Fusion Plasma Theory I. 2022

459.562 **핵융합 플라즈마이론 1 (Fusion Plasma Theory 1)**

2022년 1학기, 원자핵공학과

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Summary: This course provides the students who are interested or participate in fusion and plasma research with basic knowledge and fundamental physics focused on theoretical approach to fusion plasmas. Various physical and technological conditions required for harnessing fusion energy are first introduced, and followed by the review of three major theoretical methods of particle orbit, kinetic, MHD theories to analyze plasma and fusion systems. These theoretical approaches are applied to the discussion on equilibrium and transports of magnetic fusion plasmas to understand plasma confinement problems in fusion reactor development.

Prerequisites: Basic background in Physics including Electromagnetism

Textbooks: R.J. Goldston and P.H. Rutherford, *Introduction to Plasma Physics,* IOP Publishing Ltd 1995
J. Wesson, *Tokamaks, 3rd* Edition, Clarendon Press – Oxford (2004)
K. Miyamoto, P*lasma Physics for Nuclear Fusion*, Revised Ed., MIT Press (1989)

References: W.M. Stacey, Jr., *Fusion Plasma Analysis,* John-Wiley (1981)
J.P. Freidberg, *Ideal Magnetohydrodynamics,* Plenum (1987)
B.B. Kadomtsev, *Tokamak Plasma: A Complex Physical System,* IOP Publ. (1992)
R.D. Hazeltine and J.D. Meiss, *Plasma Confinement,* Addison Wesley (1992)

Lecture note: Download from ETL website

Classroom/time 32-109, Tue · Thurs 11:00~12:15

Grading: Midterm 40%, Final 40%, HW 10%, Attendance 5%, Class Participation 5%

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COURSE CONTENT

Week	TOPICS
1	Ch. 1. Introduction to Plasmas
2	Ch. 2. Particle Drifts in uniform fields
3	Ch. 3. " in nonuniform magnetic fields
4	Ch. 4. " in time-dependent fields
5	Ch. 6. Fluid equations
6	
7	Ch. 7. Relation between fluid and guiding center descriptions
8	Mid-term Examination
9	Ch. 8. MHD
10	
11	Ch. 9. MHD equilibrium
12	Ch. 11. Collisions
13	Ch. 12. Diffusion in Plasmas
14	Introduction to neoclassical transport
15	Final Examination

Unit 1. Single Particle Motion
Unit 2. Plasmas as Fluids
Unit 3. Collisional Process in Plasmas

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Chapter 2

Particle drifts in uniform fields

Many plasmas are immersed in externally imposed magnetic and/or electric fields. All plasmas have the potential to generate their own electromagnetic fields as well. Thus, as a first step towards understanding plasma dynamics, in this Chapter we begin by considering the behavior of charged plasmas in uniform fields, thus constructing the most fundamental aspects of a magnetized plasma. We also carefully introduce some of the mathematical formalisms that we will use throughout the book.

2.1 GYRO-MOTION

In the presence of a uniform magnetic field, the equation of motion of a charged particle is given by

$$m\dot{\mathbf{v}} = q\mathbf{v} \times \mathbf{B} \tag{2.1}$$

where q is the (signed) charge of the particle. Taking $\hat{\mathbf{z}}$ to be the direction of $\mathbf{B}$ (i.e. $\mathbf{B} = B\hat{\mathbf{z}}$ or we sometimes say $\hat{\mathbf{b}} \equiv \mathbf{B}/B$ which, in this case, is the same as $\hat{\mathbf{z}}$), we have

$$\dot{v}_x = qv_y B/m \tag{2.2}$$

$$\dot{v}_y = -qv_x B/m \tag{2.3}$$

$$\dot{v}_z = 0. \tag{2.4}$$

For a specific trajectory, we also need initial conditions at $t = 0$: these we take to be $x = x_i$, $y = y_i$, $z = z_i$, $v_x = v_{xi}$, $v_y = v_{yi}$, $v_z = v_{zi}$. If we take the time derivative of both sides of equation (2.2), we can use equation (2.3) to substitute for $\dot{v}_y$, and obtain

$$\frac{\mathrm{d}^2 v_x}{\mathrm{d}t^2} = -\left(\frac{qB}{m}\right)^2 v_x. \tag{2.5}$$

If we define $\omega_c \equiv |q|B/m$, it is clear that the solution of this equation is

$$v_x = A\cos(\omega_c t) + B\sin(\omega_c t) \tag{2.6}$$

where A and B are integration constants. Evidently ω_c, called the 'cyclotron frequency' (also sometimes called the 'Larmor frequency' or the 'gyro-frequency'), is going to prove to be a very important quantity in a magnetized plasma. It is convenient to use complex-variable notation, and rewrite equation (2.6) as

$$\begin{aligned} v_x &= \mathrm{Re}[A\exp(\mathrm{i}\omega_c t)] - \mathrm{Re}[B\mathrm{i}\exp(\mathrm{i}\omega_c t)] \\ &= \mathrm{Re}\left[(A - \mathrm{i}B)\exp(\mathrm{i}\omega_c t)\right] = \mathrm{Re}\left\{\left[v_\perp \exp(\mathrm{i}\delta)\right]\exp(\mathrm{i}\omega_c t)\right\} \\ &= \mathrm{Re}\left[v_\perp \exp(\mathrm{i}\omega_c t + \mathrm{i}\delta)\right] \end{aligned} \tag{2.7}$$

where Re indicates the real part of the subsequent expression, $v_\perp$ is an absolute speed perpendicular to **B**, and δ is a phase angle. The quantities $v_\perp$ and δ have become our new integration constants. (We will now drop the Re in this notation, since it is clear that we are dealing with real quantities.) In this formulation, $v_\perp$ and δ are chosen to match the initial velocity conditions. Equation (2.2) gives

$$v_y = \mathrm{i}(|q|/q)v_\perp \exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) = \pm \mathrm{i}v_\perp \exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) \tag{2.8}$$

where $\pm$ evidently indicates the sign of q. From the initial conditions, we now

$$v_y = \mathrm{i}(|q|/q)v_\perp \exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) = \pm \mathrm{i} v_\perp \exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) \tag{2.8}$$

where $\pm$ evidently indicates the sign of q. From the initial conditions, we now can say that $v_\perp = (v_{xi}^2 + v_{yi}^2)^{1/2}$ and $\delta = \mp\tan^{-1}(v_{yi}/v_{xi})$, where the upper sign is for positive q. Note that v_x and v_y are 90° out of phase, so we have circular motion in the plane perpendicular to **B**. Equation (2.4) indicates that v_z is a constant, and so the motion constitutes a helix along **B**. If we integrate equations (2.4), (2.7) and (2.8) in time, we obtain

$$\begin{aligned} x &= x_i - \mathrm{i}(v_\perp/\omega_c)[\exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) - \exp(\mathrm{i}\delta)] \\ y &= y_i \pm (v_\perp/\omega_c)[\exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) - \exp(\mathrm{i}\delta)] \\ z &= z_i + v_{zi}t \end{aligned} \tag{2.9}$$

where the integration constants have been chosen to match the initial position conditions.

Clearly, then, another fundamental quantity in a magnetized plasma is the length $r_L \equiv (v_\perp/\omega_c)$, called the 'Larmor radius' or 'gyro-radius'. This is the radius of the helix described by the particle as it travels along the magnetic field line. Figure 2.1 shows an electron and a proton gyro-orbit, drawn more or less to scale, for equal particle energies $W = mv_\perp^2/2$. The ratio of the two gyro-radii is the square-root of the ratio of the proton mass to the electron mass, $\sqrt{1837} \approx 43$. Note that $v_\perp$ is proportional to $(W/m)^{1/2}$, and ω_c is proportional to $1/m$, so r_L is proportional to $(mW)^{1/2}$.

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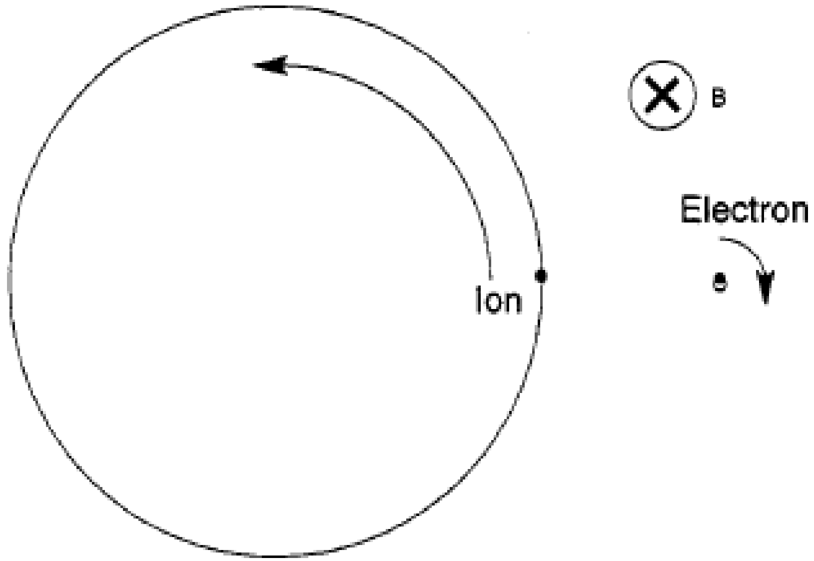

Figure 2.1. Ion and electron gyro-motion in a magnetic field. For fixed energy, the ion's gyro-orbit is much larger than the electron's. 'X' indicates that the magnetic field faces into the page.

The centers of the gyro-orbits are referred to as their 'guiding centers', or 'gyro-centers', and give a measure of a particle's average location during a gyro-orbit. Averaging equation (2.9) over a gyro-period, the guiding-center position for the particular initial values considered here is seen to be given by

$$x_{gc} = x_i + \mathrm{i}(v_\perp/\omega_c)\exp(\mathrm{i}\delta) \qquad y_{gc} = y_i \mp (v_\perp/\omega_c)\exp(\mathrm{i}\delta) \tag{2.10}$$

so that the particle's position described in terms of its guiding-center position is given by

$$\begin{aligned} x &= x_{gc} - \mathrm{i}(v_\perp/\omega_c)\exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) \\ y &= y_{gc} \pm \mathrm{i}(v_\perp/\omega_c)\exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) \\ z &= z_{gc} = z_i + v_{zi} t. \end{aligned} \tag{2.11}$$

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\begin{aligned}
x &= x_{gc} - \mathrm{i}(v_\perp/\omega_c)\exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) \\
y &= y_{gc} \pm \mathrm{i}(v_\perp/\omega_c)\exp(\mathrm{i}\omega_c t + \mathrm{i}\delta) \qquad (2.11) \\
z &= z_{gc} = z_i + v_{zi} t .
\end{aligned}
$$

Thus we can think of particle gyro-centers as sliding along magnetic field lines, like beads on a wire. Note that electrons and ions rotate around the field lines in opposite directions, with the upper sign giving the phase for positively charged particles. If you point your two thumbs along the direction of the magnetic field, the fingers of your left hand curl in the direction of rotation of positively charged ions, while those of your right hand do the same for electrons. These directions of rotation are both such that the tiny perturbation of the magnetic field inside the particle orbits, due to the current represented by the particle motion, acts to *reduce* the ambient magnetic field. High-pressure plasmas reduce the externally imposed magnetic field through the superposition of this 'diamagnetic' effect from a high density of energetic particles.

UNIT 1. Single-Particle Motion

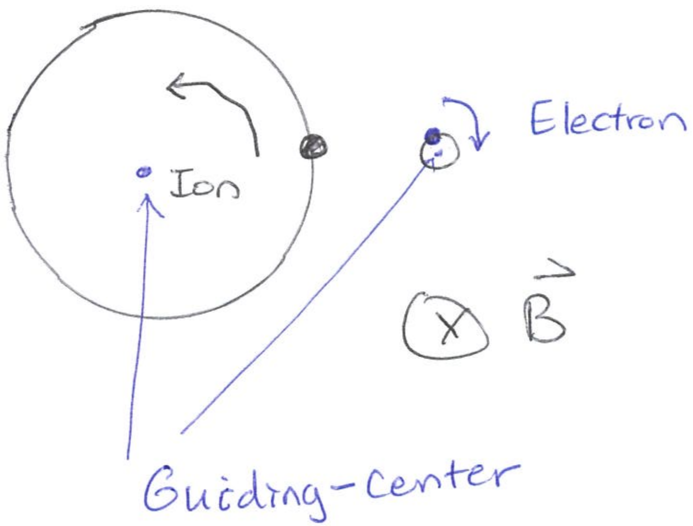
2. Particle drifts in uniform fields

2.1 GYRO-MOTION.

$$m \frac{d\vec{v}}{dt} = q\,\vec{v} \times \vec{B} \qquad (2.1)$$

for uniform $\vec{B}$ field and $\vec{E} = 0$.

It's straightforward to integrate Eq. (2.1)



- Larmor radius

  "$r_L = \frac{v_\perp}{\omega_c}$" $\propto (mW)^{1/2}$

- Cyclotron Frequency

  "$\omega_c = \frac{|q|B}{m}$".

Fundamental Scales in Magnetized Plasma

⊛ Let l : spatial scale of interest.

τ : temporal " .

* If $l \ll r_L$ and $\tau \ll \omega_c^{-1}$, the system can be regarded as "unmagnetized." ($\vec{B}$ field plays a minor role in dynamics)

* ~~If~~ $l \gg r_L$ and $\tau \gg \omega_c^{-1}$, the system is (strongly) magnetized.

This is true for many examples in fusion plasma experiments.

* For earth's magnetosphere or many geophysical systems,

$$r_{Li} \gg l \gg r_{Le} \quad \text{and} \quad \omega_{ci}^{-1} \gg \tau \gg \omega_{ce}^{-1}$$

i.e, ions are unmagnetized and electrons are magnetized.

2.2. E×B DRIFT

* Consider a uniform $\vec{B} = B\hat{z}$ and a uniform $\vec{E}$ in a different direction.

* $$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\times\vec{B}) \qquad (2.12)$$

Define $$\vec{u} \equiv \vec{v} - \frac{\vec{E}\times\vec{B}}{B^2} \qquad (2.13)$$

i.e., velocity in a frame moving with $\frac{\vec{E}\times\vec{B}}{B^2}$

Then,

* $$m\frac{d\vec{u}}{dt} = q\left(\hat{b}(\vec{E}\cdot\hat{b}) + \vec{u}\times\vec{B}\right) \qquad (2.16)$$

after substituting (2.13) to (2.12) and expanding the triple product $(\vec{E}\times\vec{B})\times\vec{B} = (\vec{E}\cdot\vec{B})\vec{B} - B^2\vec{E}$.

* We can further decompose Eq. (2.16) into // component and ⊥ component (to $\vec{B}$)

E x B Drift

- $$m \frac{du_\parallel}{dt} = q E_\parallel \quad , \quad \text{where} \quad u_\parallel \equiv \vec{u}\cdot\hat{b}, \quad E_\parallel \equiv \vec{E}\cdot\hat{b}.$$

$\rightarrow$
$$v_\parallel = \frac{qE_\parallel}{m} \cdot t + v_{\parallel i} \qquad (2.19)$$

acceleration along $\vec{B}$.   initial value.

- $\perp$ component ;
$$m \frac{d\vec{u}_\perp}{dt} = q\vec{u}_\perp \times \vec{B} \,, \qquad (2.20)$$

where $\vec{u}_\perp \equiv \vec{u} - u_\parallel \hat{b}$.

This equation describes a gyromotion as we have seen before.

$\therefore$ Guiding-center does not move in the frame moving with $\frac{\vec{E}\times\vec{B}}{B^2}$.

$\rightarrow$ • $$\vec{V}_{gc} = v_\parallel \hat{b} + \frac{\vec{E}\times\vec{B}}{B^2} \equiv v_\parallel \hat{b} + \vec{v}_E \qquad (2.21)$$

in the Lab frame

$\vec{v}_E$: ExB drift

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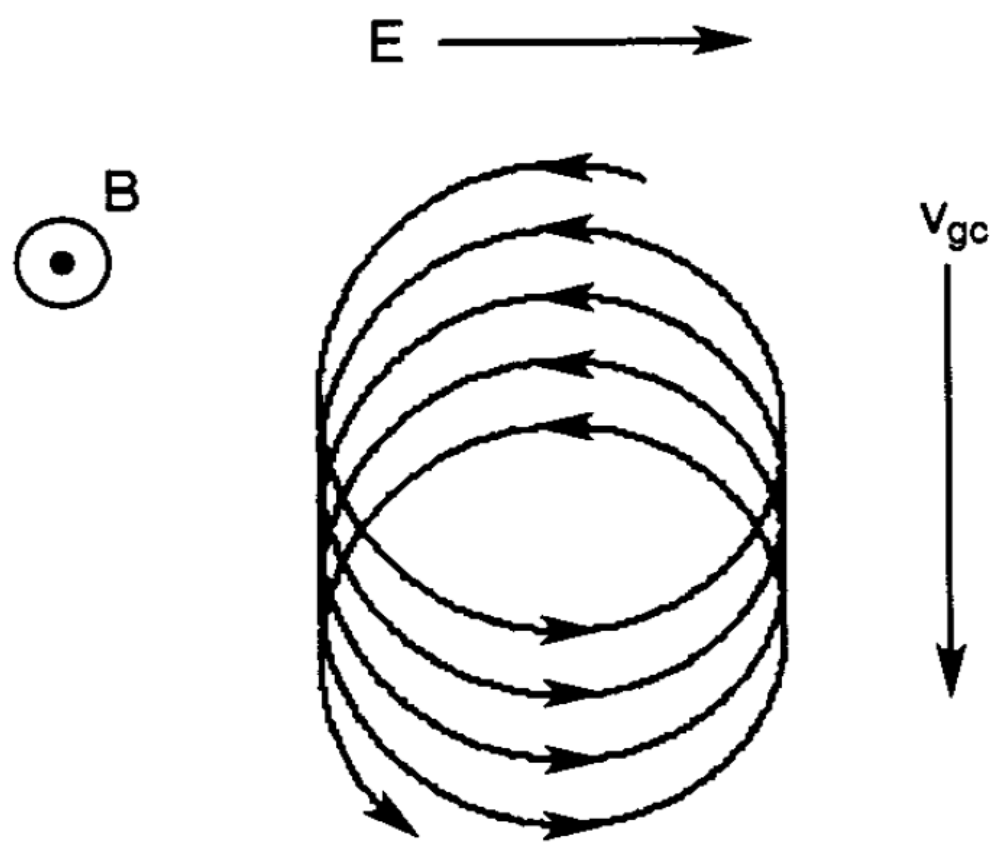

Figure 2.2. Electron **E** × **B** drift motion. The half-orbit on the left-hand side is larger than that on the right, because the electron has gained energy from the electric field. The dot indicates that the magnetic field faces out of the page.

2.3. Gravitational Drift

* $\vec{E}\times\vec{B}$ drift : $\vec{V}_E = \frac{\vec{E}\times\vec{B}}{B^2}$ independent of m and q !

We can repeat the same derivation for any other simple force on the charged particles.

$$\vec{V}_F = \left(\frac{\vec{F}}{q}\right)\times\vec{B}/B^2 \xrightarrow[\text{for } \vec{F}=m\vec{g}]{} \frac{m\,\vec{g}\times\vec{B}}{qB^2} \qquad (2.23)$$

$\vec{E}$ is a particular example.

gravitational drift

- Typically very slow.

" This expression will turn out to be useful later when we consider nonuniform $\vec{B}$. "

Ch. 3. Particle Drifts in non-uniform $\vec{B}$

3.1. ∇B drift :

* We will show later that the magnetic moment

$$\mu \equiv \frac{m v_\perp^2}{2B} = \frac{W_\perp}{B}$$ is a conserved quantity.

(actually an adiabatic invariant).

We can check $\mu = I \cdot A$ with $I = \frac{|q| \omega_c}{2\pi}$

and

$$\text{Area} = \pi r_L^2 = \frac{\pi v_\perp^2}{\omega_c^2}.$$

indeed a magnetic moment.

* ∴ For guiding-center's motion, the ⊥ energy

$W_\perp = \mu B$ acts like an effective potential,

with corresponding force $\vec{F}_{\nabla B} = -\mu \vec{\nabla} B$.

∇B Drift

* $\vec{V}_F = \frac{\vec{E}}{q} \times \vec{B} / B^2$

use $\vec{F} = \vec{F}_{\nabla B} = -\mu \vec{\nabla} B$.

to get

$$\vec{V}_{\nabla B} = \frac{-\mu \vec{\nabla} B \times \vec{B}}{q B^2} = \frac{W_\perp}{q} \frac{\vec{B} \times \vec{\nabla} B}{B^3} \qquad (3.9)$$

∇B drift

- Its direction depends on the sign of "q".
- It is independent of particles mass (for same $W_\perp$).

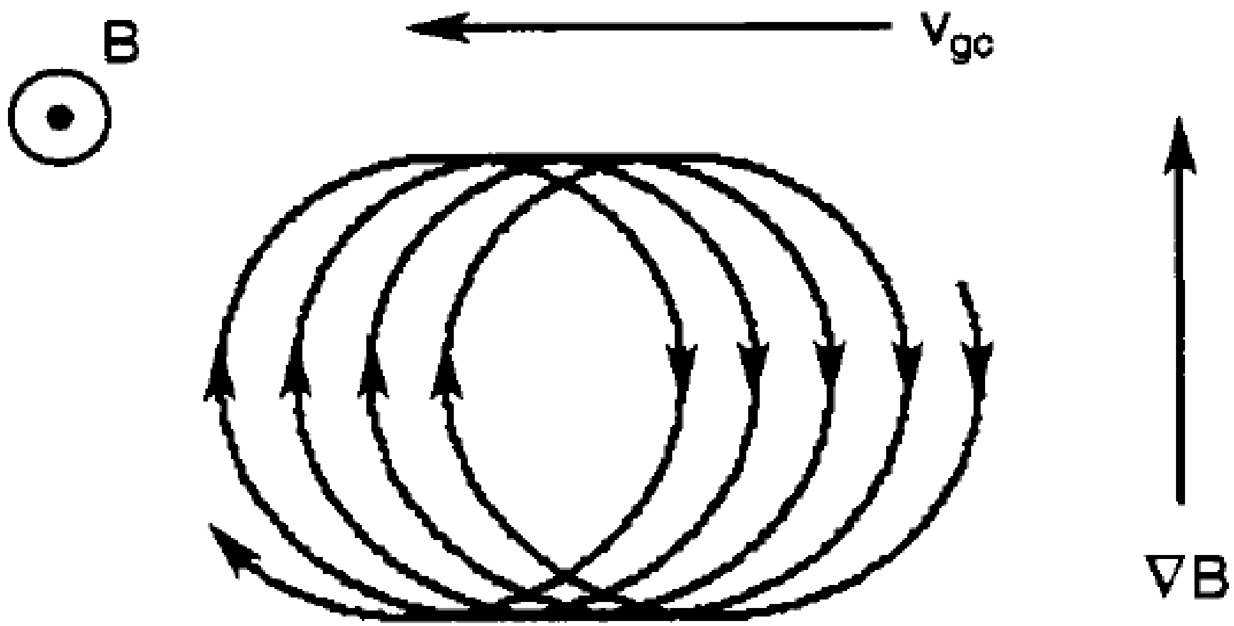

Figure 3.1. Ion ∇B drift motion. The combined effect of smaller gyro-orbits on the high-field side and larger gyro-orbits on the low-field side produces a net leftward drift of the guiding center. The dot indicates that the magnetic field faces out of the page.

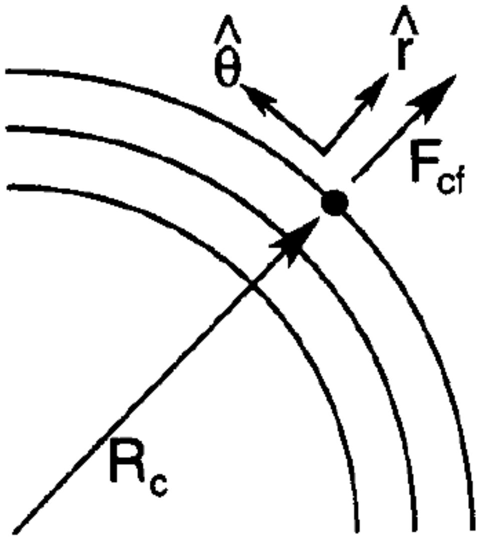

Fig. 3.2.

Take $\quad \hat{b} = \hat{\theta},$

Then $\quad \overrightarrow{v_\parallel} = v_\parallel \hat{\theta}.$

* For cylindrical geometry in Fig 3.2., "$\frac{1}{r}\frac{d}{d\theta}\hat{b} = -\hat{r}/r$"

Its generalization of any curved $\vec{B}$: $\frac{d}{ds}\hat{b} = -\vec{R}_C / {R_C}^2$, where

$\frac{d}{ds} \rightarrow (\vec{\nabla}\cdot\hat{b})\hat{b}$

$\vec{R}_C$: radius of curvature