

② BASIC CONSERVATION LAWS (Ch. 1 in Carie).

* Approaches to derive gov. eqns.

① molecular approach (statistical method)

; molecular dynamics \rightarrow macroscale phenomena.



② continuum approach

(mean-free-path of molecule \ll smallest length scale w/ physical

meaning)

- Condition for validation

; if the field variables are defined for
a sufficiently small volume (ϵ)

$$\Rightarrow \frac{1}{n} \ll \epsilon \ll L^3$$

$(n = \# \text{ of molecules / unit volume})$

$L : \text{smallest (but macroscopic) length scale w/ significant.}$

$(\sim 10^{-9} \text{ m}^3 \text{ gas molecules})$

* Reference frame -

- Eulerian vs.

- fixed in space

- fluids passing

- through a C.V.

Lagrangian .

- moving in space .

- particular mass (material vol.)
of fluid .

system .

- material (total) derivative .

- for any variable , $\dot{\alpha} = \alpha(x, y, z, t)$

$$\dot{\alpha} = \frac{\partial \alpha}{\partial t} \delta t + \frac{\partial \alpha}{\partial x} \delta x + \frac{\partial \alpha}{\partial y} \delta y + \frac{\partial \alpha}{\partial z} \delta z .$$

fraction

↓ in Lagrangian coordinate : $x = x(t)$, $y = y(t)$, $z = z(t)$,

$$\therefore \frac{D\alpha}{Dt} = \frac{\partial \alpha}{\partial t} + \frac{\partial \alpha}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial \alpha}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial \alpha}{\partial z} \cdot \frac{dz}{dt}$$

as $\frac{dx}{dt} \rightarrow 0$. $\underbrace{\frac{\partial \alpha}{\partial t}}_{\text{material derivative}} + u \underbrace{\frac{\partial \alpha}{\partial x}}_{\text{convection derivative}} + v \underbrace{\frac{\partial \alpha}{\partial y}}_{\text{convection derivative}} + w \underbrace{\frac{\partial \alpha}{\partial z}}_{\text{convection derivative}} = \frac{D\alpha}{Dt}$.

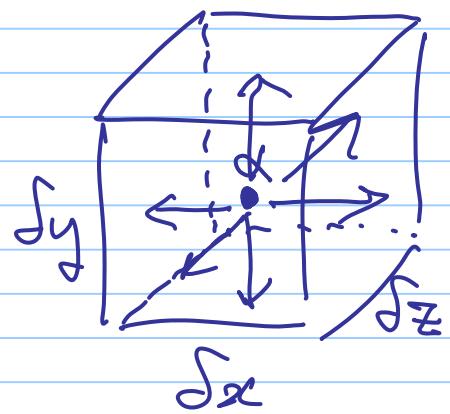
$$\therefore \frac{D\alpha}{Dt} = \frac{\partial \alpha}{\partial t} + (\bar{u} \cdot \nabla) \alpha$$

material derivative.

$$= \frac{\partial \alpha}{\partial t} + \underbrace{U_k \frac{\partial \alpha}{\partial x_k}}_{\text{time derivative}} \rightarrow \text{conductive derivative}$$

(local)

* Control Volume.



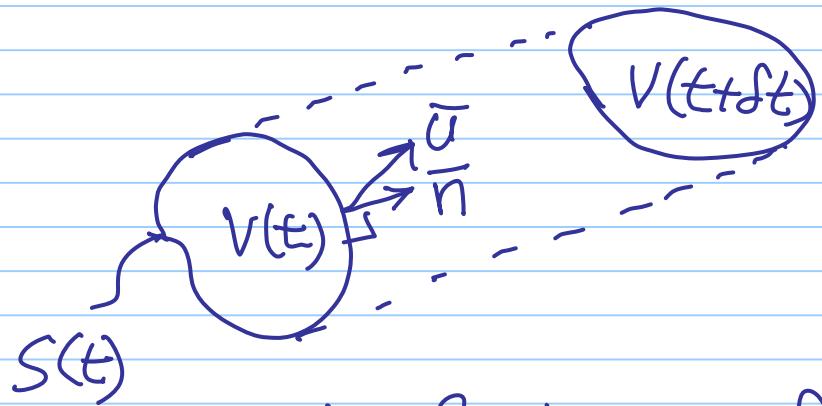
- fluid property (\mathcal{L}) is expanded to each face of the C.V.
(using Taylor Series expansion)
 - " $\Delta x, \Delta y, \Delta z \rightarrow 0$ "
 - apply conservation. → diff. eqns.

if, for an arbitrary shaped C.V. $\oint \mathcal{L} \cdot d\mathbf{d}V = 0$
↑ any differential operator.

$$\Rightarrow L \cdot d = 0$$

* Reynolds Transport theorem (RTT)

; relation between lagrangian and eulerian derivatives.



- specific mass of fluid moving at velocity, \bar{u} .
- $d = d(t)$

• rate of change of the integral of α (Lag. view)

$$\frac{D}{Dt} \int_V \alpha(t) dV = \lim_{\Delta t \rightarrow 0} \left\{ \frac{1}{\Delta t} \left[\int_{V(t+\Delta t)} d(t+\Delta t) dV - \int_{V(t)} d(t) dV \right] \right\}$$

$$-\int_{V(\epsilon)}^{} \alpha(t+ft) dV + \int_{V(\epsilon)}^{} \alpha(t+ft+ft) dV$$

$$= \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[\int_{V(\epsilon+ft)-V(\epsilon)}^{} \alpha(t+ft) dV \right] + \int_{V(\epsilon)}^{} \frac{\partial \alpha}{\partial t} dV.$$

- element of volume change,

$$\downarrow \quad dV = (\bar{A} \cdot \bar{n}) \cdot \Delta t \cdot dS.$$

$$\lim_{\Delta t \rightarrow 0} \int_{S(t)}^{} \alpha(t+ft) (\bar{A} \cdot \bar{n}) dS = \int_{S(\epsilon)}^{} \alpha(t) (\bar{A} \cdot \bar{n}) dS.$$

$$\therefore \frac{D}{Dt} \int_V \alpha(t) dV = \int_S \alpha(t) \vec{u} \cdot \vec{n} dS + \int_{V(t)} \frac{\partial \alpha}{\partial t} dV.$$

$\xrightarrow{\text{Lagrangian}}$



Gauss theorem .: $\int_V (\nabla \cdot F) dV$
(divergence)

$$V = \int_S (F \cdot \vec{n}) dA$$

$$= \int_{V(t)} \nabla \cdot (\alpha \vec{u}) dV + \int_{V(t)} \frac{\partial \alpha}{\partial t} dV$$

$$= \int_V \left[\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \vec{u}) \right] dV = \int_V \left[\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x_k} (\alpha u_k) \right] dV$$

\hookrightarrow integrand has
Eulerian derivatives!

$$\frac{D}{Dt} \int_V \alpha(t) dV = \int_V \left[\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha u) \right] dV$$

* Conservation of mass

$$\alpha = \rho$$

· for a specific mass of fluid (V : arbitrary, changing)

$$\frac{D}{Dt} \int_V \rho \cdot dV = 0 = \int_V \left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k} (\rho u_k) \right] dV$$

$$\rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k} (\rho u_k) = 0 : \text{Continuity eq.}$$

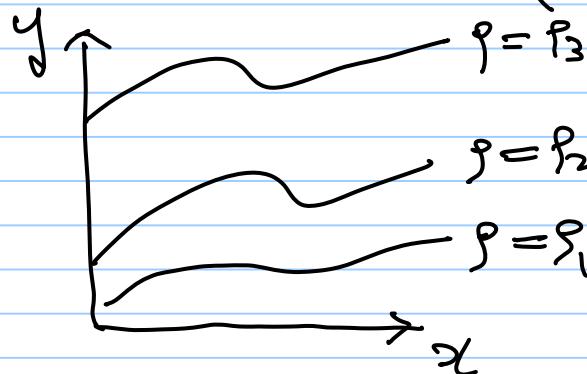
- incompressible flow

$$\frac{D\phi}{Dt} = 0$$

$$\frac{\partial \phi}{\partial t} + U_k \frac{\partial \phi}{\partial x_k} + \rho \frac{\partial u_k}{\partial x_k} = \rho \frac{\partial u_k}{\partial x_k} = 0$$

$$\rightarrow \frac{\partial u_k}{\partial x_k} = 0.$$

- What happens for stratified flow?
(w/ density gradient)



$$\rightarrow \frac{\partial \phi}{\partial x} \neq 0, \frac{\partial \phi}{\partial y} \neq 0.$$

$$\rightarrow \frac{D\phi}{Dt} = 0 \text{ for following the lines of const. } \rho.$$