)$$

11-7. Space Intersection by Collinearity

$$b_{14}\,dX_A + b_{15}\,dY_A + b_{16}\,dZ_A = J + v_{x_a} \qquad (11\text{-}9)$$

$$b_{24}\,dX_A + b_{25}\,dY_A + b_{26}\,dZ_A = K + v_{y_a} \qquad (11\text{-}10)$$

❖ Two equations of this form can be written for point a of the left photo, and two more for point a of the right photo; hence four equations result, and the three unknowns dX_A, dY_A, and dZ_A can be computed in a least squares solution

⇨ These corrections are added to the initial approximations to obtain revised values for X_A, Y_A, and Z_A

⇨ The solution is then repeated until the magnitudes of the corrections become negligible

❖ Because the equations have been linearized using Taylor's theorem, initial approximations are required for each point whose object space coordinates are to be computed

⇨ For these calculations, with normal aerial photography vertical photos can be assumed, and the initial approximations can be determined by using the parallax equations [Eqs. (8-5) through (8-7)]

11-7. Space Intersection by Collinearity

- Because the equations have been linearized using Taylor's theorem, initial approximations are required for each point whose object space coordinates are to be computed
  - ⇨ For these calculations, with normal aerial photography vertical photos can be assumed, and the initial approximations can be determined by using the parallax equations [Eqs. (8-5) through (8-7)]

- Because the $X, Y,$ and Z coordinates for both exposure stations are known, for making these H can be taken as the average of Z_{L1} and Z_{L2}, and B is computed from

$$B = \sqrt{(X_{L_2} - X_{L_1})^2 + (Y_{L_2} - Y_{L_1})^2} \qquad \text{(a)}$$

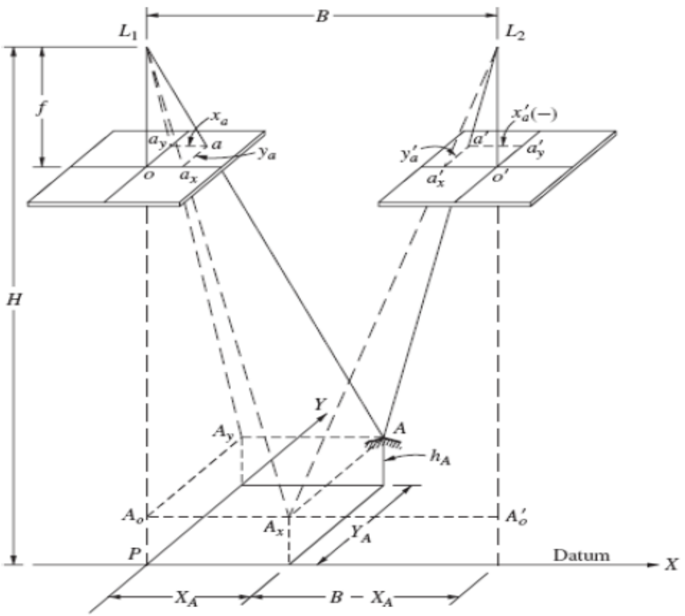

Figure 8−10. Geometry of an overlapping pair of vertical photographs.

$$h_A = H - \frac{Bf}{p_a} \qquad \text{(8-5)}$$

$$X_A = B\frac{x_a}{p_a} \qquad \text{(8-6)}$$

$$Y_A = B\frac{y_a}{p_a} \qquad \text{(8-7)}$$

11-7. Space Intersection by Collinearity

❖ The coordinates that result from Eqs. (8-6) and (8-7) are in the arbitrary system (Let these coordinates be designated as x' and y')

❖ To convert them to the X and Y ground system, coordinate transformation Eqs. (11-8) can be used

❖ For this transformation, the two exposure stations can serve as the control because their X and Y coordinates are known in the ground system, and their x' and y' coordinates in the parallax system are

$$x'_{L_1} = y'_{L_1} = y'_{L_2} = 0, \; and \;\; x'_{L_2} = B$$

❖ Since there are four equations and four unknowns, the transformation parameters can be solved for directly and applied to the imaged points to get the horizontal coordinate initial approximations as shown in Eqs. (11-11)

$$X_A = \frac{X_{L_2} - X_{L_1}}{B} x'_a - \frac{Y_{L_2} - Y_{L_1}}{B} y'_a + X_{L_1}$$

(11-11)

$$Y_A = \frac{X_{L_2} - X_{L_1}}{B} y'_a - \frac{Y_{L_2} - Y_{L_1}}{B} x'_a + Y_{L_1}$$

11-8. Analytical Stereomodel

- Aerial photographs for most applications are taken so that adjacent photos overlap by more than 50 percent
- Two adjacent photographs that overlap in this manner form a *stereopair*, and object points that appear in the overlap area constitute a *stereomodel*
- The mathematical calculation of three-dimensional ground coordinates of points in the stereomodel by analytical photogrammetric techniques forms an *analytical stereomodel*
- The process of forming an analytical stereomodel involves three primary steps: ***interior orientation, relative orientation,*** and ***absolute orientation***
- After these three steps are achieved, points in the analytical stereomodel will have object coordinates in the ground coordinate system
- These points can then be used for many purposes, such as digital mapping, serving as control for orthophoto production, or DEM generation
- The three orientation steps can be performed as distinct mathematical operations, or it is possible to combine them in a simultaneous solution

11-9. Analytical Interior Orientation

❖ **Interior orientation** for analytical photogrammetry is the step which mathematically recreates the geometry that existed in the camera when a particular photograph was exposed

⇨ This requires camera calibration information as well as quantification of the effects of atmospheric refraction

❖ In the case of film photography, a two-dimensional (usually affine) coordinate transformation can be used to relate the comparator, or softcopy coordinates to the fiducial coordinate system as well as to correct for film distortion

❖ There is no need for fiducial measurements when using photographs from a digital camera, since it has a consistent coordinate system in the form of pixels

❖ For both film and digital photography, the lens distortion and principal-point information from **camera calibration** are then used to **refine the coordinates** so that they are correctly related to the principal point and free from lens distortion

❖ Finally, **atmospheric refraction corrections** can be applied to the photo coordinates to complete the refinement and, therefore, finish the interior orientation

11-10. Analytical Relative Orientation

❖ Analytical relative orientation is the process of determining the relative angular attitude and positional displacement between the photographs that existed when the photos were taken

⇨ This involves defining certain elements of exterior orientation and calculating the remaining ones

❖ In analytical relative orientation, it is common practice to fix the exterior orientation elements $\omega, \varphi, \kappa, X_L$, and Y_L of the left photo of the stereopair to zero values

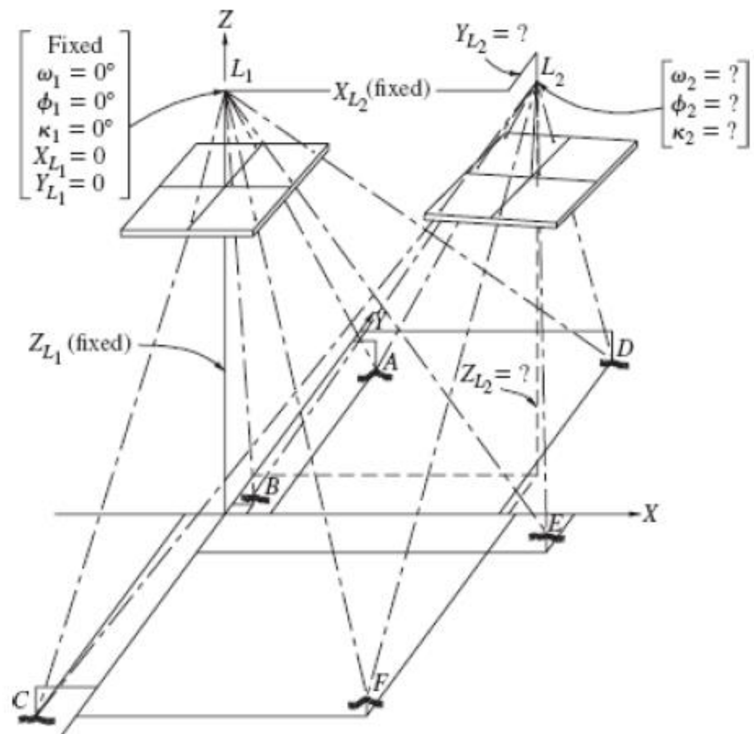

Figure 11-4. Analytical relative orientation of a stereopair.

11-10. Analytical Relative Orientation

- Analytical relative orientation is the process of determining the relative angular attitude and positional displacement between the photographs that existed when the photos were taken
  - ⇨ This involves defining certain elements of exterior orientation and calculating the remaining ones

- Also for convenience, Z_L of the left photo $((Z_{L_1}))$ is set equal to f , and X_L of the right photo (X_{L_2}) is set equal to the photo base b (With these choices for Z_{L_1} and X_{L_2}, initial approximations for the unknowns are more easily calculated, as will be explained later)
  - ⇨ This leaves five elements of the right photo that must be determined. Figure 11-4 illustrates a stereomodel formed by analytical relative orientation

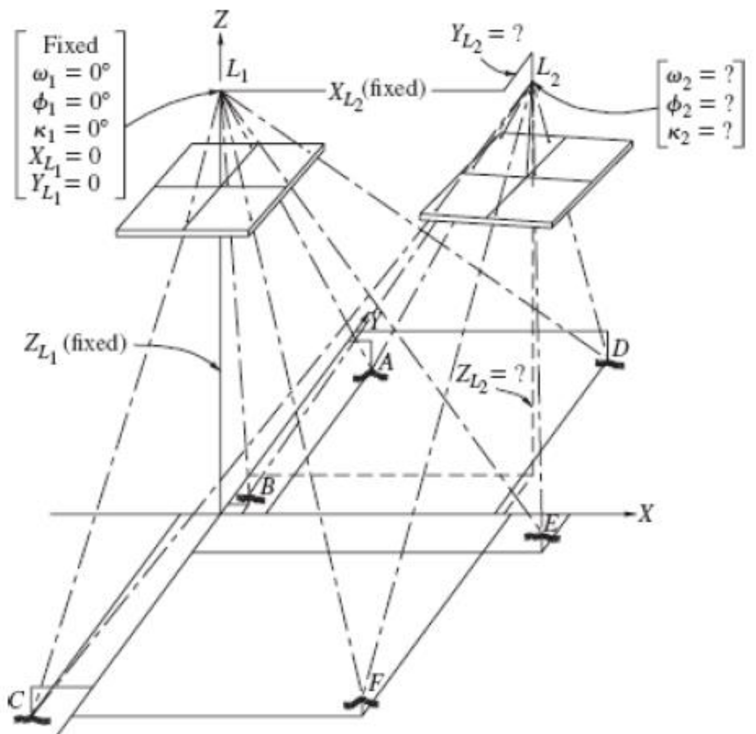

Figure 11-4. Analytical relative orientation of a stereopair.

11-10. Analytical Relative Orientation

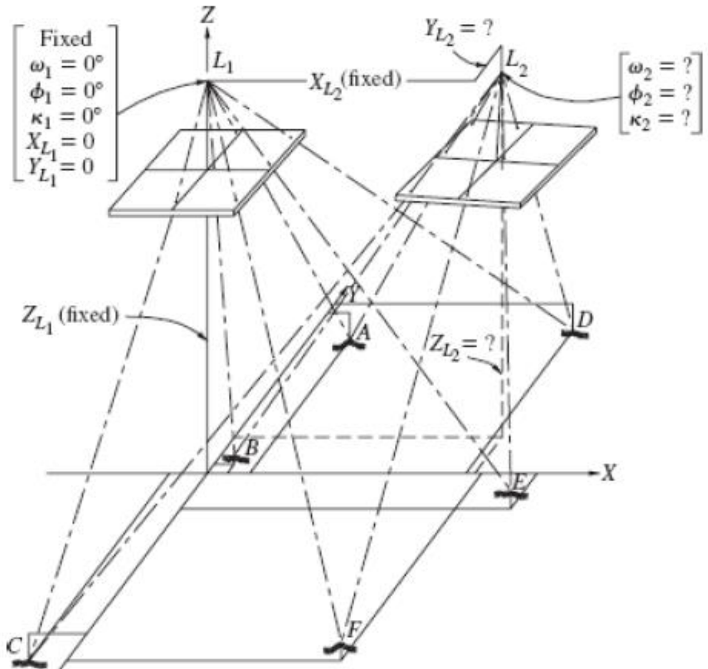

Figure 11–4. Analytical relative orientation of a stereopair.

- ❖ Although the coplanarity condition equation can be used for analytical relative orientation, <u>the collinearity condition is more commonly applied</u>
- ❖ In applying collinearity, each object point in the stereomodel contributes four equations
  - a pair of equations for the x and y photo coordinates of its image in the left photo
  - a pair of equations for the x and y photo coordinates of its image in the right photo

- ❖ In addition to the five unknown orientation elements, each object point adds three more unknowns which are their X, Y, and Z coordinates in the stereomodel
- ❖ Thus each point used in relative orientation results in a net gain of one equation for the overall solution, and therefore at least five object points are required for a solution
- ❖ If six or more points are available, an improved solution is possible through least squares

11-10. Analytical Relative Orientation

$$
A = \begin{array}{c}
\begin{matrix} d\omega_2 & d\phi_2 & d\kappa_2 & dY_{L_2} & dZ_{L_2} & dX_A & dY_A & dZ_A & dX_B & dY_B & dZ_B \end{matrix} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & (b_{a_{14}})_1 & (b_{a_{15}})_1 & (b_{a_{16}})_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (b_{a_{24}})_1 & (b_{a_{25}})_1 & (b_{a_{26}})_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{b_{14}})_1 & (b_{b_{15}})_1 & (b_{b_{16}})_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{b_{24}})_1 & (b_{b_{25}})_1 & (b_{b_{26}})_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{a_{11}})_2 & (b_{a_{12}})_2 & (b_{a_{13}})_2 & (-b_{a_{15}})_2 & (-b_{a_{16}})_2 & (b_{a_{14}})_2 & (b_{a_{15}})_2 & (b_{a_{16}})_2 & 0 & 0 & 0 \\
(b_{a_{21}})_2 & (b_{a_{22}})_2 & (b_{a_{23}})_2 & (-b_{a_{25}})_2 & (-b_{a_{26}})_2 & (b_{a_{24}})_2 & (b_{a_{25}})_2 & (b_{a_{26}})_2 & 0 & 0 & 0 \\
(b_{b_{11}})_2 & (b_{b_{12}})_2 & (b_{b_{13}})_2 & (-b_{b_{15}})_2 & (-b_{b_{16}})_2 & 0 & 0 & 0 & (b_{b_{14}})_2 & (b_{b_{15}})_2 & (b_{b_{16}})_2 \\
(b_{b_{21}})_2 & (b_{b_{22}})_2 & (b_{b_{23}})_2 & (-b_{b_{25}})_2 & (-b_{b_{26}})_2 & 0 & 0 & 0 & (b_{b_{24}})_2 & (b_{b_{25}})_2 & (b_{b_{26}})_2 \\
(b_{c_{11}})_2 & (b_{c_{12}})_2 & (b_{c_{13}})_2 & (-b_{c_{15}})_2 & (-b_{c_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{c_{21}})_2 & (b_{c_{22}})_2 & (b_{c_{23}})_2 & (-b_{c_{25}})_2 & (-b_{c_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{d_{11}})_2 & (b_{d_{12}})_2 & (b_{d_{13}})_2 & (-b_{d_{13}})_2 & (-b_{d_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{d_{21}})_2 & (b_{d_{22}})_2 & (b_{d_{23}})_2 & (-b_{d_{23}})_2 & (-b_{d_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{e_{11}})_2 & (b_{e_{12}})_2 & (b_{e_{13}})_2 & (-b_{e_{15}})_2 & (-b_{e_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{e_{21}})_2 & (b_{e_{22}})_2 & (b_{e_{23}})_2 & (-b_{e_{25}})_2 & (-b_{e_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{f_{11}})_2 & (b_{f_{12}})_2 & (b_{f_{13}})_2 & (-b_{f_{15}})_2 & (-b_{f_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{f_{21}})_2 & (b_{f_{22}})_2 & (b_{f_{23}})_2 & (-b_{f_{25}})_2 & (-b_{f_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

- Prior to solving the linearized collinearity equations, initial approximations for all unknown values must be determined
- For photography that was intended to be vertical, values of zero are commonly used for initial estimates of ω_2, ϕ_2, κ_2, and Y_{L_2}
- An initial value for Z_{L_2} may be selected equal to the value used for Z_{L_1}
- If the constraints that were noted earlier are used for the parameters, that is, $\omega_1 = \phi_1 = \kappa_1 = X_{L_1} = Y_{L_1} = 0,$ $Z_{L_1} = f, and \quad X_{L_2} = b$, then the scale of the stereomodel is approximately equal to photo scale

11-10. Analytical Relative Orientation

$$
A = \begin{array}{c}
\begin{matrix} d\omega_2 & d\phi_2 & d\kappa_2 & dY_{L_2} & dZ_{L_2} & dX_A & dY_A & dZ_A & dX_B & dY_B & dZ_B \end{matrix} \\
\left[\begin{matrix}
0 & 0 & 0 & 0 & 0 & (b_{a_{14}})_1 & (b_{a_{15}})_1 & (b_{a_{16}})_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (b_{a_{24}})_1 & (b_{a_{25}})_1 & (b_{a_{26}})_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{b_{14}})_1 & (b_{b_{15}})_1 & (b_{b_{16}})_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{b_{24}})_1 & (b_{b_{25}})_1 & (b_{b_{26}})_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{a_{11}})_2 & (b_{a_{12}})_2 & (b_{a_{13}})_2 & (-b_{a_{15}})_2 & (-b_{a_{16}})_2 & (b_{a_{14}})_2 & (b_{a_{15}})_2 & (b_{a_{16}})_2 & 0 & 0 & 0 \\
(b_{a_{21}})_2 & (b_{a_{22}})_2 & (b_{a_{23}})_2 & (-b_{a_{25}})_2 & (-b_{a_{26}})_2 & (b_{a_{24}})_2 & (b_{a_{25}})_2 & (b_{a_{26}})_2 & 0 & 0 & 0 \\
(b_{b_{11}})_2 & (b_{b_{12}})_2 & (b_{b_{13}})_2 & (-b_{b_{15}})_2 & (-b_{b_{16}})_2 & 0 & 0 & 0 & (b_{b_{14}})_2 & (b_{b_{15}})_2 & (b_{b_{16}})_2 \\
(b_{b_{21}})_2 & (b_{b_{22}})_2 & (b_{b_{23}})_2 & (-b_{b_{25}})_2 & (-b_{b_{26}})_2 & 0 & 0 & 0 & (b_{b_{24}})_2 & (b_{b_{25}})_2 & (b_{b_{26}})_2 \\
(b_{c_{11}})_2 & (b_{c_{12}})_2 & (b_{c_{13}})_2 & (-b_{c_{15}})_2 & (-b_{c_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{c_{21}})_2 & (b_{c_{22}})_2 & (b_{c_{23}})_2 & (-b_{c_{25}})_2 & (-b_{c_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{d_{11}})_2 & (b_{d_{12}})_2 & (b_{d_{13}})_2 & (-b_{d_{15}})_2 & (-b_{d_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{d_{21}})_2 & (b_{d_{22}})_2 & (b_{d_{23}})_2 & (-b_{d_{25}})_2 & (-b_{d_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{e_{11}})_2 & (b_{e_{12}})_2 & (b_{e_{13}})_2 & (-b_{e_{15}})_2 & (-b_{e_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{e_{21}})_2 & (b_{e_{22}})_2 & (b_{e_{23}})_2 & (-b_{e_{25}})_2 & (-b_{e_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{f_{11}})_2 & (b_{f_{12}})_2 & (b_{f_{13}})_2 & (-b_{f_{15}})_2 & (-b_{f_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{f_{21}})_2 & (b_{f_{22}})_2 & (b_{f_{23}})_2 & (-b_{f_{25}})_2 & (-b_{f_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right]
\end{array}
$$

- ❖ Thus the x and y photo coordinates of the left photo are good approximations for X and Y object space coordinates, and zeros are good approximations for Z object space coordinates, respectively

11-10. Analytical Relative Orientation

❖ Suppose that the six points of Fig. 11-4 were used in analytical relative orientation. In matrix form, the system of 24 equations involving 23 unknowns could be expressed as follows:

$$ {}_{24}A^{23}\,{}_{23}X^{1} = {}_{24}L^{1} + {}_{24}V^{1} \qquad (11\text{-}12)$$

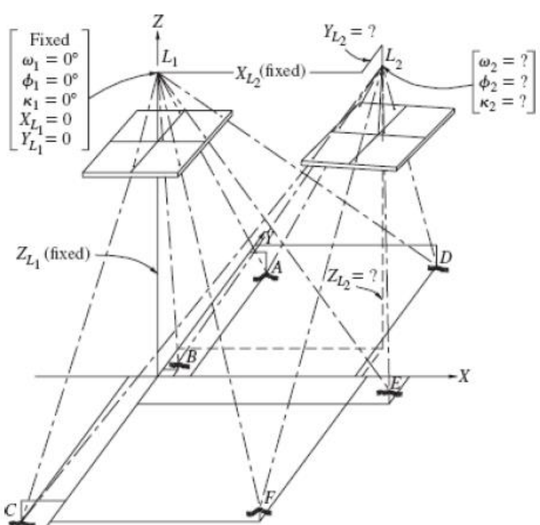

Figure 11–4. Analytical relative orientation of a stereopair.

where

$$
\begin{array}{cccccccccccc}
dX_C & dY_C & dZ_C & dX_D & dY_D & dZ_D & dX_E & dY_E & dZ_E & dX_F & dY_F & dZ_F
\end{array}
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{c_{14}})_1 & (b_{c_{15}})_1 & (b_{c_{16}})_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{c_{24}})_1 & (b_{c_{25}})_1 & (b_{c_{26}})_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (b_{d_{14}})_1 & (b_{d_{15}})_1 & (b_{d_{16}})_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (b_{d_{24}})_1 & (b_{d_{25}})_1 & (b_{d_{26}})_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (b_{e_{14}})_1 & (b_{e_{15}})_1 & (b_{e_{16}})_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (b_{e_{24}})_1 & (b_{e_{25}})_1 & (b_{e_{26}})_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{f_{14}})_1 & (b_{f_{15}})_1 & (b_{f_{16}})_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{f_{24}})_1 & (b_{f_{25}})_1 & (b_{f_{26}})_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{c_{14}})_2 & (b_{c_{15}})_2 & (b_{c_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(b_{c_{24}})_2 & (b_{c_{25}})_2 & (b_{c_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (b_{d_{14}})_2 & (b_{d_{15}})_2 & (b_{d_{16}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (b_{d_{24}})_2 & (b_{d_{25}})_2 & (b_{d_{26}})_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (b_{e_{14}})_2 & (b_{e_{15}})_2 & (b_{e_{16}})_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (b_{e_{24}})_2 & (b_{e_{25}})_2 & (b_{e_{26}})_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{f_{14}})_2 & (b_{f_{15}})_2 & (b_{f_{16}})_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (b_{f_{24}})_2 & (b_{f_{25}})_2 & (b_{f_{26}})_2
\end{bmatrix}
$$

$$
X = \begin{bmatrix} d\omega_2 \\ d\phi_2 \\ d\kappa_2 \\ dY_{L_2} \\ dZ_{L_2} \\ dX_A \\ dY_A \\ dZ_A \\ dX_B \\ dY_B \\ dZ_B \\ dX_C \\ dY_C \\ dZ_C \\ dX_D \\ dY_D \\ dZ_D \\ dX_E \\ dY_E \\ dZ_E \\ dX_F \\ dY_F \\ dZ_F \end{bmatrix}
\quad
L = \begin{bmatrix} (J_a)_1 \\ (K_a)_1 \\ (J_b)_1 \\ (K_b)_1 \\ (J_c)_1 \\ (K_c)_1 \\ (J_d)_1 \\ (K_d)_1 \\ (J_e)_1 \\ (K_e)_1 \\ (J_f)_1 \\ (K_f)_1 \\ (J_a)_2 \\ (K_a)_2 \\ (J_b)_2 \\ (K_b)_2 \\ (J_c)_2 \\ (K_c)_2 \\ (J_d)_2 \\ (K_d)_2 \\ (J_e)_2 \\ (K_e)_2 \\ (J_f)_2 \\ (K_f)_2 \end{bmatrix}
\quad
V = \begin{bmatrix} (v_{x_a})_1 \\ (v_{y_a})_1 \\ (v_{x_b})_1 \\ (v_{y_b})_1 \\ (v_{x_c})_1 \\ (v_{y_c})_1 \\ (v_{x_d})_1 \\ (v_{y_d})_1 \\ (v_{x_e})_1 \\ (v_{y_e})_1 \\ (v_{x_f})_1 \\ (v_{y_f})_1 \\ (v_{x_a})_2 \\ (v_{y_a})_2 \\ (v_{x_b})_2 \\ (v_{y_b})_2 \\ (v_{x_c})_2 \\ (v_{y_c})_2 \\ (v_{x_d})_2 \\ (v_{y_d})_2 \\ (v_{x_e})_2 \\ (v_{y_e})_2 \\ (v_{x_f})_2 \\ (v_{y_f})_2 \end{bmatrix}
$$

11-10. Analytical Relative Orientation

❖ Suppose that the six points of Fig. 11-4 were used in analytical relative orientation. In matrix form, the system of 24 equations involving 23 unknowns could be expressed as follows:

$$ {}_{24}A^{23}\,{}_{23}X^{1} = {}_{24}L^{1} + {}_{24}V^{1} \qquad (11\text{-}12)$$

where

$$
\begin{array}{cccccccccccc}
dX_C & dY_C & dZ_C & dX_D & dY_D & dZ_D & dX_E & dY_E & dZ_E & dX_F & dY_F & dZ_F
\end{array}
$$

$$
\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0\\
(b_{c_{14}})_1&(b_{c_{15}})_1&(b_{c_{16}})_1&0&0&0&0&0&0&0&0&0\\
(b_{c_{24}})_1&(b_{c_{25}})_1&(b_{c_{26}})_1&0&0&0&0&0&0&0&0&0\\
0&0&0&(b_{d_{14}})_1&(b_{d_{15}})_1&(b_{d_{16}})_1&0&0&0&0&0&0\\
0&0&0&(b_{d_{24}})_1&(b_{d_{25}})_1&(b_{d_{26}})_1&0&0&0&0&0&0\\
0&0&0&0&0&0&(b_{e_{14}})_1&(b_{e_{15}})_1&(b_{e_{16}})_1&0&0&0\\
0&0&0&0&0&0&(b_{e_{24}})_1&(b_{e_{25}})_1&(b_{e_{26}})_1&0&0&0\\
0&0&0&0&0&0&0&0&0&(b_{f_{14}})_1&(b_{f_{15}})_1&(b_{f_{16}})_1\\
0&0&0&0&0&0&0&0&0&(b_{f_{24}})_1&(b_{f_{25}})_1&(b_{f_{26}})_1\\
0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

$$
X = \begin{bmatrix} d\omega_2 \\ d\phi_2 \\ d\kappa_2 \\ dY_{L_2} \\ dZ_{L_2} \\ dX_A \\ dY_A \\ dZ_A \\ dX_B \\ dY_B \\ dZ_B \\ dX_C \\ dY_C \\ dZ_C \end{bmatrix}
\quad
L = \begin{bmatrix} (J_a)_1 \\ (K_a)_1 \\ (J_b)_1 \\ (K_b)_1 \\ (J_c)_1 \\ (K_c)_1 \\ (J_d)_1 \\ (K_d)_1 \\ (J_e)_1 \\ (K_e)_1 \\ (J_f)_1 \\ (K_f)_1 \\ (J_a)_2 \\ (K_a)_2 \\ (J_b)_2 \end{bmatrix}
\quad
V = \begin{bmatrix} (v_{x_a})_1 \\ (v_{y_a})_1 \\ (v_{x_b})_1 \\ (v_{y_b})_1 \\ (v_{x_c})_1 \\ (v_{y_c})_1 \\ (v_{x_d})_1 \\ (v_{y_d})_1 \\ (v_{x_e})_1 \\ (v_{y_e})_1 \\ (v_{x_f})_1 \\ (v_{y_f})_1 \\ (v_{x_a})_2 \\ (v_{y_a})_2 \end{bmatrix}
$$

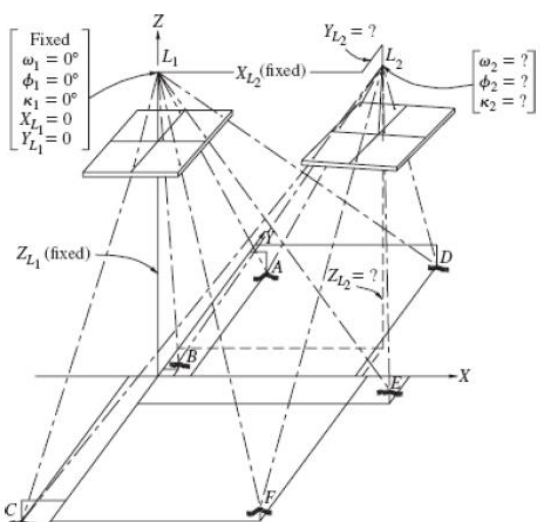

Figure 11−4. Analytical relative orientation of a stereopair.

- The subscripts a, b, c, d, e, and f correspond to the point names; subscript 1 refers to the left photo, and subscript 2 refers to the right photo
- Note that the elements of the X matrix are listed at the tops of the columns of the A matrix for illustration purposes only

11-10. Analytical Relative Orientation

❖ The following equations show the form of the $\boldsymbol{A}, \boldsymbol{X}, \boldsymbol{L}$, and $\boldsymbol{V}$ matrices when partitioned by the standard approach

$$
A=\begin{bmatrix}
{}_2 0^5 & {}_2\ddot{B}^3_{1a} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 \\
{}_2 0^5 & {}_2 0^3 & {}_2\ddot{B}^3_{1b} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 \\
{}_2 0^5 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{1c} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 \\
{}_2 0^5 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{1d} & {}_2 0^3 & {}_2 0^3 \\
{}_2 0^5 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{1e} & {}_2 0^3 \\
{}_2 0^5 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{1f} \\
{}_2\dot{B}^5_{2a} & {}_2\ddot{B}^3_{2a} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 \\
{}_2\dot{B}^5_{2b} & {}_2 0^3 & {}_2\ddot{B}^3_{2b} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 \\
{}_2\dot{B}^5_{2c} & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{2c} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 \\
{}_2\dot{B}^5_{2d} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{2d} & {}_2 0^3 & {}_2 0^3 \\
{}_2\dot{B}^5_{2e} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{2e} & {}_2 0^3 \\
{}_2\dot{B}^5_{2f} & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2 0^3 & {}_2\ddot{B}^3_{2f}
\end{bmatrix}
\quad
X=\begin{bmatrix} \dot{\Delta}_2 \\ \ddot{\Delta}_A \\ \ddot{\Delta}_B \\ \ddot{\Delta}_C \\ \ddot{\Delta}_D \\ \ddot{\Delta}_E \\ \ddot{\Delta}_F \end{bmatrix}
\quad
L=\begin{bmatrix} \varepsilon_{1a} \\ \varepsilon_{1b} \\ \varepsilon_{1c} \\ \varepsilon_{1d} \\ \varepsilon_{1e} \\ \varepsilon_{1f} \\ \varepsilon_{2a} \\ \varepsilon_{2b} \\ \varepsilon_{2c} \\ \varepsilon_{2d} \\ \varepsilon_{2e} \\ \varepsilon_{2f} \end{bmatrix}
\quad
V=\begin{bmatrix} v_{1a} \\ v_{1b} \\ v_{1c} \\ v_{1d} \\ v_{1e} \\ v_{1f} \\ v_{2a} \\ v_{2b} \\ v_{2c} \\ v_{2d} \\ v_{2e} \\ v_{2f} \end{bmatrix}
$$

Where:

$$
{}_2 0^5=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\qquad
{}_2 0^3=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
{}_2\dot{B}^5_{2p}=\begin{bmatrix} (b_{p_{11}})_2 & (b_{p_{12}})_2 & (b_{p_{13}})_2 & (-b_{p_{15}})_2 & (-b_{p_{16}})_2 \\ (b_{p_{21}})_2 & (b_{p_{22}})_2 & (b_{p_{23}})_2 & (-b_{p_{25}})_2 & (-b_{p_{26}})_2 \end{bmatrix}
\qquad
{}_2\ddot{B}^3_{ip}=\begin{bmatrix} (b_{p_{14}})_i & (b_{p_{15}})_i & (b_{p_{16}})_i \\ (b_{p_{24}})_i & (b_{p_{25}})_i & (b_{p_{26}})_i \end{bmatrix}
$$

$$
\dot{\Delta}_2=\begin{bmatrix} d\omega_2 \\ d\phi_2 \\ d\kappa_2 \\ dY_{L_2} \\ dZ_{L_2} \end{bmatrix}
\qquad
\ddot{\Delta}_p=\begin{bmatrix} dX_p \\ dY_p \\ dZ_p \end{bmatrix}
\qquad
\varepsilon_{ip}=\begin{bmatrix} (J_p)_i \\ (K_p)_i \end{bmatrix}
\qquad
v_{ip}=\begin{bmatrix} (v_{x_p})_i \\ (v_{y_p})_i \end{bmatrix}
$$

❖ In the above submatrices, $\boldsymbol{p}$ is the point designation, and $\boldsymbol{i}$ is the photo designation

❖ The prefixed subscript and postfixed superscript designate the number of rows and columns, respectively

11-10. Analytical Relative Orientation

❖ When the observation equations are partitioned in the above-described manner, the least squares solution $(A^T A)X = (A^T L)$ takes the following form

$$
\begin{bmatrix}
\dot{N}_2 & \bar{N}_a & \bar{N}_b & \bar{N}_c & \bar{N}_d & \bar{N}_e & \bar{N}_f \\
\bar{N}_a^T & \ddot{N}_a & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 \\
\bar{N}_b^T & {}_3 0^3 & \ddot{N}_b & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 \\
\bar{N}_c^T & {}_3 0^3 & {}_3 0^3 & \ddot{N}_c & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 \\
\bar{N}_d^T & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & \ddot{N}_d & {}_3 0^3 & {}_3 0^3 \\
\bar{N}_e^T & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & \ddot{N}_e & {}_3 0^3 \\
\bar{N}_f^T & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & {}_3 0^3 & \ddot{N}_f
\end{bmatrix}
\begin{bmatrix}
\dot{\Delta}_2 \\ \ddot{\Delta}_A \\ \ddot{\Delta}_B \\ \ddot{\Delta}_C \\ \ddot{\Delta}_D \\ \ddot{\Delta}_E \\ \ddot{\Delta}_F
\end{bmatrix}
=
\begin{bmatrix}
\dot{K}_2 \\ \ddot{K}_A \\ \ddot{K}_B \\ \ddot{K}_C \\ \ddot{K}_D \\ \ddot{K}_E \\ \ddot{K}_F
\end{bmatrix}
$$

Where:

$$\dot{N}_2 = \sum_{p=a}^{f} (\dot{B}_{2_p}^T \dot{B}_{2_p}) \qquad (5\times 5 \text{ submatrix})$$

$$\bar{N}_p = \dot{B}_{2_p}^T \ddot{B}_{2_p} \qquad (5\times 3 \text{ submatrix})$$

$$\ddot{N}_p = (\ddot{B}_{1_p}^T \ddot{B}_{1_p}) + (\ddot{B}_{2_p}^T \ddot{B}_{2_p}) \qquad (3\times 3 \text{ submatrix})$$

$$\dot{K}_2 = \sum_{p=a}^{f} (\dot{B}_{2_p}^T \varepsilon_{2_p}) \qquad (5\times 1 \text{ submatrix})$$

$$\ddot{K}_p = (\ddot{B}_{1_p}^T \varepsilon_{1_p}) + (\ddot{B}_{2_p}^T \varepsilon_{2_p}) \qquad (3\times 1 \text{ submatrix})$$

$${}_3 0^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

11-11. Analytical Absolute Orientation

- For a small stereomodel such as that computed from one stereopair, analytical absolute orientation can be performed using a three-dimensional conformal coordinate transformation

- This requires a minimum of two horizontal and three vertical control points, but additional control points provide redundancy, which enables a least squares solution

- It is important for the ground system to be a true cartesian coordinate system, such as local vertical, since the three-dimensional conformal coordinate transformation is based on straight, orthogonal axes

- Once the transformation parameters have been computed, they can be applied to the remaining stereomodel points, including the X_L, Y_L, and Z_L coordinates of the left and right photographs
  ⇨ This gives the coordinates of all stereomodel points in the ground system