Topics in Fusion and Plasma Studies 2
(459.667, 3 Credits)

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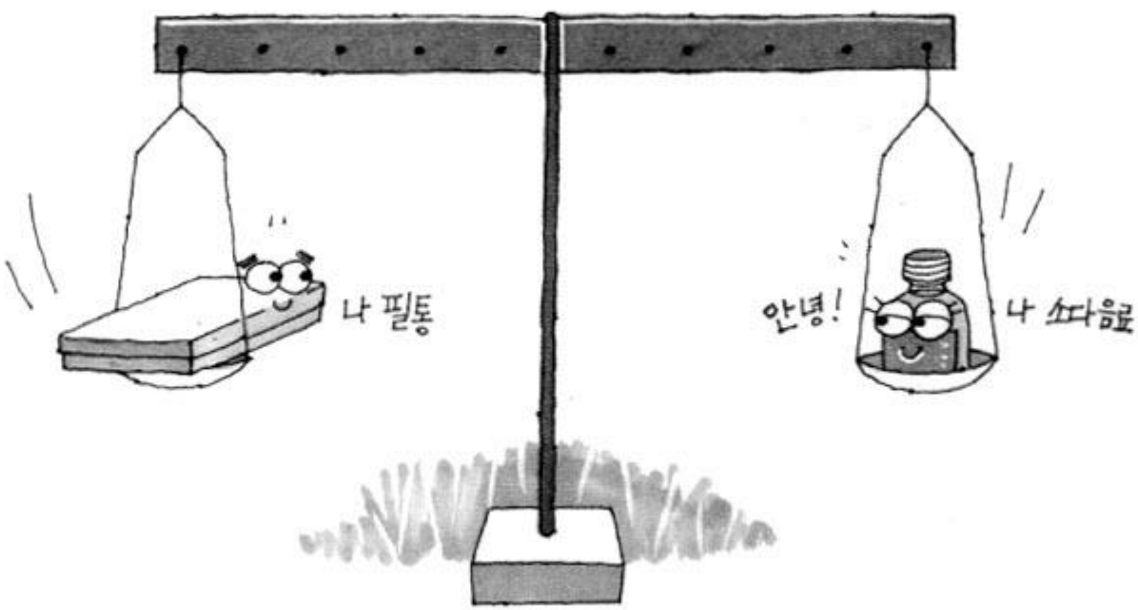
Stability: General Considerations

- **Introduction**

  - The existence of an MHD equilibrium state implies a situation where the sum of the forces acting on the plasma is zero.
  - If the plasma is perturbed from this state, the resulting perturbed forces either restore the plasma to its original equilibrium (stability) or cause a further enhancement of the initial disturbance (instability).
  - Avoidance of ideal MHD instabilities is necessary requirement for a fusion reactor.
  - Analytic linear theory is primarily concerned in this chapter.

Stability: General Considerations

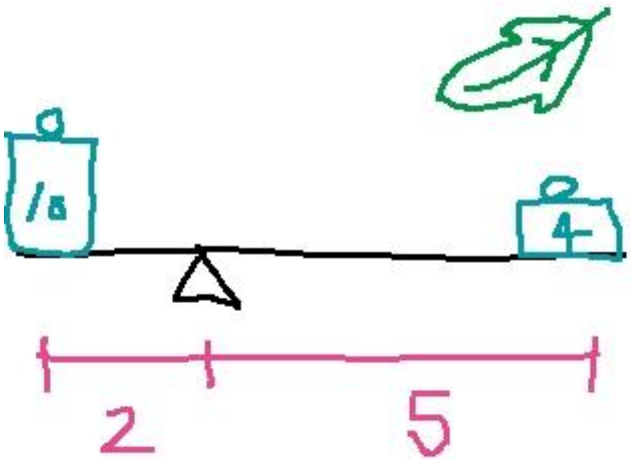
- **Definition of Stability**

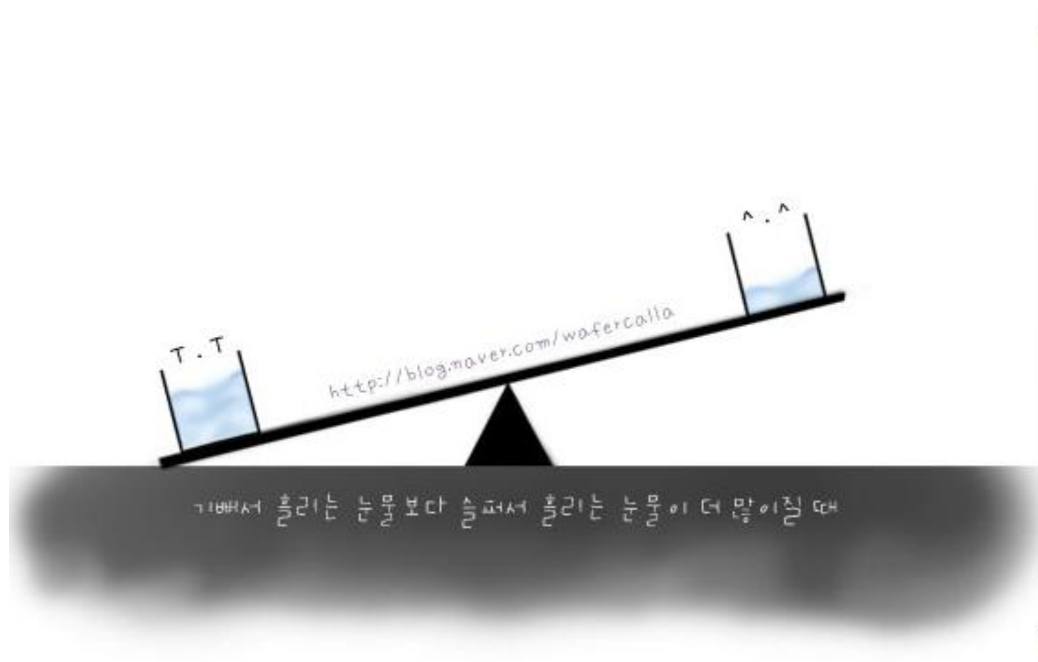


Equilibrium?
Stable?

Stability: General Considerations

- **Definition of Stability**



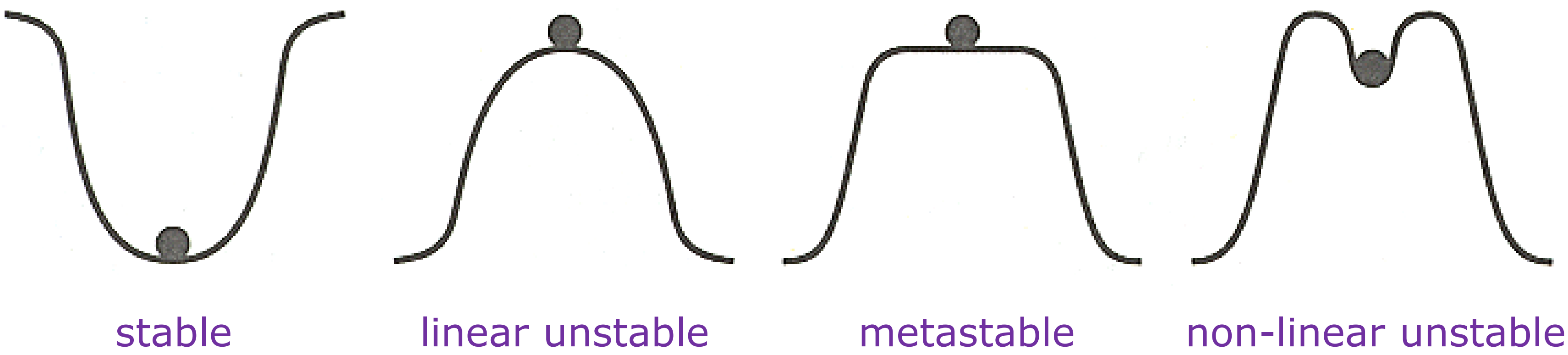


Equilibrium?

Stable?

Stability: General Considerations

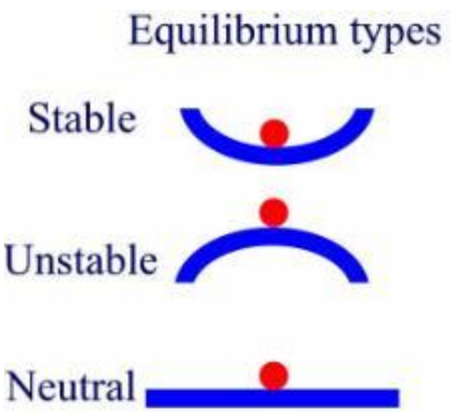
- **Definition of Stability**
  - The fact that one can find an equilibrium does not guarantee that it is stable. Ball on hill analogies:



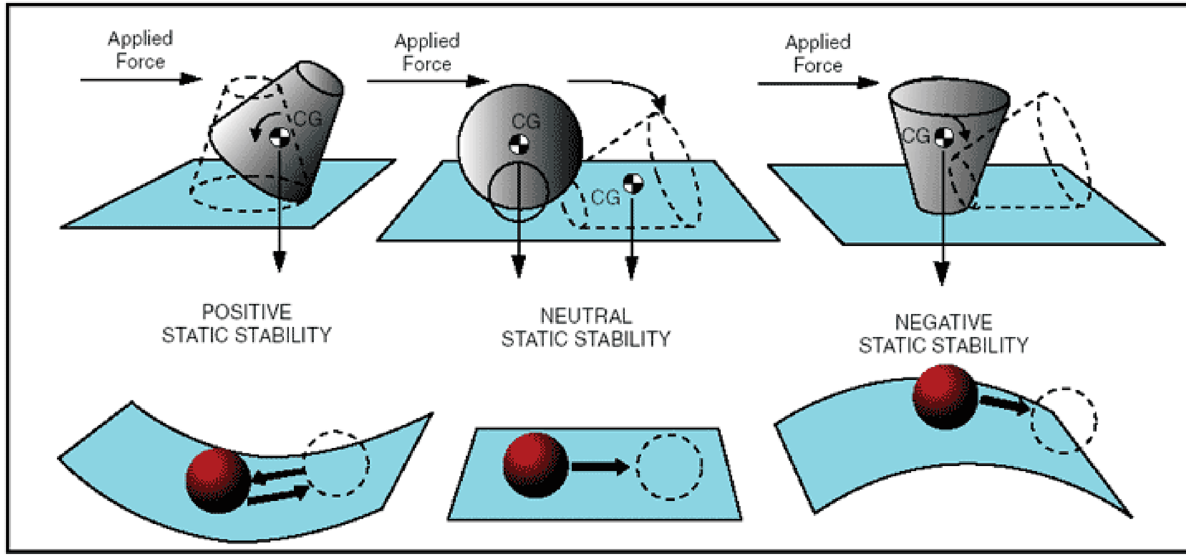
  - Generation of instability is the general way of redistributing energy which was accumulated in a non-equilibrium state.

Stability: General Considerations

- **Definition of Stability**



Marginally stable



- Often times in marginally stable plasmas, a small change in some parameter, e.g. β or I_p, transforms the system into one of type, stable or unstable.
- The condition of neutral stability defines the boundary between stability and instability.
- To a good approximation ideal MHD is closely analogous to the set of situations on the left hand side. There is no dissipation and the instabilities are so virulent that nonlinearities do not dramatically change the behaviour.

Stability: General Considerations

- **Definition of Stability**

- Definition of ideal MHD instability:
  assuming all quantities of interest linearised about their equilibrium values.

$$Q(\vec{r},t) = Q_0(\vec{r}) + \tilde{Q}_1(\vec{r},t)$$

$$\tilde{Q}_1 / |Q_0| << 1 \qquad \tilde{Q}_1(\vec{r},t) = Q_1(\vec{r})\varepsilon^{-i\omega t}$$

small 1st order perturbation

Im ω > 0: exponential instability

Im $\omega \leq$ 0: exponential stability

→ provide a simple and reliable test for stability

Stability: General Considerations

- **Waves in an Infinite Homogeneous Plasma**
  - Consider a configuration with an infinite, homogeneous and unidirectional magnetic field

$$\vec{B} = B_0 \vec{e}_z$$

$$\vec{J} = 0$$

$$p = p_0$$

$$\rho = \rho_0$$

$$\vec{v} = 0$$

$$\nabla p = \nabla \rho = \vec{J} = 0$$

$$Q(\vec{r},t) = Q_0(\vec{r}) + \tilde{Q}_1(\vec{r},t)$$

$$\tilde{Q}_1(\vec{r},t) = Q_1 \exp[-i(\omega t - \vec{k}\cdot\vec{r})]$$

$$\vec{k} = k_\perp \vec{e}_y + k_{||}\vec{e}_z$$

$$\vec{k}\cdot\vec{r} = k_\perp y + k_{||} z$$

Stability: General Considerations

- **Waves in an Infinite Homogeneous Plasma**
  - Consider a configuration with an infinite, homogeneous and unidirectional magnetic field

$$\omega\rho_1 = \rho_0(\vec{k}\cdot\vec{v}_1)$$ conservation of mass

$$\omega p_1 = \gamma p_0(\vec{k}\cdot\vec{v}_1)$$ conservation of energy

$$\omega\vec{B}_1 = -\vec{k}\times(\vec{v}_1\times\vec{B}_0)$$ Faraday's law

$$\mu_0\omega\vec{J}_1 = -i\vec{k}\times[\vec{k}\times(\vec{v}_1\times\vec{B}_0)]$$ Ampere's law

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\rho\vec{v}=0 \qquad \frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right)=0 \;\rightarrow\; \frac{dp}{dt}=-\gamma p\nabla\cdot v$$

$$\nabla\times\vec{E}=-\nabla\times\vec{v}\times\vec{B}=-\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{B}=\mu_0\vec{J}$$

Stability: General Considerations

- **Waves in an Infinite Homogeneous Plasma**

$$(\omega^2 - k_{||}^2 V_a^2) v_{1x} = 0$$

$$(\omega^2 - k_\perp^2 V_s^2 - k^2 V_a^2) v_{1y} - k_\perp k_{||} V_s^2 v_{1z} = 0$$

$$-k_\perp k_{||} V_s^2 v_{1y} + (\omega^2 - k_{||}^2 V_s^2) v_{1z} = 0$$

momentum equation

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} - \nabla p$$

$$k^2 = k_\perp^2 + k_{||}^2$$

$$V_a = (B_0^2 / \mu_0 \rho_0)^{1/2}$$ Alfvén speed

$$V_s = (\gamma p_0 / \rho_0)^{1/2}$$ adiabatic sound speed

Setting the determinant of this system to zero: dispersion relation

$$\omega^2 = k_{||}^2 V_a^2$$

$$\omega^2 = \frac{1}{2} k^2 (V_a^2 + V_s^2)[1 \pm (1 - \alpha^2)^{1/2}]$$

$$\alpha^2 = 4 \frac{k_{||}^2}{k^2} \frac{V_s^2 V_a^2}{(V_s^2 + V_a^2)^2}$$

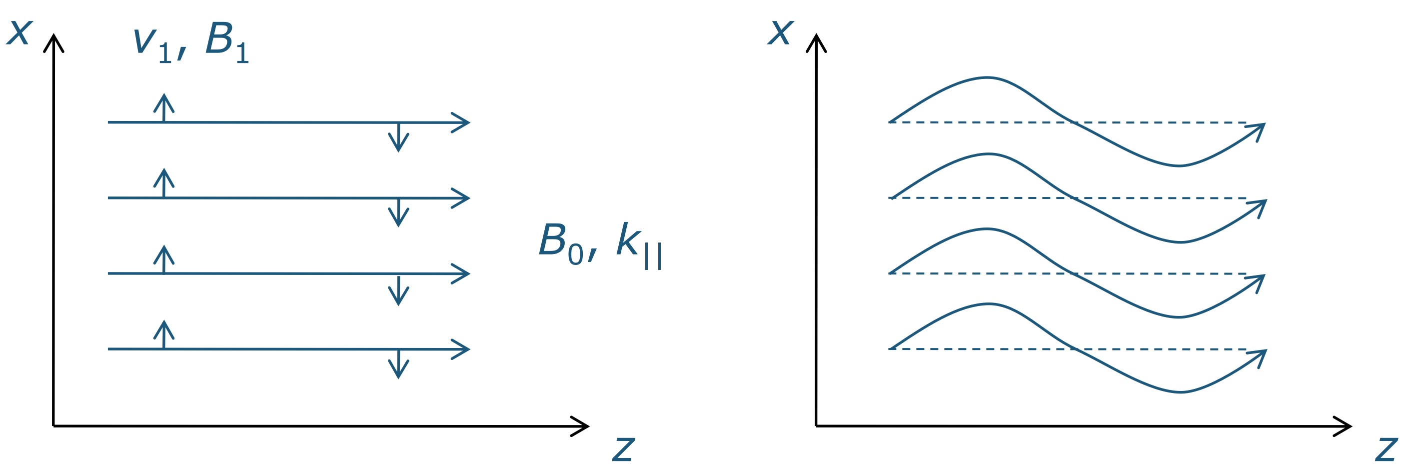
$\omega^2 \geq 0$: Im $\omega = 0$ → exponentially stable system because no sources of free energy available to drive instabilities

Stability: General Considerations

- **Waves in an Infinite Homogeneous Plasma**

1. Shear Alfvén wave $\omega^2 = k_{||}^2 V_a^2$

- Purely transverse → causing the magnetic field lines to bend
- Incompressible producing no density or pressure fluctuations
- Describing basic oscillation between perpendicular plasma kinetic energy and perpendicular line bending magnetic energy; i.e. a balance between inertia and field line tension

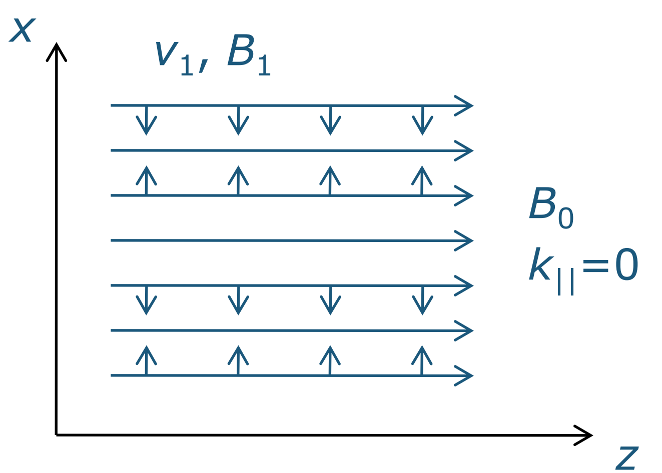


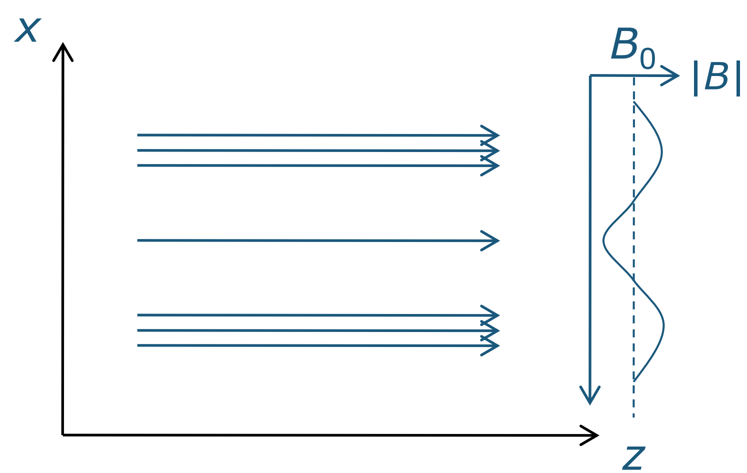
Stability: General Considerations

- **Waves in an Infinite Homogeneous Plasma**

2. Fast magnetoacoustic wave $\omega^2 = \omega_f^2 = \frac{1}{2}k^2(V_a^2 + V_s^2)[1 + (1-\alpha^2)^{1/2}]$

- Nearly transverse
- Both the magnetic field and the plasma pressure compressed
- Compressional Alfvén wave $\omega_f^2 \approx (k_\perp^2 + k_{||}^2)V_a^2$ where $\beta \sim V_s^2 / V_a^2 << 1$
- Most of the compression involves the magnetic field not the plasma.
- Basic oscillation between perpendicular plasma kinetic energy (plasma inertia) and compressional (field line pressure) plus line bending (field line tension) magnetic energy.





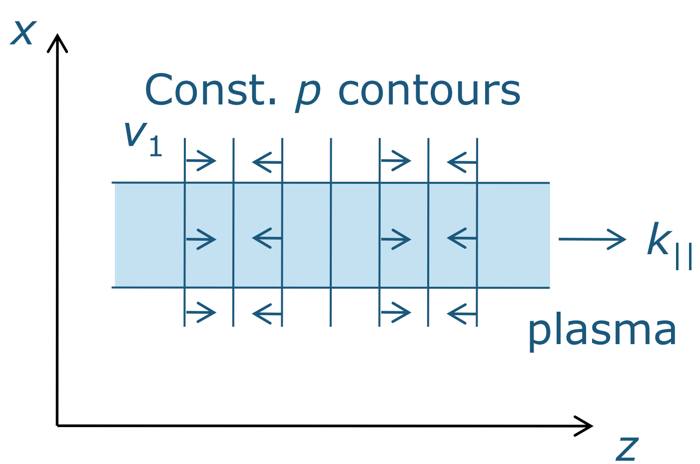
Stability: General Considerations

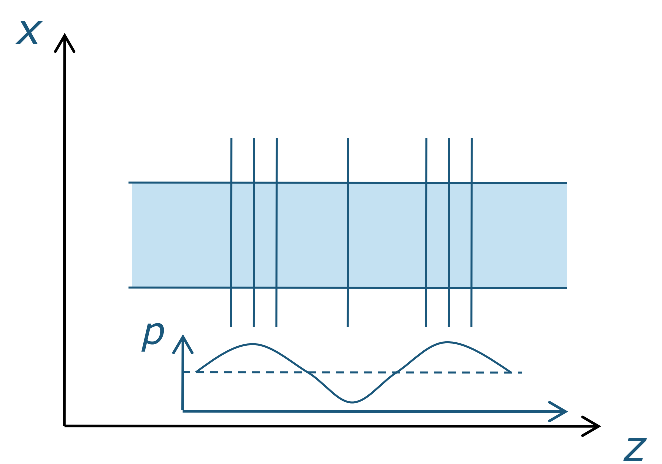
- **Waves in an Infinite Homogeneous Plasma**

3. Slow magnetoacoustic wave $\omega^2 = \omega_s^2 = \frac{1}{2}k^2(V_a^2 + V_s^2)[1-(1-\alpha^2)^{1/2}]$

- Nearly longitudinal
- Both the magnetic field and the plasma pressure compressed
- Sound wave $\omega_s^2 \approx k_{||}^2 V_s^2$ where $\beta \sim V_s^2 / V_a^2 << 1$
- Basic oscillation between parallel plasma kinetic energy (plasma inertia) and plasma internal energy (plasma pressure)
- Dispersion relation is identical to that of the ion acoustic wave of two-fluid theory.

Problem 8.3





Stability: General Considerations

- **Waves in an Infinite Homogeneous Plasma**

- Basic wave propagation characteristics of an ideal MHD plasma described
- In the homogeneous geometry, all are stable.
- In inhomogeneous geometries each of these waves is modified and can couple to one another.
- The most unstable perturbations almost always involve the shear Alfvén wave.

Stability: General Considerations

- **General Linearized Stability Equations**

Energy Principle: an elegant and powerful procedure for testing ideal MHD stability in arbitrary 3-D geometry

1. Initial value problem using the general linearised equations of motion
2. Normal-mode eigenvalue problem
3. Transformed into a variational principle
4. Reduced to the energy principle

- Initial Value Formulation

$$\vec{J}_0 \times \vec{B}_0 = \nabla p_0$$

$$\mu_0 \vec{J}_0 = \nabla \times \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\vec{v}_0 = 0$$

$$Q(\vec{r},t) = Q_0(\vec{r}) + \tilde{Q}_1(\vec{r},t) \qquad \tilde{Q}_1 / |Q_0| << 1$$

linearized

$$\tilde{v}_1 = \frac{\partial \xi}{\partial t}$$

ξ: displacement of the plasma away from its equilibrium position

Aim: to express all perturbed quantities in terms of ξ and then obtain a single equation describing the time evolution of ξ

Stability: General Considerations

- **General Linearized Stability Equations**
  - Initial Value Formulation

$$\xi(\vec{r},0) = \tilde{B}_1(\vec{r},0) = \tilde{\rho}_1(\vec{r},0) = \tilde{p}_1(\vec{r},0) = 0$$

$$\frac{\partial \xi(\vec{r},0)}{\partial t} \equiv \tilde{v}_1(\vec{r},0) \neq 0$$

initial data

Integrated with respect to time

$$\tilde{\rho}_1 = -\nabla \cdot (\rho_0 \xi)$$ conservation of mass

$$\tilde{p}_1 = -\xi \cdot \nabla p_0 - \gamma p_0 \nabla \cdot \xi$$ conservation of energy

$$\tilde{B}_1 = \nabla \times (\xi \times \vec{B}_0)$$ Faraday's law

$$\nabla \cdot \tilde{B}_1 = 0$$

Stability: General Considerations

- **General Linearized Stability Equations**
  - Initial Value Formulation

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \vec{F}(\xi)$$ momentum equation

$$\vec{F}(\xi) = \vec{J} \times \tilde{B}_1 + \tilde{J}_1 \times \vec{B} - \nabla \tilde{p}_1$$ force operator

$$= \frac{1}{\mu_0}(\nabla \times \vec{B}) \times \tilde{Q} + \frac{1}{\mu_0}(\nabla \times \tilde{Q}) \times \vec{B} + \nabla(\xi \cdot \nabla p + \gamma p \nabla \cdot \xi)$$

$$\tilde{Q} \equiv \tilde{B}_1 = \nabla \times (\xi \times \vec{B})$$

$$\xi(\vec{r},0) = 0, \quad \frac{\partial \xi(\vec{r},0)}{\partial t} = \tilde{v}_1(\vec{r},0)$$ + Boundary conditions

Formulation of the generalized stability equations as an initial value problem

Stability: General Considerations

- **General Linearized Stability Equations**
  - Normal-Mode Formulation

$$\tilde{Q}_1(\vec{r},t) = Q_1(\vec{r})\exp(-i\omega t)$$

$$\rho_1 = -\nabla\cdot(\rho\xi)$$ conservation of mass

$$p_1 = -\xi\cdot\nabla p - \gamma p\nabla\cdot\xi$$ conservation of energy

$$\vec{Q} \equiv \vec{B}_1 = \nabla\times(\xi\times\vec{B})$$ Faraday's law

$$-\omega^2\rho\xi = \vec{F}(\xi)$$ normal-mode formulation

$$\vec{F}(\xi) = \frac{1}{\mu_0}(\nabla\times\vec{B})\times\tilde{Q} + \frac{1}{\mu_0}(\nabla\times\tilde{Q})\times\vec{B} + \nabla(\xi\cdot\nabla p + \gamma p\nabla\cdot\xi)$$

- An eigenvalue problem for the eigenvalue ω^2
- Assumed that for the problems of interest the eigenvalues are discrete and distinguishable so that the concept of exponential stability is valid.
- To obtain a more complete understanding, additional detailed knowledge of **F** is required.

Stability: General Considerations

- **Properties of the Force Operator F**
  - Self-Adjointness of **F**

  - major impact on both the analytic and the numerical formulation of linearized MHD stability

$$\int \eta \cdot \vec{F}(\xi) d\vec{r} = \int \xi \cdot \vec{F}(\eta) d\vec{r}$$

$$\int \eta \cdot \vec{F}(\xi) d\vec{r} = -\int d\vec{r}[\frac{1}{\mu_0}(\vec{B}\cdot\nabla\xi_\perp)\cdot(\vec{B}\cdot\nabla\eta_\perp) + \gamma p(\nabla\cdot\xi)(\nabla\cdot\eta)$$

$$+\frac{B^2}{\mu_0}(\nabla\cdot\xi_\perp + 2\xi_\perp\cdot\kappa)(\nabla\cdot\eta_\perp + 2\eta_\perp\cdot\kappa)$$

$$-\frac{4B^2}{\mu_0}(\xi_\perp\cdot\kappa)(\eta_\perp\cdot\kappa) + (\eta_\perp\xi_\perp : \nabla\nabla)(p + \frac{B^2}{2\mu_0})]$$

$$\xi = \xi_\perp + \xi_{||}\vec{b}, \quad \eta = \eta_\perp + \eta_{||}\vec{b}$$

$\vec{n}\cdot\xi = \vec{n}\cdot\eta = 0$ BC: plasma surrounded by a perfectly conducting wall

Stability: General Considerations

- **Properties of the Force Operator F**
  - Real ω^2

$$-\omega^2 \rho \xi = \vec{F}(\xi)$$

Dot product with $\xi^*(\mathbf{r})$ and integrating over the plasma volume

$$\omega^2 \int \rho |\xi|^2 d\vec{r} = -\int \xi^* \cdot \vec{F}(\xi) d\vec{r}$$

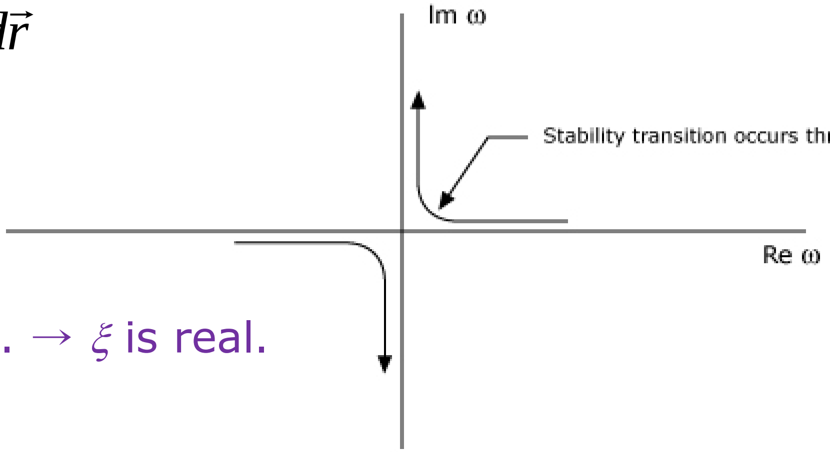
$\omega^2 > 0$: pure oscillation (stable)
$\omega^2 < 0$: exponentially grow (unstable)

$$\omega^{*2} \int \rho |\xi|^2 d\vec{r} = -\int \xi \cdot \vec{F}(\xi^*) d\vec{r}$$

$$(\omega^2 - \omega^{*2}) \int \rho |\xi|^2 d\vec{r} = 0$$

$$\omega^2 = \omega^{*2}$$

ω^2 is purely real. $\rightarrow$ ξ is real.



In ideal MHD the self-adjointness of **F** guarantees that at any stability boundary Im ω =0, the Re ω must also be zero simultaneously.

Stability: General Considerations

- **Properties of the Force Operator F**
  - Orthogonality of the Normal Modes

$$-\omega_m^2 \rho \xi_m = \vec{F}(\xi_m)$$

$$-\omega_n^2 \rho \xi_n = \vec{F}(\xi_n)$$

$$(\omega_n^2 - \omega_m^2)\int \rho \xi_m \cdot \xi_n d\vec{r} = 0$$

$$\int \rho \xi_m \cdot \xi_n d\vec{r} = 0$$ for $n \neq m$, $\omega_m^2 \neq \omega_n^2$

orthogonal with weight function ρ

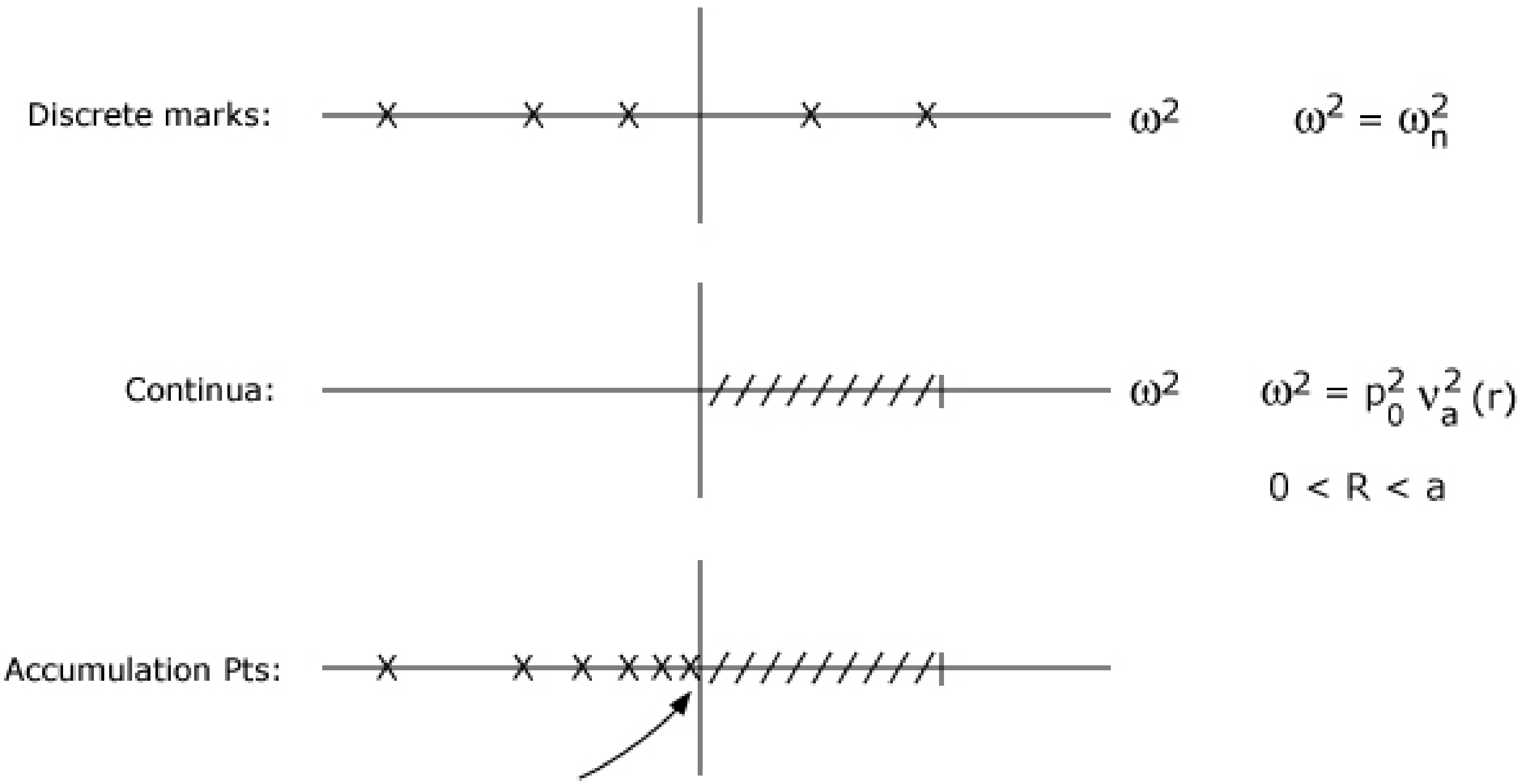
Stability: General Considerations

- **Properties of the Force Operator F**
  - Spectrum of **F**

- In general F exhibits both discrete eigenvalues and continua
- Spectrum: $(\vec{F}/\rho-\lambda)^{-1}$ (initial conditions)
- The points where $(\vec{F}/\rho-\lambda)^{-1}$ do not exist define the spectrum of **F**.
- Continua significantly complicate MHD analysis for general initial value problems. They require more than just picking up the pole contributions from the displace transform.
- However the continua lie on stable side of the spectrum and thus do not affect stability.
- Accumulation points: these provide a simple necessary condition for stability.

Stability: General Considerations

- **Properties of the Force Operator F**



Stability: General Considerations

- **Elements of Variational Calculus**

  Construction of the Variational Principle

  Classic eigenvalue problem

$$\frac{d}{dx}\left(f \frac{\partial y}{\partial x} \right) + (\lambda - g)y = 0 \qquad \lambda\text{: eigenvalue}$$

$$y(0) = y(1) = 0$$

Method of solution

- Analytical methods if *f*, *g* are sufficiently simple.
- Power series expansions
- Asymptotic expansions
- Transform methods
- Numerical methods
- Variational calculus

Stability: General Considerations

- **Elements of Variational Calculus**

Construction of the Variational Principle

$$\frac{d}{dx}\left(f\,\frac{\partial y}{\partial x}\right) + (\lambda - g)y = 0$$

$$y(0) = y(1) = 0$$

Multiplied by y
and integrated over the region $0 \le x \le 1$

$$\lambda = \frac{\int (fy'^2 + gy^2)dx}{\int y^2 dx}$$

Why is this variational?
- Substitute all allowable trial function $y(x)$ into the equation above.
- When resulting λ exhibits an extremum (maximum, minimum, saddle point) then λ and y are actual eigenvalue and eigenfunction.

Stability: General Considerations

- **Elements of Variational Calculus**

Construction of the Variational Principle

Proof

- assume $y_0(x)$ as a trial function yielding λ_0.
- Modify y by a small but arbitrary perturbation

$$y(x) = y_0(x) + \delta y(x) \longrightarrow \lambda = \lambda_0 + \delta\lambda$$

$$\delta y(0) = \delta y(1) = 0$$

$$\frac{N_0 + N_1 + N_2}{D_0 + D_1 + D_2} = \frac{N_0 + N_1 + N_2}{D_0}\left(1 - \frac{D_1}{D_0} - \frac{D_2}{D_0} + \frac{D_1^2}{D_0^2} + \cdots\right)$$

$$\delta\lambda = \frac{\int [f(y_0+\delta y)'^2 + g(y_0+\delta y)^2]dx}{\int (y_0+\delta y)^2 dx} - \frac{\int (f y_0'^2 + g y_0^2)dx}{\int y_0^2 dx}$$

For small δy

$$\delta\lambda \approx \frac{2\int [\delta y' f y_0' + \delta y y_0 (g - \lambda_0)]dx}{\int y_0^2 dx} = -\frac{2\int \delta y[(f y_0')' + (\lambda_0 - g) y_0]dx}{\int y_0^2 dx}$$

Stability: General Considerations

- **Elements of Variational Calculus**

Construction of the Variational Principle

At an extremum $\delta\lambda$ = 0 for arbitrary δy, implying $$\frac{d}{dx}\left(f\frac{\partial y}{\partial x}\right)+(\lambda-g)y=0$$

$$(fy_0')'+(\lambda_0-g)y_0=0$$ equivalent to the original eigenvalue equation

Infinite number of integral relations for λ possible but not variational

Example: $$\lambda=\frac{\int\left\{hfy'^2+[hg-(fy')'/2]y^2\right\}dx}{\int hy^2dx}$$ multiplied by $h(x)y(x)$

$$(hfy_0')'+\left[\lambda_0h-hg+\frac{(fh')'}{2}\right]y_0=0$$

$\delta\lambda$ = 0 does not satisfy the original equation unless $h(x)$=1
→ not variational

Stability: General Considerations

- **Elements of Variational Calculus**

  Construction of the Variational Principle

  Since $\delta\lambda = 0$ when *y* coincides with a true eigenfunction, this implies that an estimate for λ using a guess (trial function) for *g* is more accurate than the trial function itself.

$$\lambda = \lambda_0 + \frac{\int [f(\delta y')^2 + g(\delta y)^2]dx}{\int y_0^2 dx} + O[(\delta y)^3] = \lambda_0 + O(\varepsilon^2)$$

$$y = y_0 + O(\varepsilon)$$

Stability: General Considerations

- **Elements of Variational Calculus**

Generalized Boundary Conditions

$$y(0)=0, \quad y'(1)=Ay(1)$$

$$\lambda = \frac{\int (fy'^2 + gy^2)dx - fy'y\big|_{x=1}}{\int y^2 dx}$$

$$\delta\lambda = -\frac{2\int \delta y[(fy_0')' + (\lambda_0 - g)y_0]dx + f(y_0'\delta y - y_0\delta y')\big|_{x=1}}{\int y_0^2 dx}$$

$$\longrightarrow \qquad (fy_0')' + (\lambda_0 - g)y_0 = 0 \qquad y_0'(1) = Ay_0(1), \quad \delta y'(1) = A\delta y(1)$$

- Proper variational principle equivalent to the original eigenvalue problem.
- Although using trial functions which satisfy $y'(1)=Ay(1)$ is not unexpected, it is often difficult to implement practically.

Stability: General Considerations

- **Elements of Variational Calculus**

  Generalized Boundary Conditions

  More elegant and more convenient alternative variational principle: replace $y'(1)$ with $Ay(1)$

$$\lambda = \frac{\int (fy'^2 + gy^2)dx - Afy^2\big|_{x=1}}{\int y^2 dx}$$

$$\longrightarrow \quad \delta\lambda = -\frac{2\int \delta y[(fy_0')' + (\lambda_0 - g)y_0]dx + 2f\delta y(y_0' - Ay_0)\big|_{x=1}}{\int y_0^2 dx}$$

$$\longrightarrow \quad (fy_0')' + (\lambda_0 - g)y_0 = 0 \qquad y_0'(1) = Ay_0(1)$$

- If we choose trial functions which allow $y(1)$ to float freely, then the variational principle forces the trial function to satisfy $y'(1)=Ay(1)$.
- This is the natural boundary condition. It has the important advantage of allowing trial functions to be substituted that do not automatically satisfy this condition.

Stability: General Considerations

- **Variational Formulation**

Application of the variational principle to MHD

$$-\omega^2 \rho \xi = \vec{F}(\xi)$$ normal-mode formulation

$$\omega^2 = \frac{\delta W(\xi^*, \xi)}{K(\xi^*, \xi)}$$ dot product with ξ^* then integrated over the plasma volume

$$\lambda = \frac{\int (f y'^2 + g y^2) dx}{\int y^2 dx}$$

$$\delta W(\xi^*, \xi) = -\frac{1}{2} \int \xi^* \cdot \vec{F}(\xi) d\vec{r}$$

$$= -\frac{1}{2} \int \xi^* \cdot [\frac{1}{\mu_0} (\nabla \times \vec{Q}) \times \vec{B} + \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{Q} + \nabla(\gamma p \nabla \cdot \xi + \xi \cdot \nabla p)] d\vec{r}$$

$$K(\xi^*, \xi) = \frac{1}{2} \int \rho |\xi|^2 d\vec{r}$$

Any allowable function ξ for which ω^2 becomes an extremum is an eigenfunction of the ideal MHD normal mode equations with eigenvalue ω^2.

Stability: General Considerations

- **Variational Formulation**

Proof

$$\xi \to \xi + \delta\xi, \quad \omega^2 \to \omega^2 + \delta\omega^2 \qquad \delta\omega^2 = 0$$

$$\omega^2 + \delta\omega^2 = \frac{\delta W(\xi^*,\xi) + \delta W(\delta\xi^*,\xi) + \delta W(\xi^*,\delta\xi) + \delta W(\delta\xi^*,\delta\xi)}{K(\xi^*,\xi) + K(\delta\xi^*,\xi) + K(\xi^*,\delta\xi) + K(\delta\xi^*,\delta\xi)}$$

$$\delta\omega^2 = \frac{\delta W(\delta\xi^*,\xi) + \delta W(\xi^*,\delta\xi) - \omega^2[K(\delta\xi^*,\xi) + K(\xi^*,\delta\xi)]}{K(\xi^*,\xi)}$$

Using self-adjoint property

$$K(\xi^*,\delta\xi) = K(\delta\xi,\xi^*), \quad \delta W(\xi^*,\delta\xi) = \delta W(\delta\xi,\xi^*)$$

Since is $\delta\xi$ arbitrary and $\delta\omega^2$ =0 (extremum)

$$\int d\vec{r}\{\delta\xi^* \cdot [\vec{F}(\xi) + \omega^2\rho\xi] + \delta\xi \cdot [\vec{F}(\xi^*) + \omega^2\rho\xi^*]\} = 0 \longrightarrow \; -\omega^2\rho\xi = \vec{F}(\xi)$$

Demonstrated that the normal-mode eigenvalue equation and the variational principle are equivalent formulations for the linearised ideal MHD stability problem.

Stability: General Considerations

- **Variational Formulation**

$$\omega^2 = \frac{\delta W(\xi^*,\xi)}{K(\xi^*,\xi)}$$

$$\delta W(\xi^*,\xi) = -\frac{1}{2}\int \xi^* \cdot \vec{F}(\xi)\,d\vec{r}$$

$$K(\xi^*,\xi) = \frac{1}{2}\int \rho|\xi|^2\,d\vec{r}$$

$$-\omega^2 K + \delta W = 0$$

Conservation of energy

Kinetic energy

- Change in potential energy associated with the perturbation

- Equal to the work done against the force **F**(ξ) in displacing the plasma by an amount ξ.

References

- *http://www.free-online-private-pilot-ground-school.com/Aeronautics.html*
- *http://serc.carleton.edu/introgeo/models/EqStBOT.html*