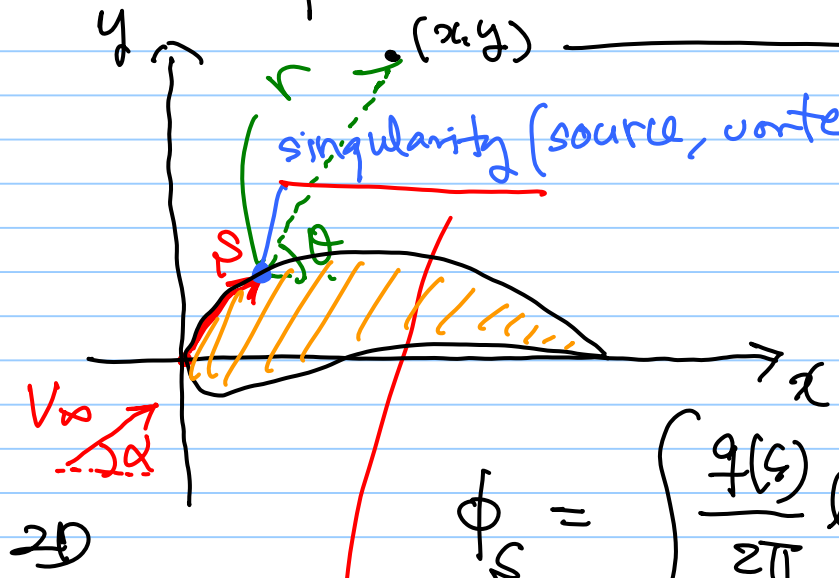
⑧ PANEL METHOD

노트 제목 2019-05-08

- Surface singularity method to solve 2D/3D potential flow.
  - approximate the surface of the body w/ a series of panel.
  - distribute singularities (sources, vortices, ...) on each panel
  - Get ϕ's (or ψ's) from all singularities.
- Advantages.
  - No need to generate grids.
  - low cost.

· applicable to various geometries



$$\phi = \phi_\infty + \phi_s + \phi_v \quad \text{— (1)}$$

vel. potential (ϕ); free-stream (ϕ_∞); source (ϕ_s); vortex (ϕ_v)

$$\phi_s = \int \frac{q(s)}{2\pi} \ln r \cdot ds, \quad \phi_v = -\int \frac{\gamma(s)}{2\pi} \theta \cdot ds$$

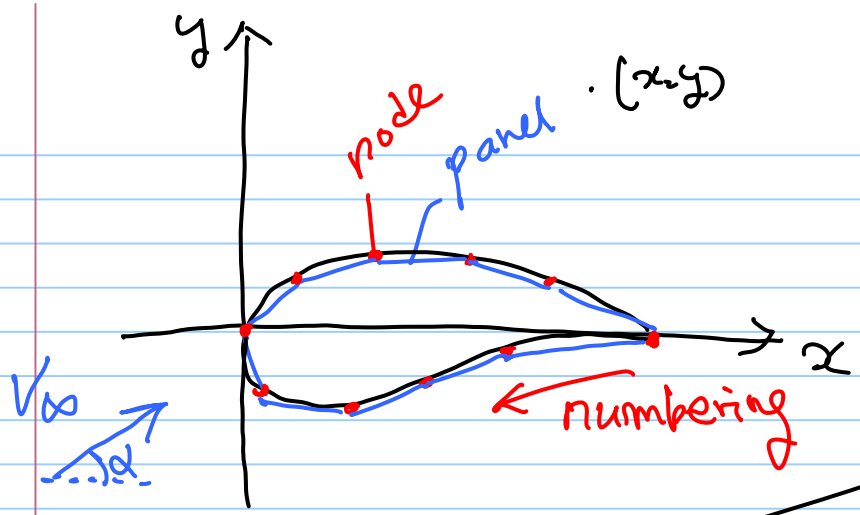
q, γ : strengths of source, vortex (varying with s)

can be distributed continuously or discretely.

- We need to find $q(s)$ and $\gamma(s)$ to satisfy
  - i) flow tangency boundary condition.
  - ii) kutta condition.

→ Method proposed by Hess and Smith (1966).

1. Constant vortex strength over the entire airfoil.
2. Use kutta condition to determine γ.
3. allow $q(s)$ to vary from panel to panel.
   → such that flow tangency condition is satisfied everywhere.



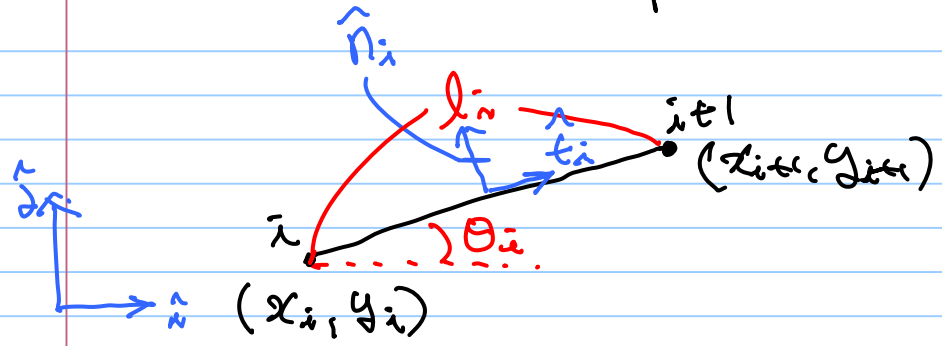
Discretize ① as

$$\phi(x,y) = \underbrace{V_\infty(x\cos\alpha + y\sin\alpha)}_{\phi_\infty} + \sum_{j=1}^{N}\int_j \left[\underbrace{\frac{q(s)}{2\pi}\ln r}_{\phi_s} - \underbrace{\frac{\gamma}{2\pi}\theta}_{\phi_v}\right] ds \quad \text{— ②}$$

- Assume that $q(s)$ is constant within each panel.

$(N+1)$ unknowns: $N \times q_j$, γ $\longrightarrow$ $(N+1)$ equations: flow tangency for N panels (Kutta Condition.

· Consider i^{th} panel.



$$\begin{cases} \sin\theta_i = (y_{i+1} - y_i)/l_i \\ \cos\theta_i = (x_{i+1} - x_i)/l_i \end{cases}$$

$$\begin{cases} \hat{n}_i = -\sin\theta_i\,\hat{i} + \cos\theta_i\,\hat{j} \\ \hat{t}_i = \cos\theta_i\,\hat{i} + \sin\theta_i\,\hat{j} \end{cases}$$

· midpoint of the panel, $\bar{x}_i = \frac{1}{2}(x_i + x_{i+1})$

$$\bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$$

↳ let's define, $u_i = u(\bar{x}_i, \bar{y}_i)$

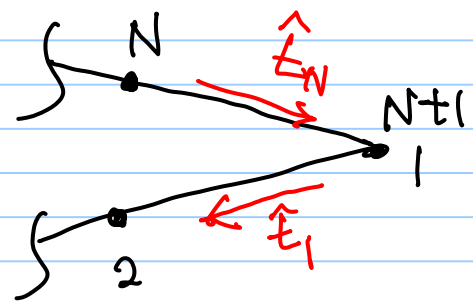
$$v_i = v(\bar{x}_i, \bar{y}_i)$$

* flow tangency condition : $\bar{u}\cdot\vec{n} = 0$ for each panel.

: $-u_i \sin\theta_i + v_i \cos\theta_i = 0$ for $i = 1, \cdots, N$.

* Kutta condition. (flow leaves the TE smoothly)

$$\Downarrow$$



$$\bar{u}\cdot\hat{t}\Big|_1 = -\bar{u}\cdot\hat{t}\Big|_N .$$

$$(u_1\hat{i} + v_1\hat{j})\cdot(-\cos\theta_1\hat{i} + \sin\theta_1\hat{j})$$

$$= -(u_N\hat{i} + v_N\hat{j})\cdot(-\cos\theta_N\hat{i} + \sin\theta_N\hat{j})$$

$$\rightarrow u_1\cos\theta_1 + v_1\sin\theta_1 = -u_N\cos\theta_N - v_N\sin\theta_N . \quad (3)$$

- Next, calculate u_i, v_i, using superposition of all sources and vortices located at each panel midpoint.

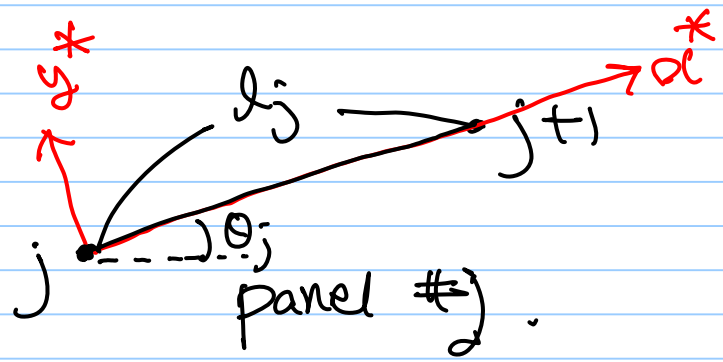
from eq. 2.

$$u_i = V_\infty \cos\alpha + \sum_{j=1}^{N} q_j \underline{\underline{u_{s_{ij}}}} + \gamma \sum_{j=1}^{N} \underline{\underline{u_{v_{ij}}}} \quad \text{— (4)}$$

$$v_i = V_\infty \sin\alpha + \sum_{j=1}^{N} q_j \underline{\underline{v_{s_{ij}}}} + \gamma \sum_{j=1}^{N} \underline{\underline{v_{v_{ij}}}}$$

※ $u_{s_{ij}}$: u_i induced by source (unit strength) located at jth panel.

· in a local coordinate system



based on (x, y) coord.

$$\left(\begin{aligned} u &= u^* \cos\theta_j - v^* \sin\theta_j \\ v &= u^* \sin\theta_j + v^* \cos\theta_j \end{aligned} \right. \quad \text{— (5)}$$

based on (x^*, y^*) coord.

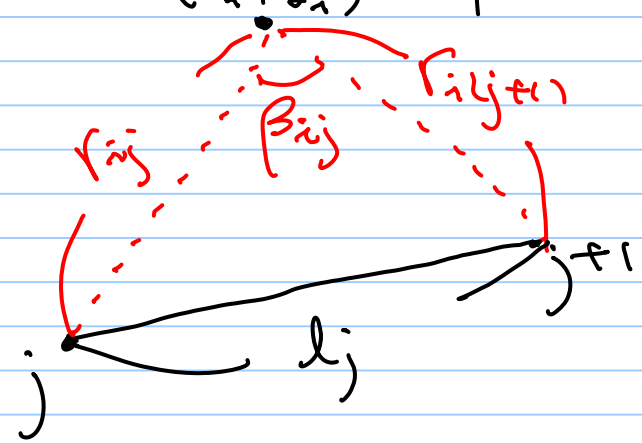
· Q) u^*_{sij} or v^*_{sij} ??

$$\left(\phi = \frac{m}{2\pi} \ln r, = \frac{m}{2\pi} \ln\left(\sqrt{x^2+y^2}\right) \rightarrow u \sim \frac{\partial \phi}{\partial x} \right)$$

$$u^*_{sij} = \frac{1}{2\pi} \int_0^{l_j} \frac{x_i^* - t}{(x_i^* - t)^2 + y_i^{*2}} dt, \quad v^*_{sij} = \frac{1}{2\pi} \int_0^{l_j} \frac{y_i^*}{(x_i^* - t)^2 + y_i^{*2}} dt \quad \text{— (6)}$$

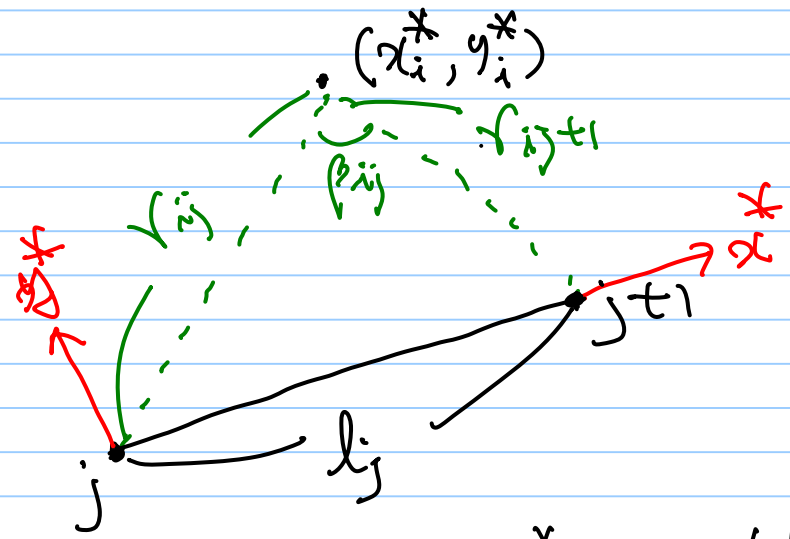
$$= -\frac{1}{2\pi}\ln\left[(x_i^*-t)^2+y_i^{*2}\right]^{1/2}\Bigg|_0^{l_j} \;\vdots\; = \frac{1}{2\pi}\tan^{-1}\left(\frac{y_i^*}{x_i^*-t}\right)\Bigg|_0^{l_j}$$

$$= -\frac{1}{2\pi}\ln\frac{r_{i(j+1)}}{r_{ij}} \qquad = \frac{\beta_{ij}}{2\pi}$$



- Panels. (#, positions, ← designed) → source, vortex, ...

$\boxed{\text{BC's}}$ → strength, (unknowns)



$$\rightarrow \quad u^*_{sij} = -\frac{1}{2\pi}\ln\frac{r_{ij+1}}{r_{ij}} \qquad \text{— (6)}$$

$$v^*_{sij} = \frac{\beta_{ij}}{2\pi}, \quad (\beta_{ii} = \pi)$$

Similary,

$$u^*_{vij} = -\frac{1}{2\pi}\int_0^{l_j}\frac{y^*}{(x^*-t)^2+y^{*2}}\,dt = \frac{\beta_{ij}}{2\pi} \qquad \text{— (7)}$$

$$v^*_{vij} = -\frac{1}{2\pi}\int_0^{l_j}\frac{x^*-t}{(x^*-t)^2+y^{*2}}\,dt = \frac{1}{2\pi}\ln\frac{r_{ij+1}}{r_{ij}}$$

Then, the flow tangency condition $(-u_i \sin\theta_i + v_i \cos\theta_i = 0)$ is re-written as:

$$\sum_{j=1}^{N} A_{ij} q_j + A_{i(N+1)} \gamma = b_i$$

$$A_{ij} = -u_{sij} \sin\theta_i + v_{sij} \cos\theta_i$$

$$= -u^*_{sij}(\cos\theta_j \sin\theta_i - \sin\theta_j \cos\theta_i) + v^*_{sij}(\sin\theta_j \sin\theta_i + \cos\theta_j \cos\theta_i)$$

$$\rightarrow 2\pi A_{ij} = \sin(\theta_i - \theta_j) \ln \frac{r_{ij+1}}{r_{ij}} + \cos(\theta_i - \theta_j) \beta_{ij}$$

similarly, $$2\pi A_{i(N+1)} = \sum_{j=1}^{N} \cos(\theta_j - \theta_i) \cdot \ln \frac{r_{ij+1}}{r_{ij}} - \sin(\theta_i - \theta_j) \beta_{ij}$$

& $b_i = V_\infty \sin(\theta_i - \alpha)$.

$\Rightarrow$ "N" equations, + "Kutta condition".

$$\sum_{j=1}^{N} A_{(N+1)j}\, q_j + A_{(N+1)(N+1)}\, \gamma = b_{N+1}.$$

$$2\pi A_{(N+1)(N+1)} = \sum_{k=1,N} \sum_{j=1}^{N} \sin(\theta_k - \theta_j) \frac{r_{k(j+1)}}{r_{kj}} + \cos(\theta_k - \theta_j)\beta_{kj}$$

$$b_{N+1} = -V_\infty \cos(\theta_1 - \alpha) - V_\infty \cos(\theta_N - \alpha)$$

$$Ax = b, \quad \begin{bmatrix} A_{11} & & \\ & \ddots & \\ & & A_{(N+1)(N+1)} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \\ \gamma \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_{N+1} \end{bmatrix}.$$

- Once this is solved, for example,
  - the tangential velocity at the midpoint of

each panel:

$$V_{t_i} = V_\infty \cdot \cos(\theta_i - \alpha) + \sum_{j=1}^{N} \frac{q_j}{2\pi}\left[\sin(\theta_i-\theta_j)\beta_{ij} - \cos(\theta_i-\theta_j)\ln\frac{r_{ij+1}}{r_{ij}}\right]$$

$$+ \frac{\gamma}{2\pi}\sum_{j=1}^{N}\left[\sin(\theta_i-\theta_j)\ln\frac{r_{ij+1}}{r_{ij}} + \cos(\theta_i-\theta_j)\beta_{ij}\right]$$

pressure coefficient, $C_p \equiv \dfrac{P-P_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - \dfrac{V_{t_i}^2}{V_\infty^2}$

Bernoulli eq.

HW #? Design your own Joukowski airfoil w/ thickness & camber. (due to final).

At $\alpha = 2°, 5°, 10°, 15°, 30°$, using panel method,