

446.328 Mechanical System Analysis

기계시스템해석

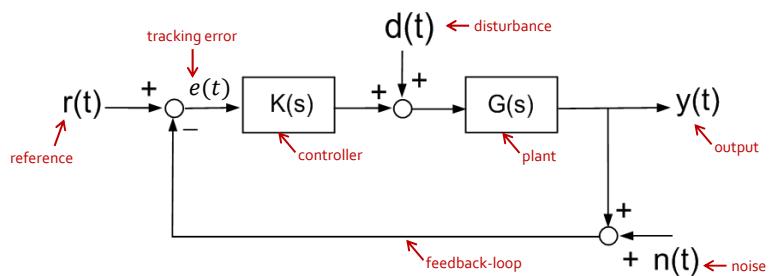
- lecture 23-24-

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Feedback Control System



- * goal: design the controller $K(s)$ s.t.
1) the closed-loop system is stable
2) $e(t) \rightarrow 0$ as quickly and robustly as possible even with $d(t)$

signal propagation

$$Y(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} R(s) + \frac{G(s)}{1 + G(s)K(s)} D(s) - \frac{G(s)K(s)}{1 + G(s)K(s)} N(s)$$

nominal tracking disturbance effect noise effect

- closed-loop TF: $H(s) = \frac{G(s)K(s)}{1+G(s)K(s)}$, loop transfer function $L(s) = G(s)K(s)$

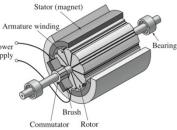
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Benefits of Feedback

system dynamics

block diagram



<u>open-loop control</u>	$W = \frac{k_m k_a}{R_a(sI+b)+k_m^2} R + \frac{R_a}{R_a(sI+b)+k_m^2} T_d$ <p style="text-align: center;">nominal tracking disturbance effect</p>
<u>closed-loop control</u>	$W = \frac{k_m k_a}{R_a(sI+b)+k_m k_a+k_m^2} R + \frac{R_a}{R_a(sI+b)+k_m k_a+k_m^2} T_d - \frac{k_m k_a}{R_a(sI+b)+k_m k_a+k_m^2} N$ <p style="text-align: center;">controller disturbance effect noise effect</p>

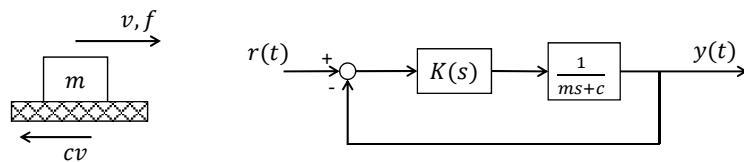
* benefit of feedback (with large k_a):

- 1) disturbance rejection; 2) robustness against uncertainty; 3) transient shaping (i.e., pole)

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P-Control of 1st Order System



$$m\dot{v} + cv = f$$

input:

output: v

* Proportional control $K(s) = k_p$

$$Y(s) = \frac{k_p}{ms + (k_p + c)} R(s) + \frac{1}{ms + (k_p + c)} d(s)$$


 no oscillation w/
step input $r(t)$.

- closed-loop stability for any $k_p > 0$

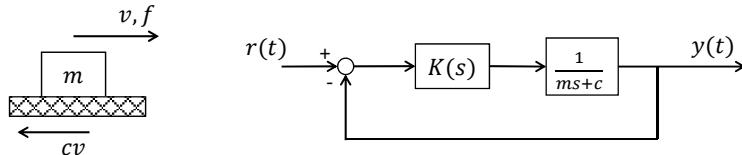
- if $r(t) = r$, $y(t) \rightarrow \frac{k_p}{k_n + c}r$ ($\approx r$ if k_p is very big) \rightarrow offset!

- faster convergence with a larger k_p ($\tau = \frac{m}{k_p + c}$)

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PI-Control of 1st Order System



$$m\ddot{v} + cv = f$$

* Proportional-Integral control

$$K(s) = k_p + k_I/s$$

$$Y(s) = \frac{k_p s + k_I}{ms^2 + (k_p + c)s + k_I} R(s)$$

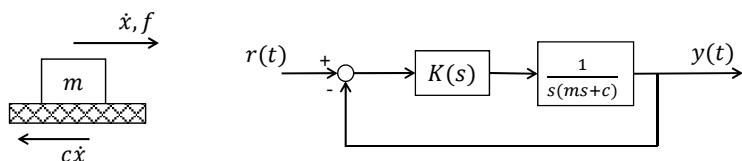
I-control:
 - to combat steady-state error
 - a large $k_I \rightarrow$ oscillation!

- closed-loop stability of $k_p > -c, k_I > 0$
- step tracking: if $r(t) = r, y(t) \rightarrow r$
- perfect disturbance rejection: if $d(t) = d, y_d \rightarrow 0$ due to the I-action
- transient shaping: critically-damped for fast, yet, no-oscillatory behavior
 (via $ms^2 + (k_p + c)s + k_I = m(s + p)^2$)
- ramp tracking: if $r(t) = at, e(t) \rightarrow ca/k_I \rightarrow$ need order increase...

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P-Control of 2nd Order System



$$m\ddot{x} + c\dot{x} = f$$

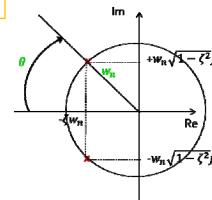
input: f

output: x

* Proportional-Integral control

$$K(s) = k_p$$

$$Y(s) = \frac{k_p}{ms^2 + cs + k_p} R(s)$$

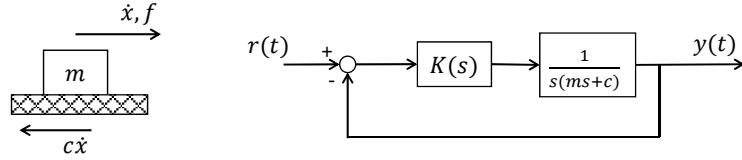


- closed-loop stability for any $k_p > 0$
- step tracking: if $r(t) = r, y(t) \rightarrow r \leftarrow$ with P-control, due to increase of plant's order
- disturbance rejection error: if $d(t) = d, y_d \rightarrow d/k_p \leftarrow$ need order increase
- transient shaping: can assign $\zeta = \frac{c}{2\sqrt{mk_p}} = 1$, but can't assign w_n together
- ramp tracking: if $r(t) = at, e(t) \rightarrow ca/k_p \leftarrow$ need PI-control

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PI-Control of 2nd Order System



$$m\ddot{x} + c\dot{x} = f$$

input: f

output: x

* Proportional-Integral control

$$K(s) = k_p + k_I/s$$

$$Y(s) = \frac{k_p s + k_I}{ms^3 + cs^2 + k_p s + k_I} R(s)$$

- closed-loop stability: if $ck_p > mk_I$

- step tracking: if $r(t) = r$, $y_r(t) \rightarrow r$

- perfect disturbance rejection: if $d(t) = d$, $y_d \rightarrow 0$

- perfect ramp tracking: if $r(t) = at$, $e(t) \rightarrow 0$

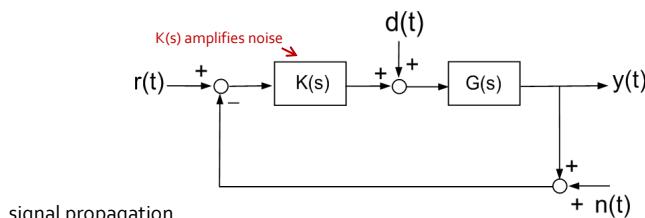
- transient shaping → PID-control $K(s) = k_p + \frac{k_I}{s} + k_D s \rightarrow H(s) = \frac{k_D s^2 + k_p s + k_I}{ms^3 + (c+k_p)s^2 + k_p s + k_I}$

impulse with step input: low-freq. approximation $k_D s \approx \frac{k_D s}{\tau s + 1}$



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Fundamental Limitation of Feedback Control



$$Y(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} R(s) + \frac{G(s)}{1 + G(s)K(s)} D(s) - \frac{G(s)K(s)}{1 + G(s)K(s)} N(s)$$

robustness & faster response w/ large K

disturbance rejection w/ large K

also → 1 with large K!!!
→ full-effect of noise?

⇒ can't achieve good reference tracking and good noise attenuation at the same time, unless $r(t)$ and $n(t)$ are in different frequency band

ex) dc-motor ($I = 1, k_m = 1, R_a = 1, b = 0.5, k_a = 50$)

$$H(s) = \frac{k_m k_a}{R_a(sI+b)+k_m^2+k_m k_a}$$

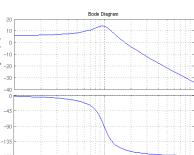
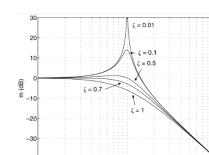
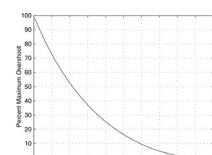
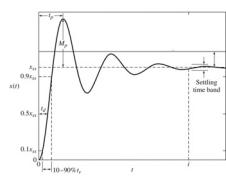
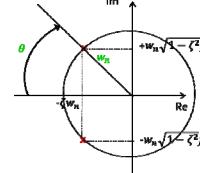
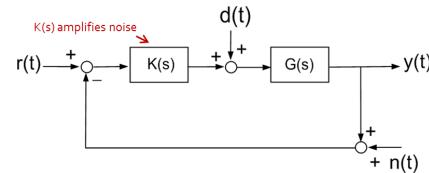
1) for dc-tracking: $H(0) = \frac{k_m k_a}{R_a b + k_m^2 + k_m k_a} \approx 0.97$

2) for noise attenuation: $H(jw)|_{w=60Hz} \approx 0.13$

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Observations



- ⇒ Ts: how quickly the steady-state is attained ($\zeta\omega_n$)
- ⇒ Tr: how fast the system's initial response is
- ⇒ no overshoot if $\zeta \geq 1$
- ⇒ bandwidth $2\log_{10}|H(j\omega_c)| = -3\text{dB}$: how agile system tracking can be
- ⇒ resonance: booming or instability, typically need to avoid particularly high-freq.

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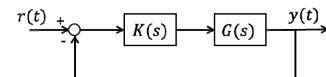
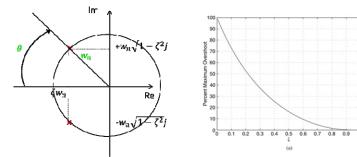
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Feedback Control Design Example - I

choose k, p s.t.

- 1) closed-loop system is stable
- 2) PO $\leq 5\%$
- 3) $T_s < 4\text{sec}$

$$G(s) = \frac{k}{s(s + p)}$$



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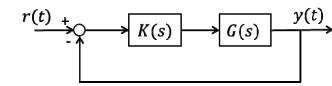
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Example 10.8.3

- stabilization of unstable system

$$G(s) = \frac{1}{s^2 - 4}$$

- PID control



Feedback Control Design Example

choose k, k_1 s.t.

- 1) closed-loop system is stable;
- 2) PO w.r.t. step $r(t) \leq 10\%$
- 3) minimize steady-state error for ramp input
- 4) minimize the effect of step disturbance
- 5) stability margin, actuator saturation: $0 \leq k \leq 100, 0 \leq k_1 \leq 20$

Feedback Control Design Example

choose k, k_1 s.t.

- 1) closed-loop system is stable;
- 2) PO w.r.t. step $r(t) \leq 10\%$
- 3) minimize steady-state error for ramp input
- 4) minimize the effect of step disturbance
- 5) stability margin, actuator saturation: $0 \leq k \leq 100, 0 \leq k_1 \leq 20$

Example 10.3.2

- thermal system modeling and control

Example 10.4.1 & 10.4.2

- fluid power system and hydraulic control implementation

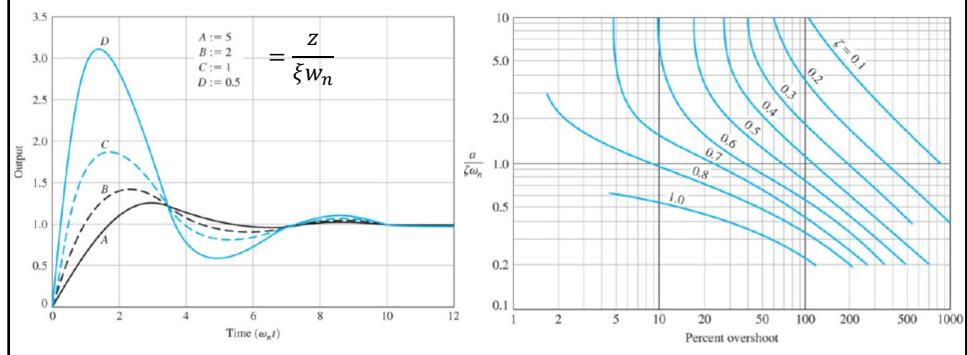
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Effect of Additional Poles & Zeros

$$H(s) = \frac{s + z}{s + p} \frac{w_n^2/z}{(s^2 + 2\zeta w_n s + w_n^2)}$$

- 1) if $p \geq 10\zeta w_n \Rightarrow$ 2nd order poles are dominant; effect of p negligible
- 2) if $z \geq 10\zeta w_n \Rightarrow$ effect of z negligible
- 3) effect of p, z become significant when they are close to the 2nd order poles
(or slower than them)



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Next Lecture

- no more lecture!