

Carrier dynamics

How are charges generated/recombined and how are they moving?

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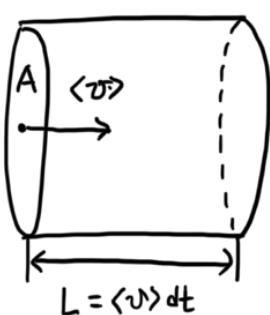
graph LR
 A[① Drift
② Diffusion] --> B[Current]
 C[③ Generation
④ Recombination] --> D[Direct thermal
Indirect thermal
Optical]

```

Continuity equation!

## ① Drift current

Electrical current due to electrons moving with an average speed  $\langle v \rangle$  with a volume density  $n$



- ▷ Average speed :  $\langle v \rangle = \frac{1}{N} \sum_{\vec{v}} v_{\vec{j}}$
  - ▷ Volume swept by electrons in  $dt$  :  $dV = A \langle v \rangle dt$
  - ▷ # of electrons in the volume :  $dN = n dV = n A \langle v \rangle dt$

▷ Total charge carried by these e's :  $dQ = -q dN = -q n A \langle v \rangle dt$

▷ Current density by these  $e^-$ 's :  $J \triangleq \frac{I}{A} = \frac{1}{A} \frac{dQ}{dt} = -q n \langle v \rangle$

▷ Current density in 3D :  $\vec{J} = -q n \langle \vec{v} \rangle$

• Average electronic velocity  $\langle \vec{v} \rangle$

- In the absence of  $\vec{E}$ -field :  $\sum_{\vec{g}} \vec{v}_g = 0 \rightarrow \langle \vec{v} \rangle = 0 \rightarrow \vec{J} = 0$

- In the presence of  $\vec{E}$ -field :

▷ Velocity at time  $t$  :  $\vec{v} = \vec{v}_0 - \frac{e \vec{E}}{m} t \quad (0 \leq t \leq \tau)$

▷ Average velocity :  $\langle \vec{v} \rangle = \langle \vec{v}_0 \rangle - \frac{e \vec{E}}{m} \langle t \rangle = - \frac{e \vec{E}}{m} \tau$  Collision time

• Current density :  $\vec{J} = -ne \langle \vec{v} \rangle = \left( \frac{ne^2 \tau}{m} \right) \vec{E} \triangleq G \vec{E}$

$\vec{J} = G \vec{E}, \quad G = \frac{ne^2 \tau}{m}$  Ohm's law &  $G$  : conductivity.

$\langle \vec{v} \rangle = - \frac{e \tau}{m} \vec{E} \triangleq -\mu_n \vec{E}$   $\mu_n$  : electron mobility.

$\vec{J} = \frac{ne^2 \tau}{m} \vec{E} = en \left( \frac{e \tau}{m} \right) \vec{E} = en \mu_n \vec{E}$  Drift current.

Hole :  $\langle \vec{v}_h \rangle = \mu_h \vec{E}, \quad \vec{J}_h = ep \mu_p \vec{E}$

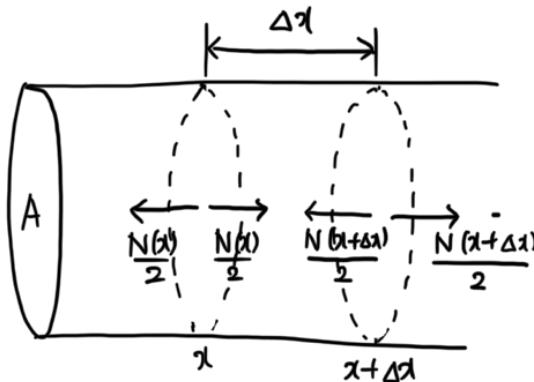
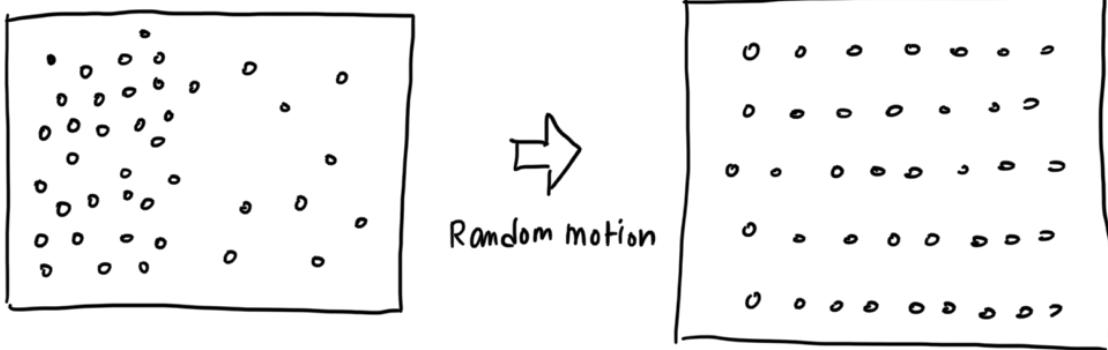
Electron :  $\langle \vec{v}_e \rangle = \mu_n \vec{E}, \quad \vec{J}_e = en \mu_n \vec{E}$

$\vec{J} = \vec{J}_h + \vec{J}_e = e(p \mu_p + n \mu_n) \vec{E} = G_{tot} \vec{E}$

## ② Diffusion current

• Derived from the first Fick's law

• Differential equations that describe the flux of a substance and its concentration.



During the time interval  $\Delta t$ ,  
the net flux through an area  $A$   
at  $x$  is:

$$\begin{aligned}\tilde{n}(x) &= \frac{1}{A} \frac{\frac{N(x)}{2} - \frac{N(x+\Delta x)}{2}}{\Delta t} \cdot \frac{\Delta x^2}{\Delta x^2} \\ &= \frac{\Delta x^2}{2\Delta t} \frac{\frac{N(x)}{A\Delta x} - \frac{N(x+\Delta x)}{A\Delta x}}{\Delta x}\end{aligned}$$

Here,  $\frac{\Delta x^2}{2\Delta t} \triangleq D$  and  $\frac{N}{A\Delta x} \triangleq n$  ← concentration ( $\text{cm}^{-3}$ )

and let  $\Delta x \rightarrow 0$

$$\text{Then, } \lim_{\Delta x \rightarrow 0} \tilde{n}(x) = D \lim_{\Delta x \rightarrow 0} \frac{n(x) - n(x+\Delta x)}{\Delta x} = -D \frac{dn}{dx}$$

If the particle is an electron, then a charge flux (a current) is

$$J(x) = (-e) \tilde{n}(x) = eD \frac{dn}{dx}$$

|                                           |                                                                                                                                           |
|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| $\therefore \text{Diffusion current for}$ | $\left\{ \begin{array}{l} \text{electron : } \vec{J}_n = eD_n \nabla n. \\ \text{hole : } \vec{J}_p = -eD_p \nabla p \end{array} \right.$ |
|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|

Total current

|                                                                                                      |                                                                                         |
|------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| $\vec{J}_p = \vec{J}_{p,\text{drift}} + \vec{J}_{p,\text{diff}} = e\mu_{pp} \vec{E} - eD_p \nabla p$ | $\vec{J}_n = \vec{\tau} \dots \pm \vec{\tau} \dots - e\mu_{nn} \vec{E} + eD_n \nabla n$ |
|------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|

$$v_n = J_{n,\text{drift}} + J_{n,\text{diff}} = e \mu_n n \mathcal{E} + e v_n v''$$

$$\vec{J} = \vec{J}_p + \vec{J}_n$$

### \* Einstein relationship

- In the semiconductor under equilibrium conditions, (in 1D for simplicity)

$$J_{n,\text{drift}} + J_{n,\text{diff}} = e \mu_n n \mathcal{E} + e D_n \frac{dn}{dx} = 0. \quad \dots \textcircled{1}$$

- The potential energy of an electron in the conduction band

$$-qV = E_c - E_{\text{ref}} \quad \text{and since } \mathcal{E} = -\nabla V = -\frac{dV}{dx},$$

$$\mathcal{E} = \frac{1}{e} \frac{dE_c}{dx} = \frac{1}{e} \frac{dE_v}{dx} = \frac{1}{e} \frac{dE_i}{dx} \quad \dots \textcircled{2}$$

- The electron concentration  $n = n_i \exp\left(\frac{E_F - E_i}{k_B T}\right)$

$$\rightarrow \frac{dn}{dx} = -\frac{n_i}{k_B T} \exp\left(\frac{E_F - E_i}{k_B T}\right) \cdot \frac{dE_i}{dx} \quad \left(\because \frac{dE_F}{dx} = 0\right)$$

$$= -\frac{e}{k_B T} n \mathcal{E} \quad \dots \textcircled{3}$$

↑  
positional invariance of  
Fermi level under equilibrium

- If we substitute ③ into ①, we get

$$e \mu_n n \mathcal{E} + e D_n \frac{dn}{dx} = e \mu_n n \mathcal{E} - e D_n \frac{e}{k_B T} n \mathcal{E} = 0$$

$$\rightarrow e n \mathcal{E} \left( \mu_n - D_n \frac{e}{k_B T} \right) = 0$$

$$\rightarrow \text{since } \mathcal{E} \neq 0, \quad \boxed{\frac{D_n}{\mu_n} = \frac{k_B T}{e}}$$

$\therefore$  Einstein relationships for

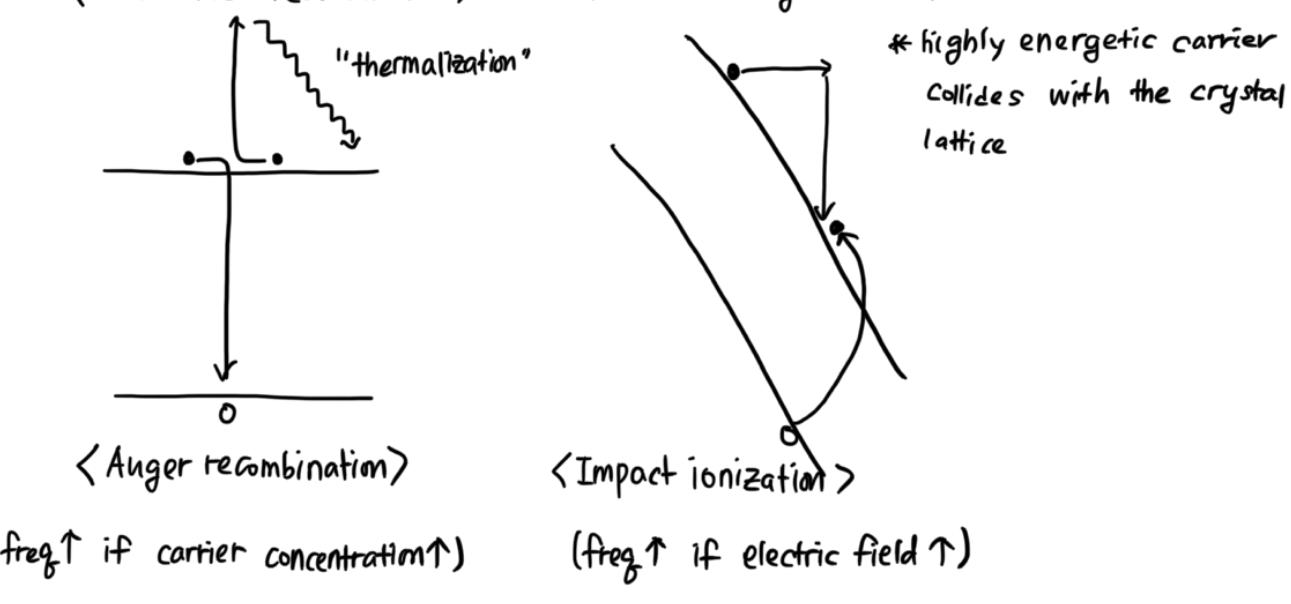
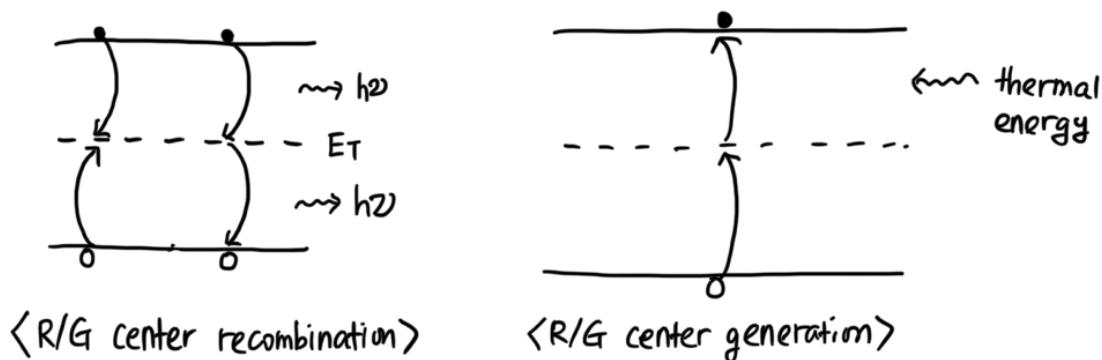
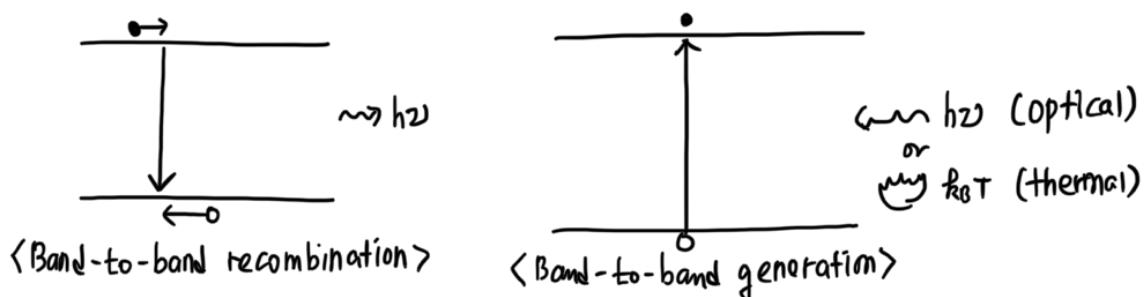
$$\text{hole : } \frac{D_p}{\mu_p} = \frac{k_B T}{e}$$

$$\text{electron : } \frac{D_n}{\mu_n} = \frac{k_B T}{e}.$$

### (3) & (4) Recombination & Generation.

R-G : Nature's order-restoring mechanism.

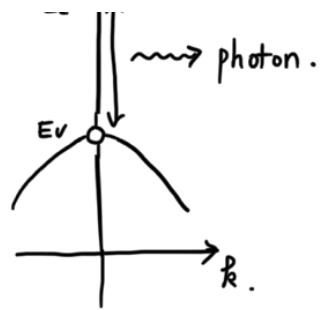
The means whereby the carrier excess and deficit inside the semiconductor is stabilized or eliminated



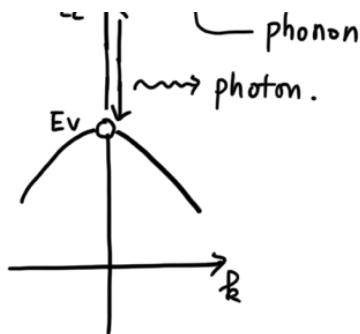
\*Momentum conservation.

In any R-G process, both energy and momentum must be conserved.





Direct-gap  
semiconductor  
(e.g. GaAs)



Indirect-gap  
Semiconductor  
(e.g. Si or Ge).

photon-assisted transition : vertical line

phonon-assisted transition : horizontal line

$\therefore$  photon (a massless particle) carries negligible momentum.

$$P = \sqrt{\left(\frac{E}{c}\right) - m^2 c^2} \quad (\text{Einstein's energy-momentum relation})$$

$$\lim_{m \rightarrow 0} P = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \approx 0 \quad (\because h = 6.6 \times 10^{-34} \text{ J.s})$$

$\lambda = 300 - 700 \text{ nm}$  for visible

$\therefore$  the thermal energy associated with lattice vibrations (phonons)  $\cong 10-50 \text{ meV}$   
Whereas the phonon momentum is comparatively large.

- Bond-to-band recombination in an indirect-gap semiconductor (e.g. Si and Ge) is totally negligible compared to R-G center recombination.
- B-B-B recombination in a direct-gap semiconductor proceeds at a much faster rate, which is the process producing light from LEDs and Lasers.
- R-G center recomb/generation is, in most cases, a dominant mechanism in both direct and indirect semiconductors.

e.g.) n-type semiconductor.

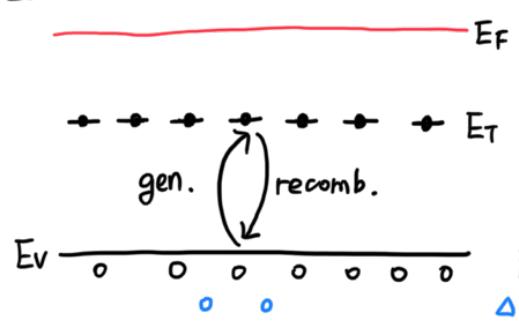


$\Delta n$

$n_0 \cong N_D$

• Recombination

$\partial P |_{- \dots - n}$



$$\frac{\partial P}{\partial t} \Big|_R = -N_T P \frac{\zeta_p}{\tau_p}$$

proportionality  
constant

In equilibrium,

$$\frac{\partial P}{\partial t} \Big|_{R,eq} = -N_T P_0 C_p$$

$$\frac{\partial P}{\partial t} \Big|_{G,eq} = -\frac{\partial P}{\partial t} \Big|_{R,eq} = N_T P_0 C_p$$

• Generation

$$\frac{\partial P}{\partial t} \Big|_G = \frac{\partial P}{\partial t} \Big|_{G,eq} = N_T P_0 C_p$$

Majority carriers remain unperturbed.

Minority carriers change greatly by perturbation.

• Recombination and Generation together.

$$\begin{aligned} \frac{\partial P}{\partial t} \Big|_G + \frac{\partial P}{\partial t} \Big|_R &= -N_T P C_p + N_T P_0 C_p = -N_T C_p (P - P_0) \\ &= -N_T C_p \Delta P \quad (\Delta P = P - P_0) \\ &\stackrel{\Delta P}{=} -\frac{\Delta P}{\zeta_p} \quad \left( \frac{1}{\zeta_p} = N_T C_p \right) \\ &\qquad\qquad\qquad \uparrow \\ &\qquad\qquad\qquad \text{minority carrier lifetime.} \\ &\qquad\qquad\qquad \text{in n-type semiconductor} \end{aligned}$$

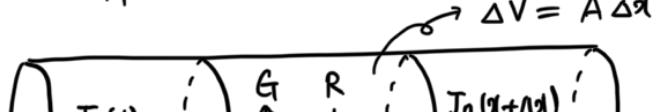
|                                                                                   |                                                                         |
|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------|
| $\therefore \text{for } \begin{cases} \text{n-type} \\ \text{p-type} \end{cases}$ | $\frac{\partial P}{\partial t} \Big _{R-G} = -\frac{\Delta P}{\zeta_p}$ |
|                                                                                   | $\frac{\partial n}{\partial t} \Big _{R-G} = -\frac{\Delta n}{\zeta_n}$ |

Under low-level injection conditions

### Continuity equation

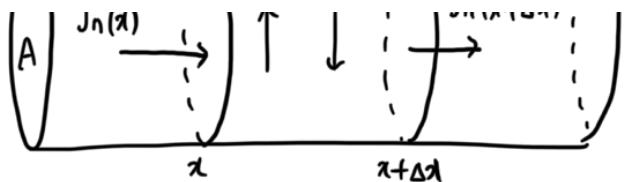
What processes do affect  $e^-$  and  $h^+$  concentrations as functions of time and space?

e.g.) n-type semiconductor.



Drift  
 Diffusion  
 Recombination  
 Generation

\*only R-G center recomb/generation



Mechanisms considered:

- $\frac{\partial N}{\partial t} = ? = G \Delta V - R \Delta V + \frac{A}{e} (J_n(x) - J_n(x + \Delta x))$

Divide both sides by  $\Delta V = A \Delta x$

- $\frac{\partial n}{\partial t} = (G - R) - \frac{1}{e} \frac{J_n(x) - J_n(x + \Delta x)}{\Delta x}$

let  $\Delta x \rightarrow 0$

- $\frac{\partial n}{\partial t} = - \frac{\Delta n}{\tau_n} + \frac{1}{e} \frac{\partial J_n}{\partial x}$

Here,  $J_n = J_{n0, \text{drift}} + J_{n0, \text{diff}} + J_{\Delta n, \text{drift}} + J_{\Delta n, \text{diff}}$

$$= e M_n \Delta n \Sigma + e D_n \frac{\partial \Delta n}{\partial x}$$

and  $\frac{\partial n}{\partial t} = \frac{\partial}{\partial t} (n_0 + \Delta n) = \frac{\partial \Delta n}{\partial t}$

$$\therefore \frac{\partial \Delta n}{\partial t} = - \frac{\Delta n}{\tau_n} - \frac{1}{e} \left( M_n \Delta n \Sigma + D_n \frac{\partial \Delta n}{\partial x} \right)$$