3.2. Curvature Drift

* Now, consider a curved magnetic field.

A simple example is given in Fig 3.2.

In a rotating frame in θ-direction, a charged particle will feel a "centrifugal force."

$$\vec{F}_{cf} = \frac{m v_{\parallel}^2}{R_c}\hat{r} = m v_{\parallel}^2 \frac{\vec{R}_c}{R_c^2}, \quad (3,10)$$

where $\vec{R}_c$ is the radius of curvature vector

in general, $$\frac{\vec{R}_c}{R_c^2} = -(\hat{b}\cdot\vec{\nabla})\hat{b} \quad (3,12)$$

* Once again, using $\vec{V}_F = \frac{\vec{F}\times\vec{B}}{qB^2}$ for any simple force,

$$\boxed{\vec{V}_{curv} = \frac{m v_{\parallel}^2}{qB^2}\frac{\vec{R}_c\times\vec{B}}{R_c^2} = \frac{2W_{\parallel}}{qB^2}\frac{\vec{R}_c\times\vec{B}}{R_c^2} = \frac{2W_{\parallel}}{qB^2}\vec{B}\times(\hat{b}\cdot\vec{\nabla})\hat{b}} \quad (3,11)\ (3.13)$$

Curvature drift

∇B and Curvature Drifts in vacuum Fields

* If $\vec{\nabla}\times\vec{B} = 0$, $(\vec{\nabla}B)_\perp = -B\,\vec{R}_c/R_c^2 = (\vec{B}\cdot\vec{\nabla})\hat{b}$
(vacuum with no $\vec{J}$). (3.14)

Then, $$\vec{V}_{curv.} = \pm\frac{v_\parallel^2}{\omega_c}\frac{\vec{B}\times\vec{\nabla}B}{B^2} = \frac{2W_\parallel}{q}\frac{\vec{B}\times\vec{\nabla}B}{B^3} \quad (3.15)$$

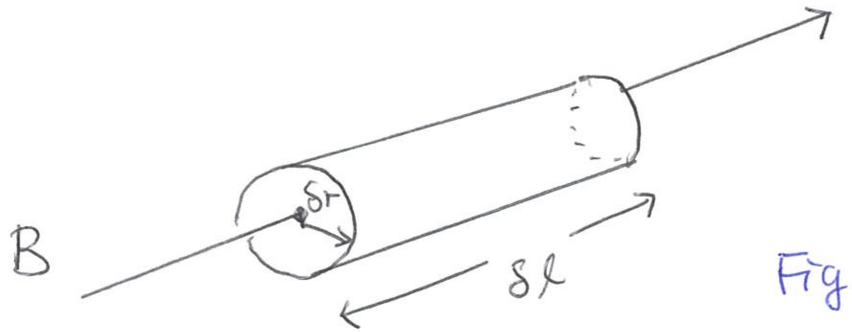
$$\left(\text{cf. } \vec{V}_{grad} = \frac{W_\perp}{q}\frac{\vec{B}\times\vec{\nabla}B}{B^3}\right).$$

i.e., it has almost the same expression as $\vec{V}_{grad}$.

* For bi-Maxwellian (anisotropic) plasma, $\langle W_\parallel\rangle = T_\parallel/2$
and
$\langle W_\perp\rangle = T_\perp$.

$$\therefore \langle \vec{V}_{curv} + \vec{V}_{grad}\rangle = \frac{T_\parallel + T_\perp}{q}\frac{\vec{B}\times\vec{\nabla}B}{B^3} \quad (3.16)$$

3.3. Conservation of μ : static $\vec{B}$ field



(r, θ, z)

Locally $\vec{B} = B\hat{z}$

Fig 3.4.

* Consider a differential cylindrical volume around a magnetic field line somewhere in the system where the $\vec{B}$ fields are converging (eg. Fig 3.3, see the next slide).

* $\vec{\nabla} \cdot \vec{B} = 0 \rightarrow$ apply Gauss's law to the differential volume.

$$\pi (\delta r)^2 \delta l \frac{dB}{dz} + 2\pi \, \delta r \, \delta l \langle B_r \rangle = 0 \qquad (3.17)$$

Flux through the ends, the sides

$$\therefore \quad \langle B_r \rangle = -\frac{\delta r}{2} \frac{dB}{dz} \qquad (3.18)$$

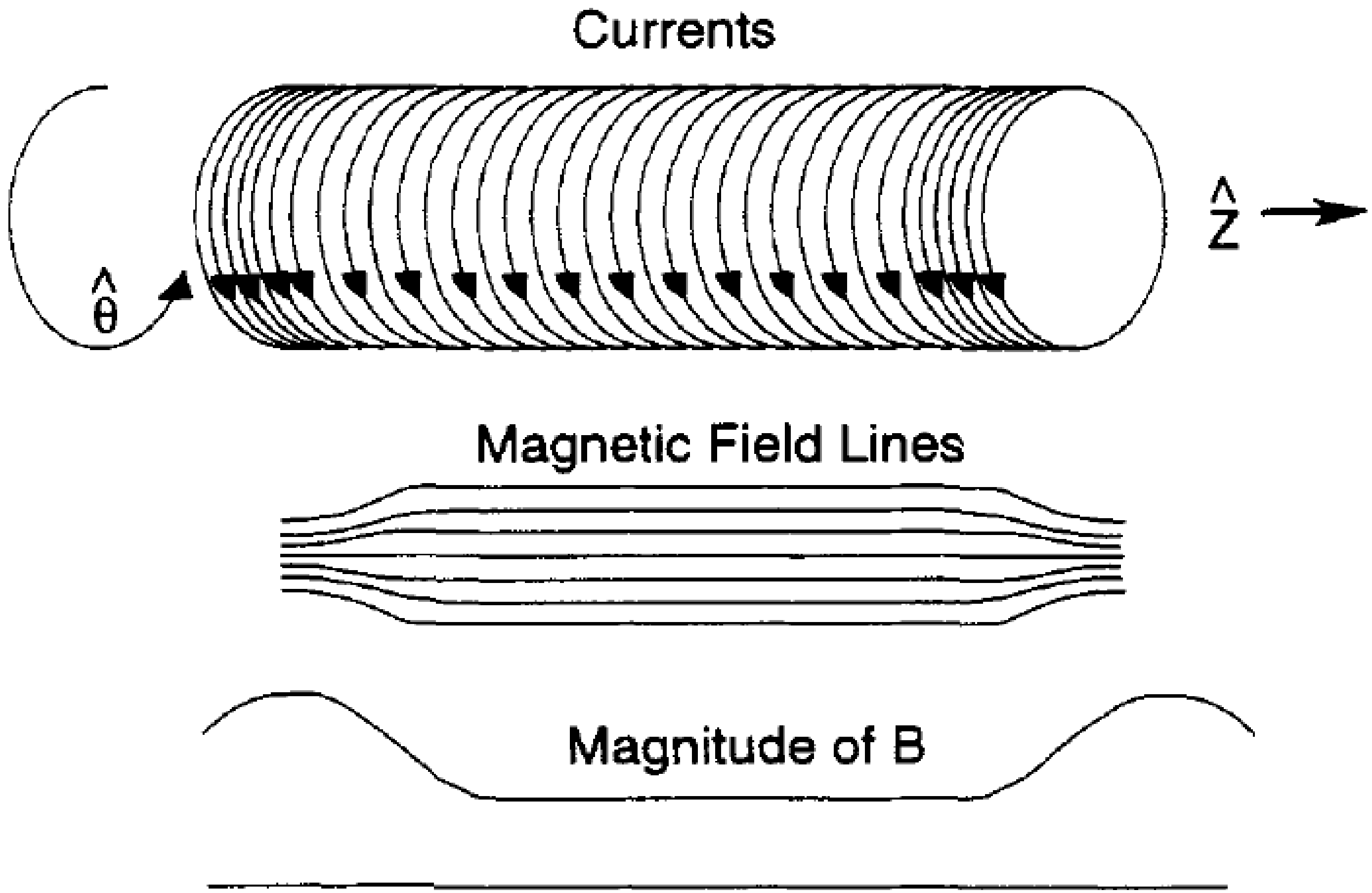


Figure 3.3. Currents in a solenoidal winding and the resulting 'mirror' magnetic fields inside the solenoid, shown schematically.

* Now, suppose that δr is chosen to be r_L of a particle whose guiding center lies on the axis of the cylinder.

* The Lorentz force on the particle is

$$\vec{F}_{mag} = q\, \vec{V}_\perp \times \langle B_r \rangle \hat{r} \rightarrow \text{along } \hat{b} \cong \hat{z}.$$

$$\therefore \langle F_\parallel \rangle = -\frac{|q| V_\perp^2}{2\omega_c} \frac{dB}{dz} = -\frac{W_\perp}{B} \frac{dB}{dz} = -\mu \frac{dB}{dz} \qquad (3.19)$$

The force in the direction opposite to the field gradient for both electrons and ions.

"Mirror Force."

* General Expression :

$$\boxed{m \frac{dV_\parallel}{dt} = -\mu \frac{dB}{ds}} \qquad (3.20)$$

s : distance along the field.

* Multiplying Eq. (3.20) with $v_{\parallel} = \frac{ds}{dt}$, we obtain

$$\frac{d}{dt}\left(\frac{m v_{\parallel}^2}{2}\right) = -\mu \frac{dB}{ds}\frac{ds}{dt} = -\mu \frac{dB}{dt} \qquad (3.21).$$

* In the presence of static (time-independent!) $\vec{B}$ field, the kinetic energy of a single particle should be conserved.

$$\therefore \frac{d}{dt}\left(\frac{m v_{\parallel}^2}{2} + \frac{m v_{\perp}^2}{2}\right) = \frac{d}{dt}\left(\frac{m v_{\parallel}^2}{2} + \mu B\right) = 0. \qquad (3.22)$$

* From Eqs. (3.21) and (3.22), we ~~conclud~~ conclude that

$$\boxed{\frac{d\mu}{dt} = 0} \qquad (3.24).$$

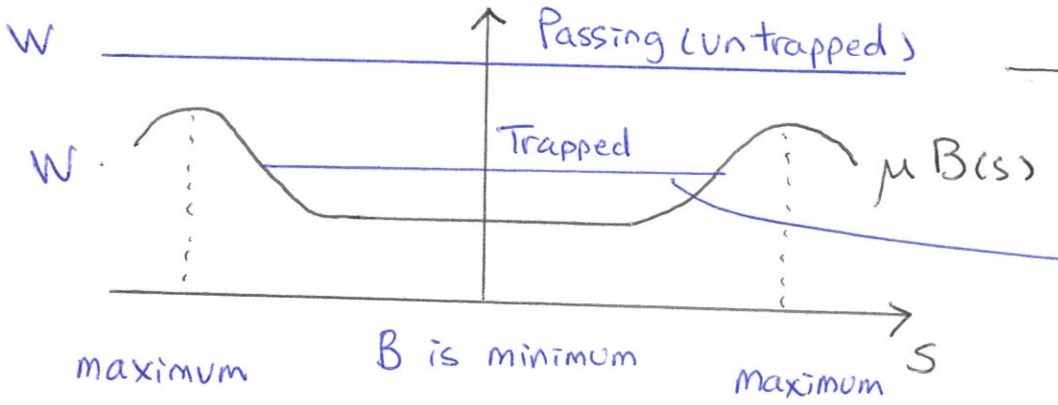
Invariance of "μ": The velocity component $v_{\perp}$ increases as the particle moves along $\vec{B}$ field into a region of higher B so as to keep "$W_{\perp}/B$" constant. → $v_{\parallel}$ decreases.

3.4. Magnetic Mirrors

* $$\frac{m v_\parallel^2}{2} = W - \underline{\mu B} \qquad (3.25)$$

origin : $\perp$ kinetic energy

acts as an effective potential energy as a ptl moves along $\vec{B}$, because μ = constant and B is a function of position (varies along $\hat{b}$).



Passing (untrapped) → $\mu < \frac{W}{B_{max}}$.

Trapped → $\mu > \frac{W}{B_{max}}$.

* For marginally trapped particles,

$$W_\perp \text{ (midplane)} = \mu B_{min} = W B_{min}/B_{max},$$

$$\therefore W_\parallel \text{ ('')}/W = (1 - B_{min}/B_{max}).$$

* → $$v_\perp(\text{midplane})/v = (B_{min}/B_{max})^{1/2}$$
$$v_\parallel(\ \text{"}\ )/v = (1 - B_{min}/B_{max})^{1/2} \qquad (3.26)$$

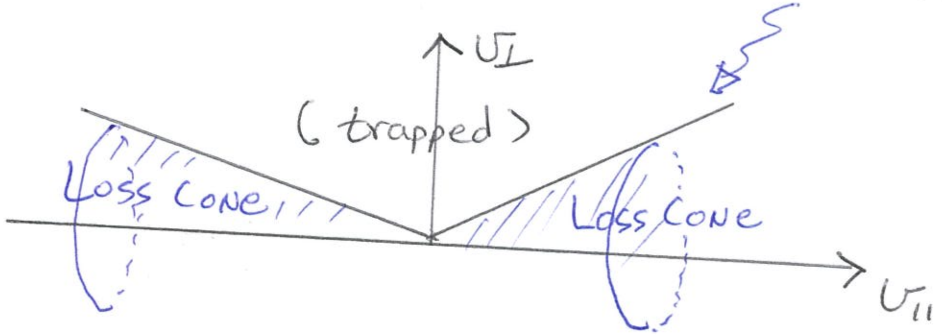


Fig. 3.5.

* Loss cone is independent of "q" and "m" of particles.

* But collisions can change the direction of ptls' velocity and scatter them into the loss cone.

* For $T_e \approx T_i$, electrons collide more frequently and will be lost preferentially.

* As a consequence, positive electric potential develops and confines the electrons with low-energy in the loss cone electrostatically.

* This electric field builds up to the point where it keeps the net outflux of electrons balanced with the slower outflux of ions.

→ Collisional scattering of ions sets the pace for particle loss.

* However, more energetic electrons will still escape over the "top" of the "electrostatic potential well".

* Therefore, electron thermal losses tend to dominate the energy balance of mirror-trapped plasmas.

$\boxed{\text{Homework}}$

Problem 3.3. on page 32, Problem 3.5. on page 41.