16. Mining-induced surface subsidence
16.1 Types and effects of mining-induced subsidence

- **Subsidence**
  - Lowering of the ground surface following underground extraction of an orebody.
  - Types: continuous (trough) subsidence and discontinuous subsidence
- **Continuous subsidence**
  - Smooth surface substance without any step changes
  - Displacement is of elastic orders in magnitude.
  - Associated with the extraction of thin, horizontal or flat-dipping orebodies overlain by weak, non-brittle sedimentary strata.
16.1 Types and effects of mining-induced subsidence

- **Discontinuous subsidence**
  - Large surface displacement over limited surface area with steps or discontinuities in the surface profile.
  - Crown holes: arising from the collapse of the roofs of generally abandoned and shallow open workings (a).
  - Chimney caving (piping, funneling, sinkholes): involves the progressive migration of an unsupported mining cavity to the surface with a similar size and shape of the original excavation (b).
  - Plug subsidence: sudden chimney formation controlled by structural features such as a dyke or a fault (c).
16.1 Types and effects of mining-induced subsidence

- Discontinuous subsidence affects smaller area but causes more disastrous consequences.
- Prediction of the subsidence profile of continuous subsidence, and occurrence and areal extent of discontinuous subsidence are important to the planning of mining operations.
16.2 Chimney caving

- Chimney caving mechanisms

1) Progressive mechanism in weathered or weak rock or in previously caved rock
   - Fallen materials bulk and fill the stope void to prevent further subsidence unless the stope is initially large and open or the materials are drawn from it.

2) Progressive mechanism controlled by regular discontinuities

3) Plug subsidence mechanism controlled by one or more major structural features: shows rigid body displacement without breaking up or dilation and it is not affected by draw control.
16.2 Chimney caving

Plug subsidence controlled by dykes
16.2 Chimney caving

- Limiting equilibrium analysis of chimney caving

1) General block geometry
- Helpful in estimating ultimate collapse conditions of chimney caving by the 1st and 3rd mechanism
- Factor of safety:

\[
\frac{Q}{W} = \frac{\int_{0}^{p} \int_{0}^{z} \tau \, dz \, dp}{W}
\]

- Applying Coulomb shear strength with groundwater pressure:

\[
Q = \int_{0}^{p} \int_{0}^{z} \{c' + [k \gamma z - u(z, p)] \tan \phi'\} \, dz \, dp
\]
16.2 Chimney caving

2) Rectangular block geometry

- Groundwater pressure force: \( \frac{\gamma_w z'^2}{3} = \frac{\gamma_w (z-d)^2}{3} \)

- Total weight: \( W = \gamma ab \cos \alpha \left( h - b \sin \alpha \right) \frac{2}{2} \)

- Shear resistance: \( Q = 2Q_{BCGF} + Q_{DCGH} + Q_{ABFE} \)

For \( 0 \leq d \leq h - b \sin \alpha \)

\[
Q_{BCGF} = \int_0^{b \cos \alpha} \left[ \int_0^{z'} (c + k \gamma z \tan \phi') \, dz - \frac{\gamma_w (z-d)^2}{3} \tan \phi' \right] \, dx
\]

\[
= Q_1 - \frac{\gamma_w \tan \phi'}{3} b \cos \alpha \left[ h^2 - h b \sin \alpha + \frac{b^2}{3} \sin^2 \alpha \right. \\
\left. - d \left( 2h - b \sin \alpha - d \right) \right]
\]

where

\[
Q_1 = \frac{b \cos \alpha}{2} \left[ c \left( 2h - b \sin \alpha \right) + k \gamma \tan \phi' \left( h^2 - h b \sin \alpha + \frac{b^2}{3} \sin^2 \alpha \right) \right]
\]
16.2 Chimney caving

\[ Q_{DCGH} = a \left[ c'(h - b \sin \alpha) + \frac{k \gamma}{2} \tan \phi' (h - b \sin \alpha)^2 \right. \]
\[ \left. - \frac{\gamma_w \tan \phi'}{3} (h - b \sin \alpha - d)^2 \right] \]

\[ Q_{ABFE} = a \left[ c'h + \frac{k \gamma h^2}{2} \tan \phi' - \frac{(h - d)^2}{3} \gamma_w \tan \phi' \right] \quad \text{for } 0 \leq d \leq h \]

For \( h - b \sin \alpha \leq d \leq h \)

\[ Q_{BCGF} = Q_1 - \frac{\gamma_w \tan \phi'}{9 \tan \alpha} (h - d)^3 \]

\[ Q_{DCGH} = a \left[ c'(h - b \sin \alpha) + \frac{k \gamma}{2} \tan \phi' (h - b \sin \alpha)^2 \right] \quad \text{for } h - b \sin \alpha \leq d \]

\[ Q_{ABFE} = a \left[ c'h + \frac{k \gamma h^2}{2} \tan \phi' - \frac{(h - d)^2}{3} \gamma_w \tan \phi' \right] \quad \text{for } 0 \leq d \leq h \]

For \( h \leq d \)

\[ Q_{BCGF} = Q_1 \]

\[ Q_{DCGH} = a \left[ c'(h - b \sin \alpha) + \frac{k \gamma}{2} \tan \phi' (h - b \sin \alpha)^2 \right] \quad \text{for } h - b \sin \alpha \leq d \]

\[ Q_{ABFE} = a \left[ c'h + \frac{k \gamma h^2}{2} \tan \phi' \right] \]
16.2 Chimney caving

- Factor of safety:

For $h \leq d$

$$F = F_1 = \frac{2c'(a + b \cos \alpha)}{\gamma ab \cos \alpha} + \frac{k \tan \phi'}{2h - b \sin \alpha} \left \{ \frac{h^2 + (h - b \sin \alpha)^2}{b \cos \alpha} + \frac{2}{a} \left [ h(h - b \sin \alpha) + \frac{b^2 \sin^2 \alpha}{3} \right ] \right \}$$

For $h - b \sin \alpha \leq d \leq h$

$$F = F_1 - \frac{2\gamma_w (h - d)^2 \tan \phi'}{3\gamma b (2h - b \sin \alpha)} \left [ \sec \alpha + \frac{2(h - d)}{3a \sin \alpha} \right ]$$

For $0 \leq d \leq h - b \sin \alpha$

$$F = F_1 - \frac{2\gamma_w \tan \phi'}{3\gamma (2h - b \sin \alpha)} \left \{ \frac{h^2 + (h - b \sin \alpha)^2 - 2d(2h - b \sin \alpha - d)}{b \cos \alpha} + \frac{2}{3a} \left [ 3h(h - b \sin \alpha) + b^2 \sin^2 \alpha - 3d(2h - b \sin \alpha - d) \right ] \right \}$$

- Critical span is obtained by setting $F=1$ (refer to example at p.492)
- Safety factor decreases as the span increases, mining depth decreases, $k$ decreases and unit weight of overburden increases.
16.2 Chimney caving

- Relationship between stope span (L), rock mass cohesion ($c_m$), and safety factor

![Graph showing the relationship between stope span (L) and safety factor (F) for different values of rock mass cohesion ($c_m$). The graph includes lines for $c_m = 0.04$ MPa, $c_m = 0.14$ MPa, and $c_m = 0.2$ MPa. The graph also shows the weight density ($\gamma$) as $0.27$ MN/m$^3$.](image)
16.3 Sinkholes in carbonate rocks

- Carbonate rocks: including $\text{CO}_3^{2-}$ ion, weak and soluble in acid water, ex) limestone, dolomite
- Karst topography: landscape shaped by the dissolution of a layer or layers of soluble bedrock, usually carbonate rocks ex) doline, polje…
- Cavities develop in carbonate rocks generally above the water table where surface acid water flows downwards.
16.3 Sinkholes in carbonate rocks

- Basic concept of sinkhole: a sinkhole will form when the equilibrium of a stable arch of material above the void is disturbed.

- Sinkhole formation mechanism postulated by Jennings et al (1965).
  a) Adjacent stiff materials for abutment exist.
  b) Arching must develop in the residuum.
  c) A void must exist or develop below the arched residuum.
  d) A reservoir must exist below the arch to accept the material transported by flowing water.
  e) Disturbing force by water, for example, is required to collapse the roof.

- Development of sinkhole can be accelerated by artificially lowering the water table: a large sinkhole will not develop where the water table is high.
16.4 Discontinuous subsidence associated with caving methods of mining

- Block caving
  - Factors influencing the extent of surface geometry: the dip, shape, strength of orebody; strength of surrounding rock and overburden; structural features such as faults and dykes; mining depth and in situ stress; surface slope; prior surface mining; placement of fill; nearby underground excavations
  - Angle of break (subsidence): angle between the horizontal undercut level and the extremity of surface cracks (complement of the angle of draw)
16.4 Discontinuous subsidence associated with caving methods of mining

- Zone of influence: a zone outside the zone of angle of draw where a small-scale or micro-deformation occurs (c.f. a zone of angle of draw showing the large-scale or macro-deformation)

- Prediction of the limits of the zone of influence:
  1) Project the orebody perimeters to the surface to establish the caving area.
  2) Estimate the angle of break using an empirical method.
  3) Calibrate the estimate against observed angles of break in this or similar mines.
  4) Check the estimated angle of break using other methods
  5) Modify the current estimate of the angle of break to reflect local geological features.
  6) Use numerical modeling to check the angle of break and to estimate stress and displacements.
16.4 Discontinuous subsidence associated with caving methods of mining

Braden breccia is stronger than other rocks
16.4 Discontinuous subsidence associated with caving methods of mining

Development of surface subsidence at San Manuel mine, Arizona, USA
16.4 Discontinuous subsidence associated with caving methods of mining

- Progressive hangingwall caving
16.4 Discontinuous subsidence associated with caving methods of mining

Ideal model for limiting equilibrium analysis

3DEC model for kidd mine, Canada
16.5 Continuous subsidence due to the mining of tabular orebodies

- Concepts and definitions
  - Critical area is an extraction area of which maximum subsidence value is the overall maximum possible for the given ore seam.
16.5 Continuous subsidence due to the mining of tabular orebodies

- Subcritical area is an extraction area of which maximum subsidence value is less than the overall maximum possible for the given ore seam.
- Supercritical area is an extraction area of which maximum subsidence value is the overall maximum possible for the given ore seam and is maintained over a finite width rather than at a point.
- Angle of draw ($\zeta$): an angle between a vertical line and a line from the seam base to a point of zero surface subsidence; lower for stronger rocks.
- Critical area of a horizontal seam at depth $h$:
  \[ W_c = 2h \tan \zeta \]
- Primary parameters of interest:
  - the maximum subsidence ($S_{max}$)
  - the maximum ground tilt ($G_{max}$)
  - the maximum tensile and compressive ground strains ($+E_{max}$ and $-E_{max}$)
  - the minimum radius of ground curvature ($R_{min}$)
16.5 Continuous subsidence due to the mining of tabular orebodies

- **Empirical prediction methods**
  - The empirical methods by National Coal Board (UK) have been the most comprehensive and widely used methods for many years (*Subsidence Engineers’ Handbook*):
    
    \[
    +E_{\text{max}} = 1000 \times K_1 \times S_{\text{max}} / h \\
    -E_{\text{max}} = 1000 \times K_2 \times S_{\text{max}} / h \\
    G_{\text{max}} = 1000 \times K_3 \times S_{\text{max}} / h \\
    1/R_{\text{min}} = K_3 \times E_{\text{max}} / h \quad \text{(max. curvature)}
    \]

  - NCB’s methods met with variable success in other parts of the world because of the site-specific nature of the empirical methods.
  - Concepts developed by the NCB have been found to be applicable elsewhere.
16.5 Continuous subsidence due to the mining of tabular orebodies

- Profile function by hyperbolic tangent function:

\[
s(x) = \frac{1}{2} S_{\text{max}} \left[ 1 - \tanh \left( \frac{bx}{h} \right) \right]
\]

where \( b \) is a constant controlling the slope at the inflection point.

- Surface slope or tilt:

\[
g = \frac{ds}{dx} = \frac{bS_{\text{max}}}{2h} \sec h \left( \frac{bx}{h} \right)
\]

For \( b = 5 \), the maximum slope at the inflection point \( (x = 0) \) is

\[
G = \frac{2.5S_{\text{max}}}{h}
\]
16.5 Continuous subsidence due to the mining of tabular orebodies

- Influence functions are used to describe the surface subsidence caused by the extraction of an element dA.

\[ p(r) = w(\xi, \eta) f(r), \quad r = \sqrt{(x-\xi)^2 + (y-\eta)^2} \]

\[ s(x, y) = \int \int_A w(\xi, \eta) f\left(\sqrt{(x-\xi)^2 + (y-\eta)^2}\right) d\xi d\eta \]

One of the most widely used functions is

\[ p(r) = \frac{n S_{\max}}{B^2} \exp \left[ -n\pi \left( \frac{r}{B} \right)^2 \right] \]

where \( n \) is a parameter characterizing the strata properties, and \( B \) is the critical radius of extraction, \( B = h \tan \zeta \).
16.5 Continuous subsidence due to the mining of tabular orebodies

- Trough subsidence analyzed as elastic deformation
  - The strata above deep tabular deposits may deform elastically.
  - Surface subsidence of a completely closed excavation shows that the max. settlement is independent of the elastic constants of rock mass.
  - Berry (1963)’s calculation of subsidence in a transversely isotropic media:

\[
s(x) = \frac{m}{\pi(\alpha_1 - \alpha_2)} \left[ \alpha_1 \tan^{-1} \frac{2ah_1}{x^2 - a^2 + h_1^2} - \alpha_2 \tan^{-1} \frac{2ah_2}{x^2 - a^2 + h_2^2} \right]
\]

where \(h_{1,2} = h/\alpha_{1,2}\) and \(\alpha_{1,2}\) are real values
16.5 Continuous subsidence due to the mining of tabular orebodies

- Numerical methods
  - Numerical methods can eliminate some simplifying assumptions used for analytical approaches such as Berry’s analysis.
  - FEM, BEM, FDM etc. have been adopted for the analysis

- Relation of subsidence to face position and time
  - Typical longitudinal subsidence at point P:
16.5 Continuous subsidence due to the mining of tabular orebodies

- Design measures to limit subsidence effects
  - Giving up the extraction (70%) reduces the subsidence (80%).
16.5 Continuous subsidence due to the mining of tabular orebodies

- Relation between pillar width and subsidence

Narrow panels \((w/h < 0.33)\) with wide pillars \((w_p/h > 0.2)\)

Narrow panels \((w/h < 0.33)\) with narrow pillars \((w_p/h = 0.06)\)

Wide panels \((w/h > 0.6)\) with wide pillars \((w_p/h > 0.1)\)
16.5 Continuous subsidence due to the mining of tabular orebodies

- **Harmonic extraction** means a phased removal of the mineral by adopting at least two faces which advance at calculated distance apart. It makes the ground surface lowered smoothly and horizontal strains minimized.